

Boost 2014, UCL (London)

Q-Jets in SCET: from Q-thrust to Q-(sub)jettiness

Andrew Hornig

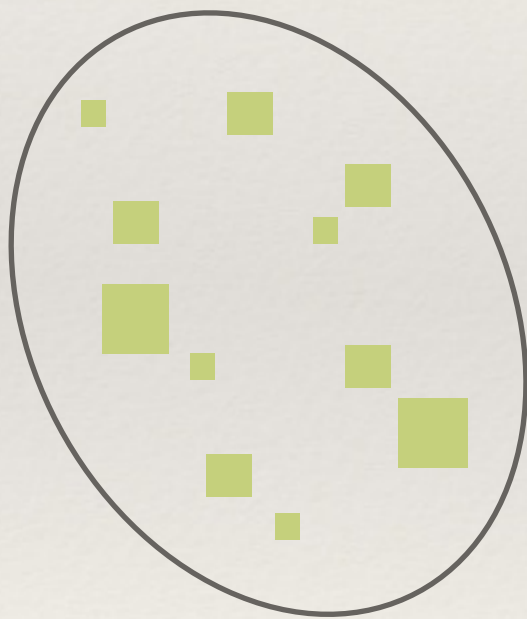
LANL

Aug 18, 2014

What is a “Q-Jet”?

- ❖ classical (deterministic) substructure analyses

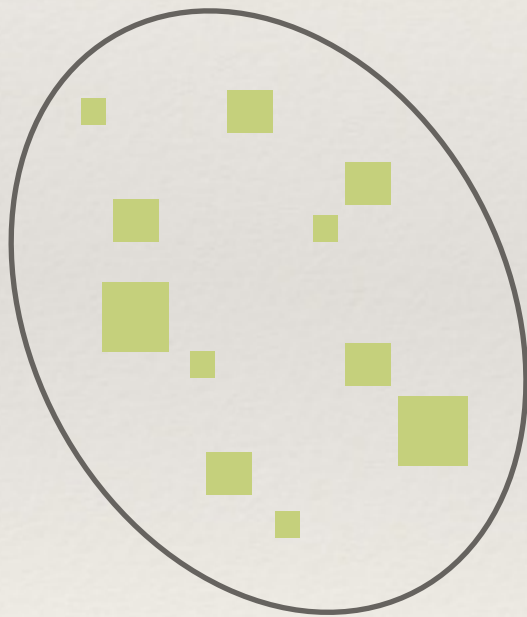
1: find jet



What is a “Q-Jet”?

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1: find jet



2: cluster into trees
(depends on algorithm)

k_T (groups soft first)

C/A (groups by angle)

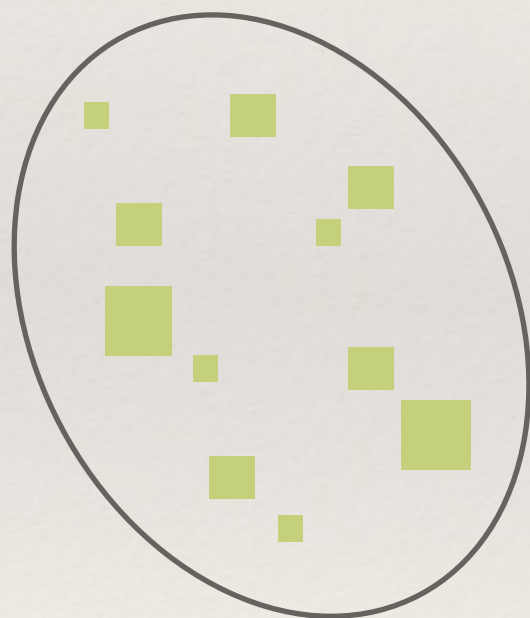
anti- k_T (groups soft last)



What is a “Q-Jet”?

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1: find jet



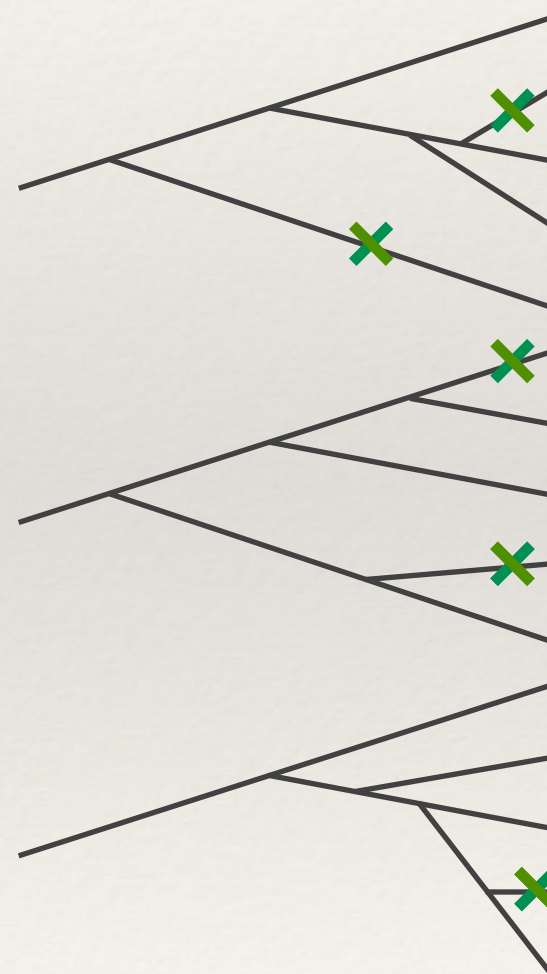
2: cluster into trees
(depends on algorithm)

k_T (groups soft first)

C/A (groups by angle)

anti- k_T (groups soft last)

3: get “groomed” m_J



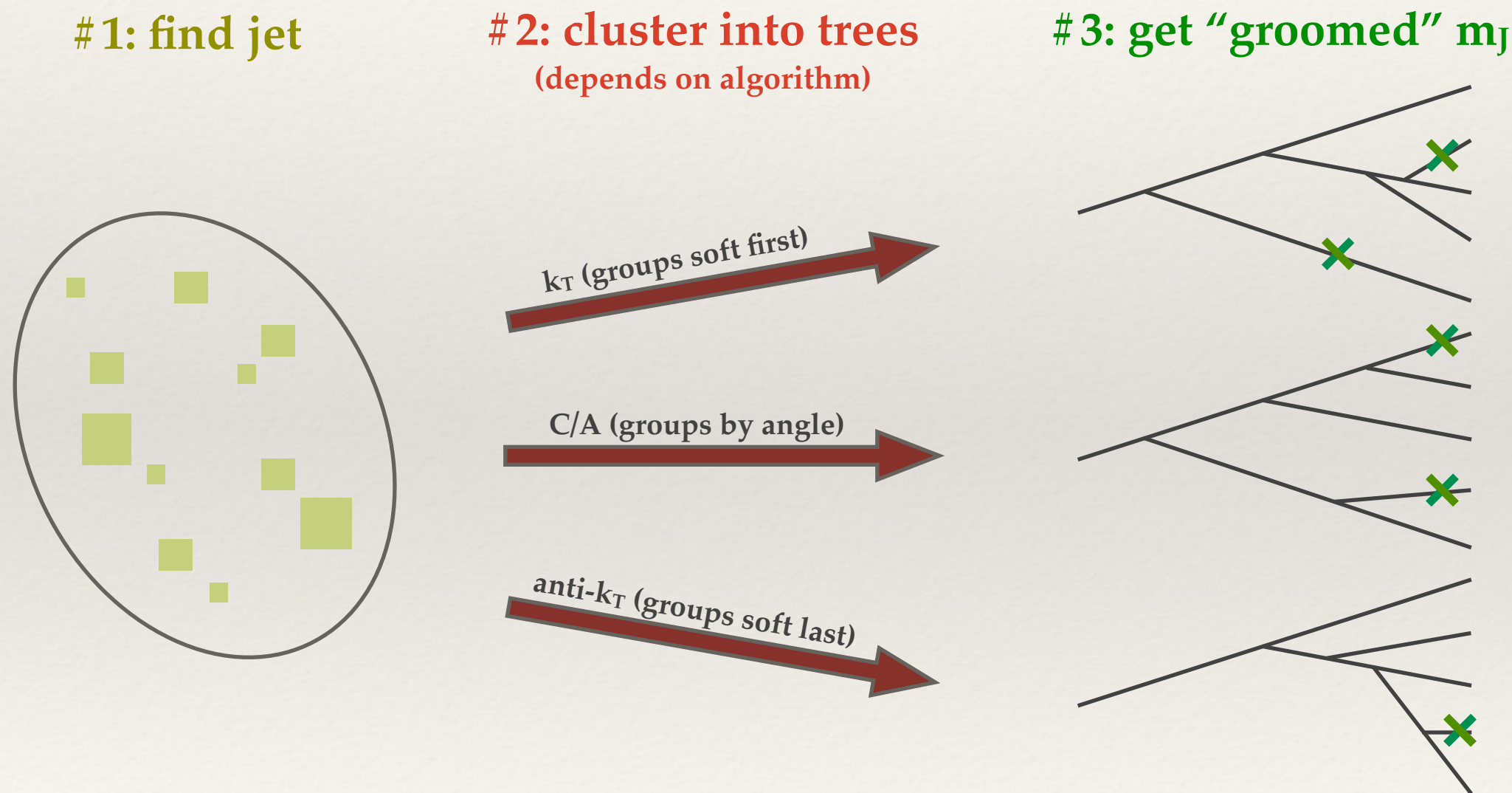
“Mass Drop+Filter”
Butterworth, Davison,
Rubin, Salam
(0802.2470)

“Pruning”
Ellis, Walsh, Vermilion
(0912.0033)

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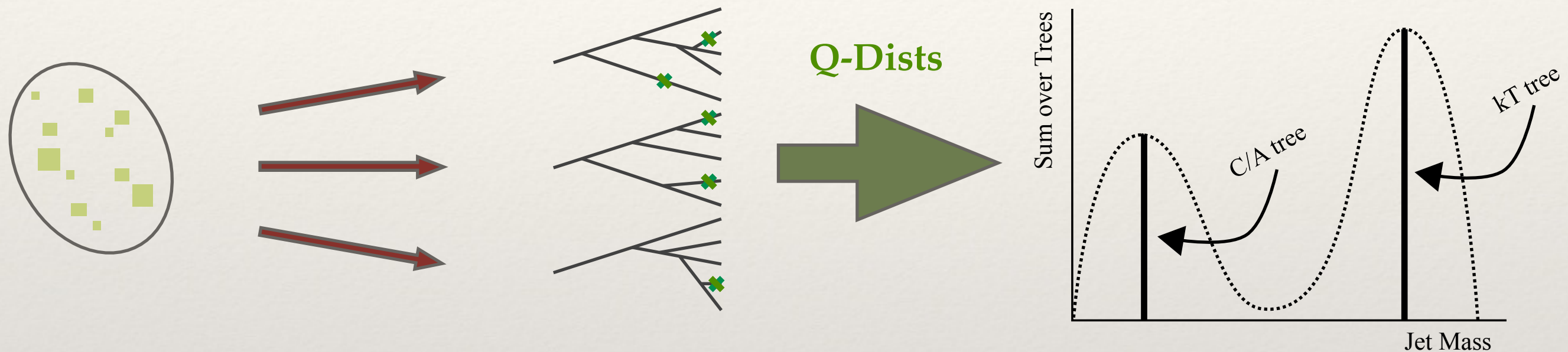
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- ❖ Q-Jets: use “all” clusterings \Rightarrow mass distribution *for each jet*

What is a “Q-Jet”?

- ❖ different for different algorithms :

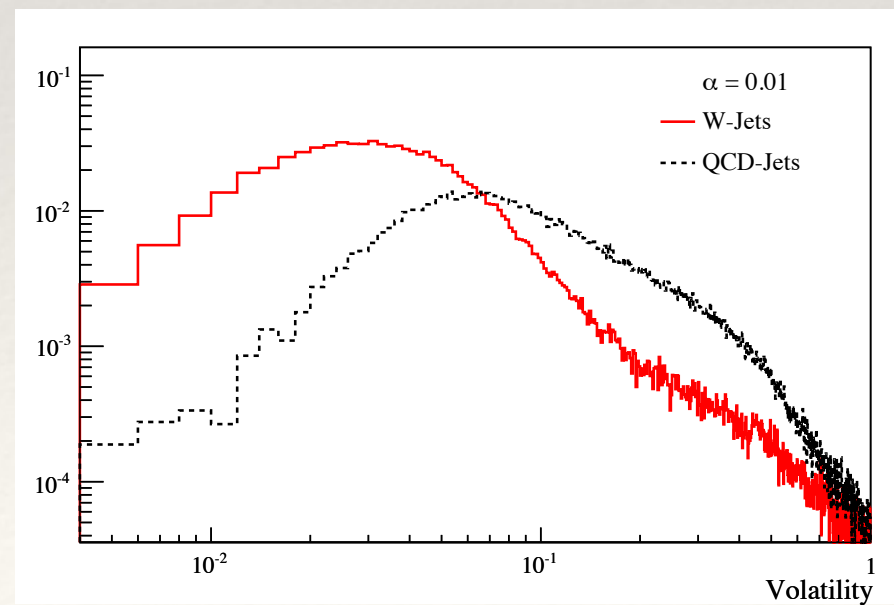


- ❖ Variation larger for QCD jets (no real m_J scale)

⇒ “Volatility”:

$$\mathcal{V} = \Gamma / \langle m \rangle$$

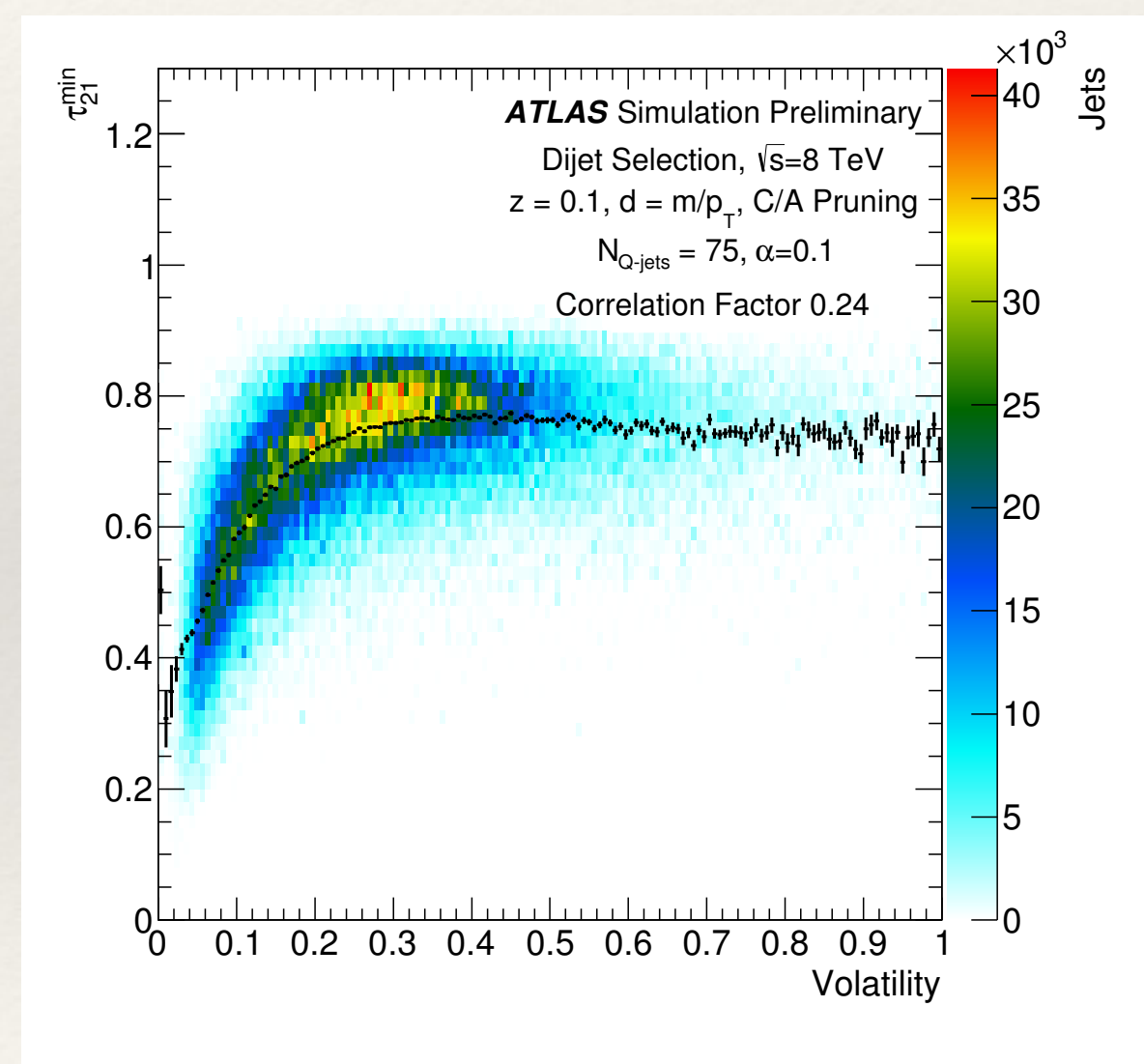
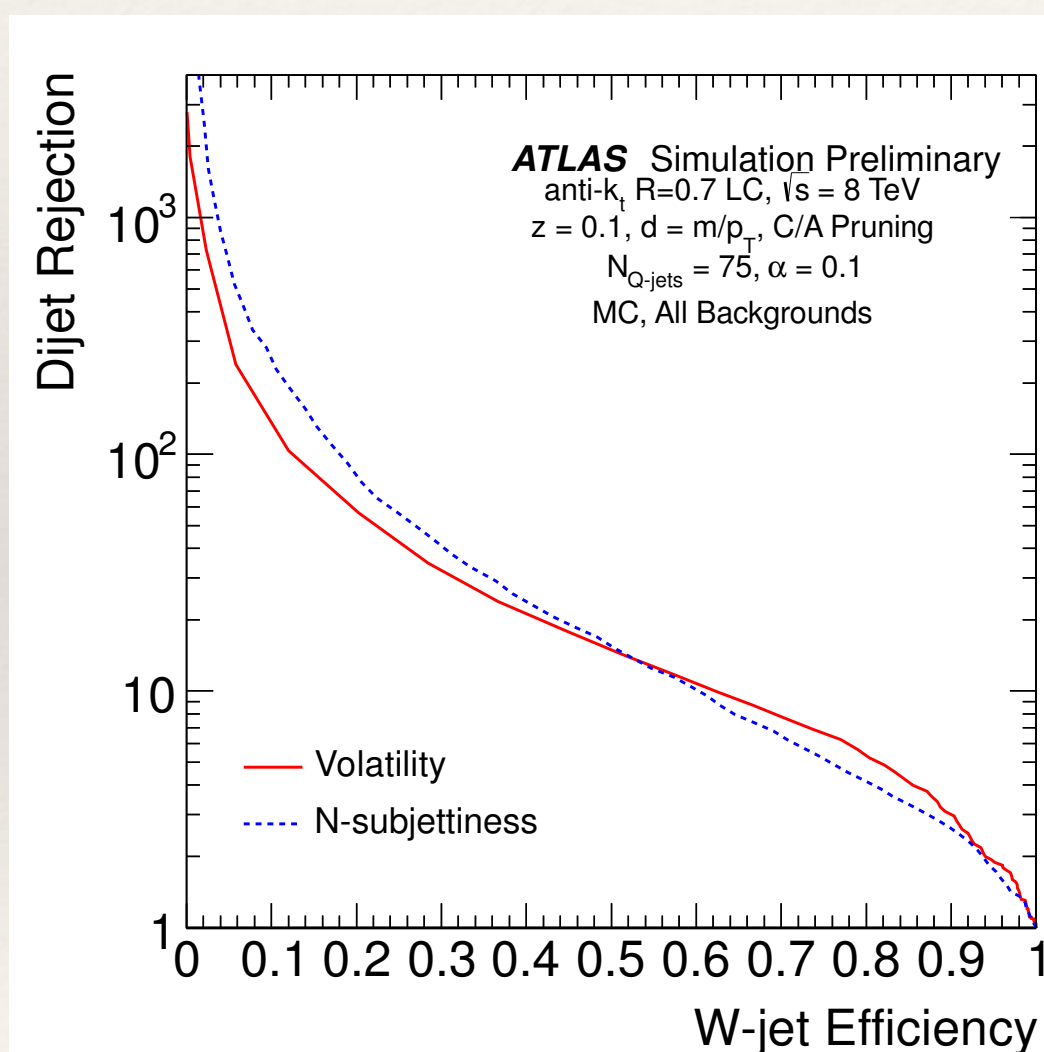
$$\Gamma \equiv \sqrt{\langle m^2 \rangle - \langle m \rangle^2}$$



Results from Last Year

- ❖ compare to standard candle (N-subjettiness):

Thaler, Tilburg 1011.2268

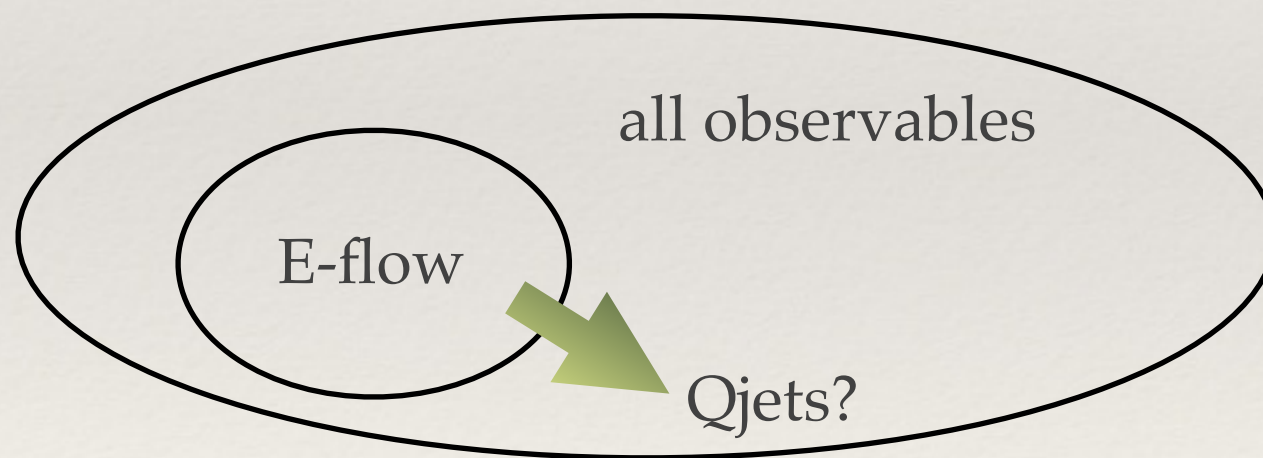


- ❖ BOOST2013 working groups: understand correlations

Why are Q-Jets different?

❖ 2 reasons:

1. NOT deterministic: probabilistic assignments
2. NOT energy-flow variable (event/jet shape): fundamentally iterative
(depends on clustering, not just particle 4-momenta)

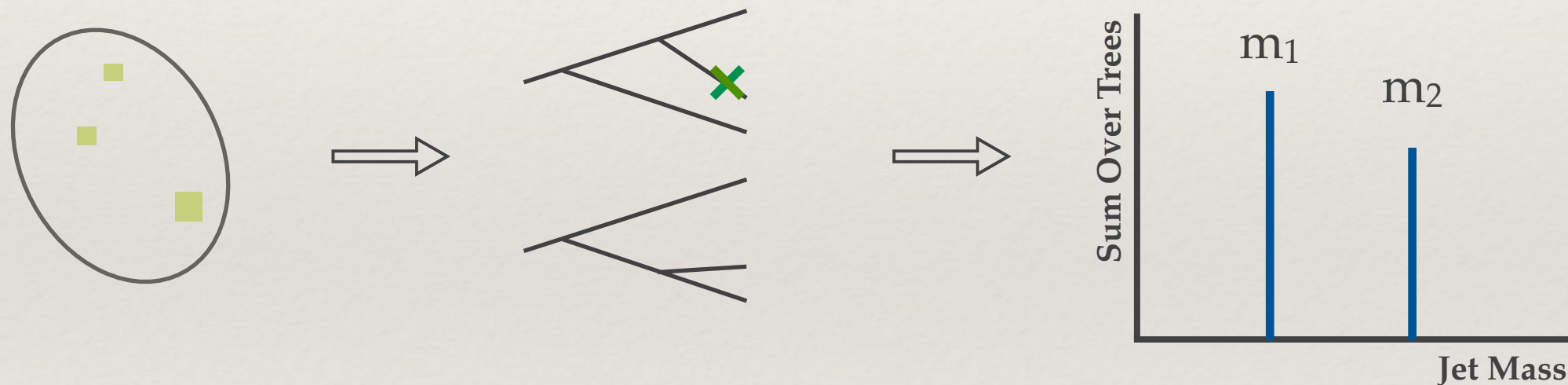


❖ Can we understand differences analytically?

Q-Jet Volatility Calculation (?)

- ❖ non-trivial mass Q-dists require at least $O(10)$ particles

\Rightarrow need $O(\alpha^{10})$ calculation....



- ❖ also not well-suited for resummation in SCET
- ❖ *both related to fact that Q-jets is recursive/iterative....*

What *can* we Calculate?

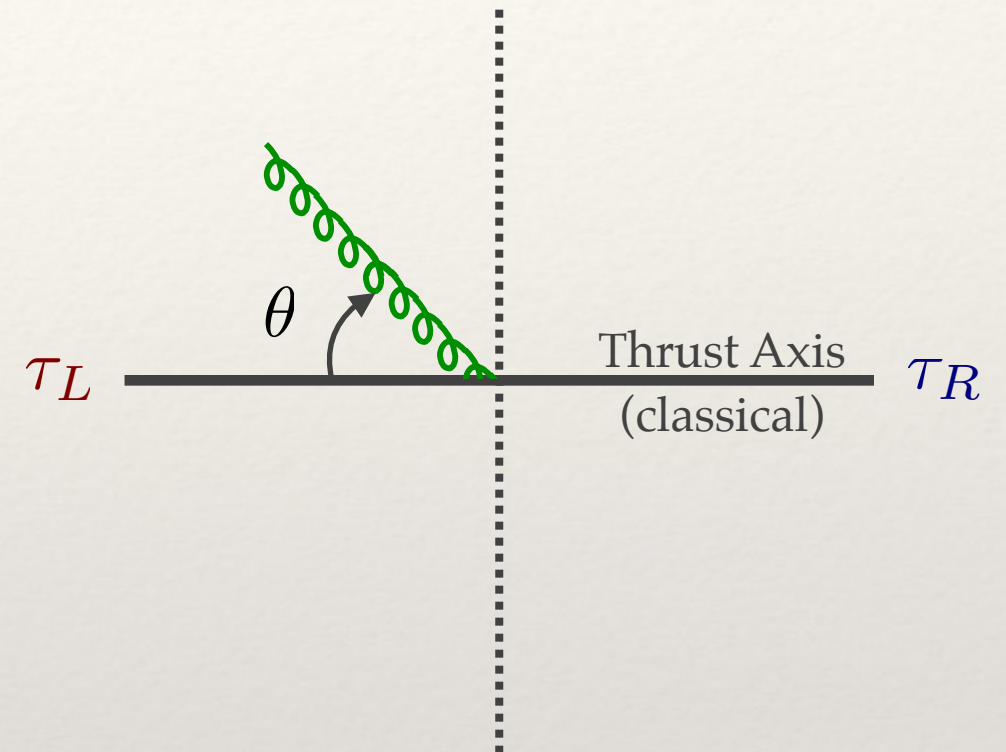
- ❖ energy-flow (non-iterative) easier
 - ❖ are probabilistic observables possible / sensible?
 - ❖ if calculable, what is effect on Non-Global Logs?
(important in general, but esp. substructure)
- ➡ define Q-thrust (then on to Q-(sub)jettiness)

Q-Thrust

- ❖ cluster L, R with some probability:

$$\underbrace{\delta(\tau_L - k^+ / Q) \Theta\left(\theta < \frac{\pi}{2}\right)}_{\rightarrow P_L(\theta)}$$

$$+ \underbrace{\delta(\tau_R - k^- / Q) \Theta\left(\theta > \frac{\pi}{2}\right)}_{\rightarrow P_R(\theta)}$$



- ❖ disentangle Q-Jets / N-subjettiness (un)correlation???
1. non-deterministic (like traditional Q-Jets)
 2. *but* now energy-flow / shape var

Q-Thrust

- ❖ probability conservation:

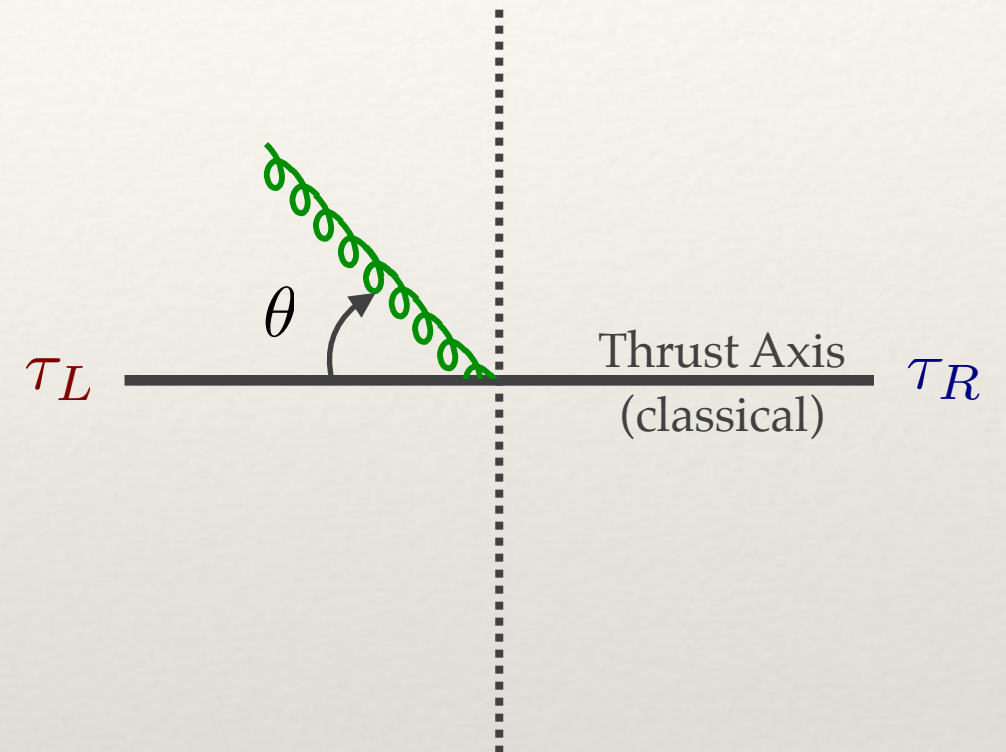
$$P_L(\theta) + P_R(\theta) = 1$$

- ❖ IR safety:

$$P_L(0) = P_R(\pi) = 1$$

- ❖ symmetry (not needed):

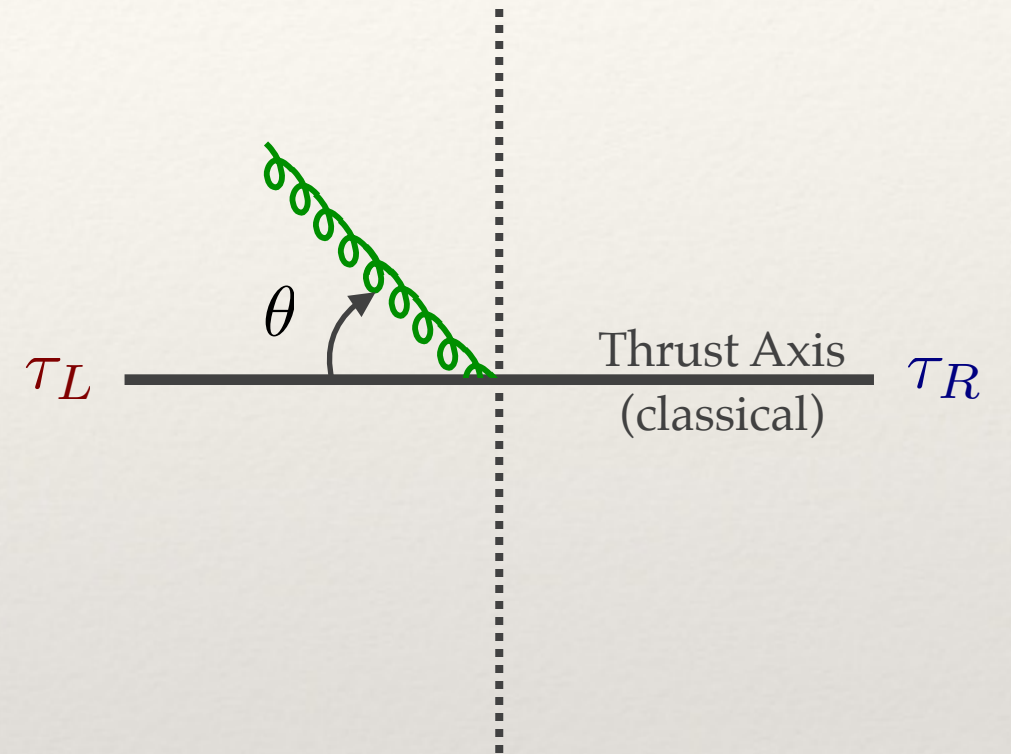
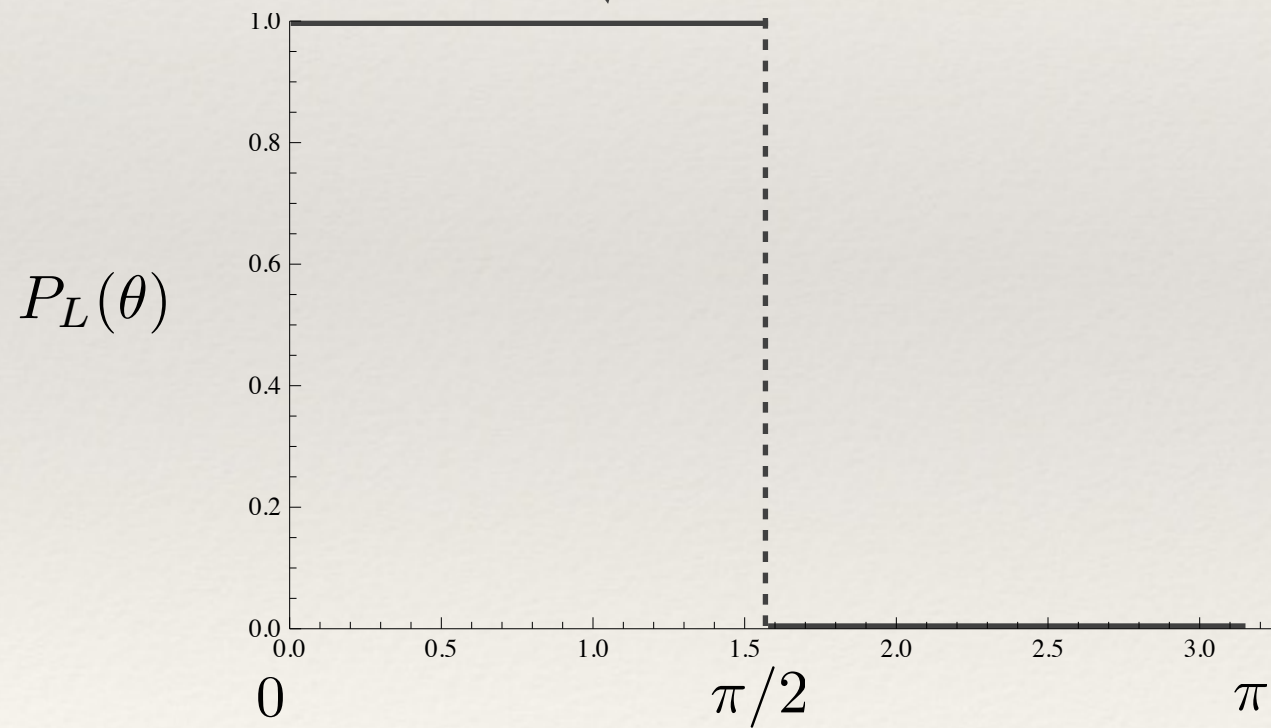
$$P_{L,R}(\theta) = 1 - P_{L,R}(\pi - \theta)$$



Q-Thrust

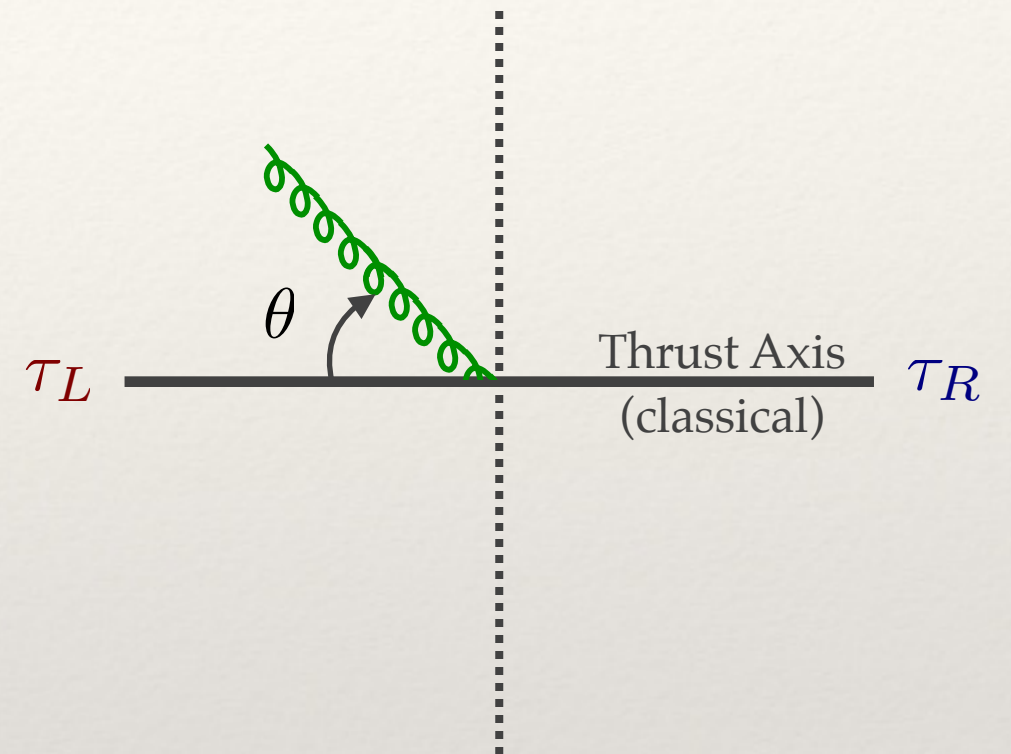
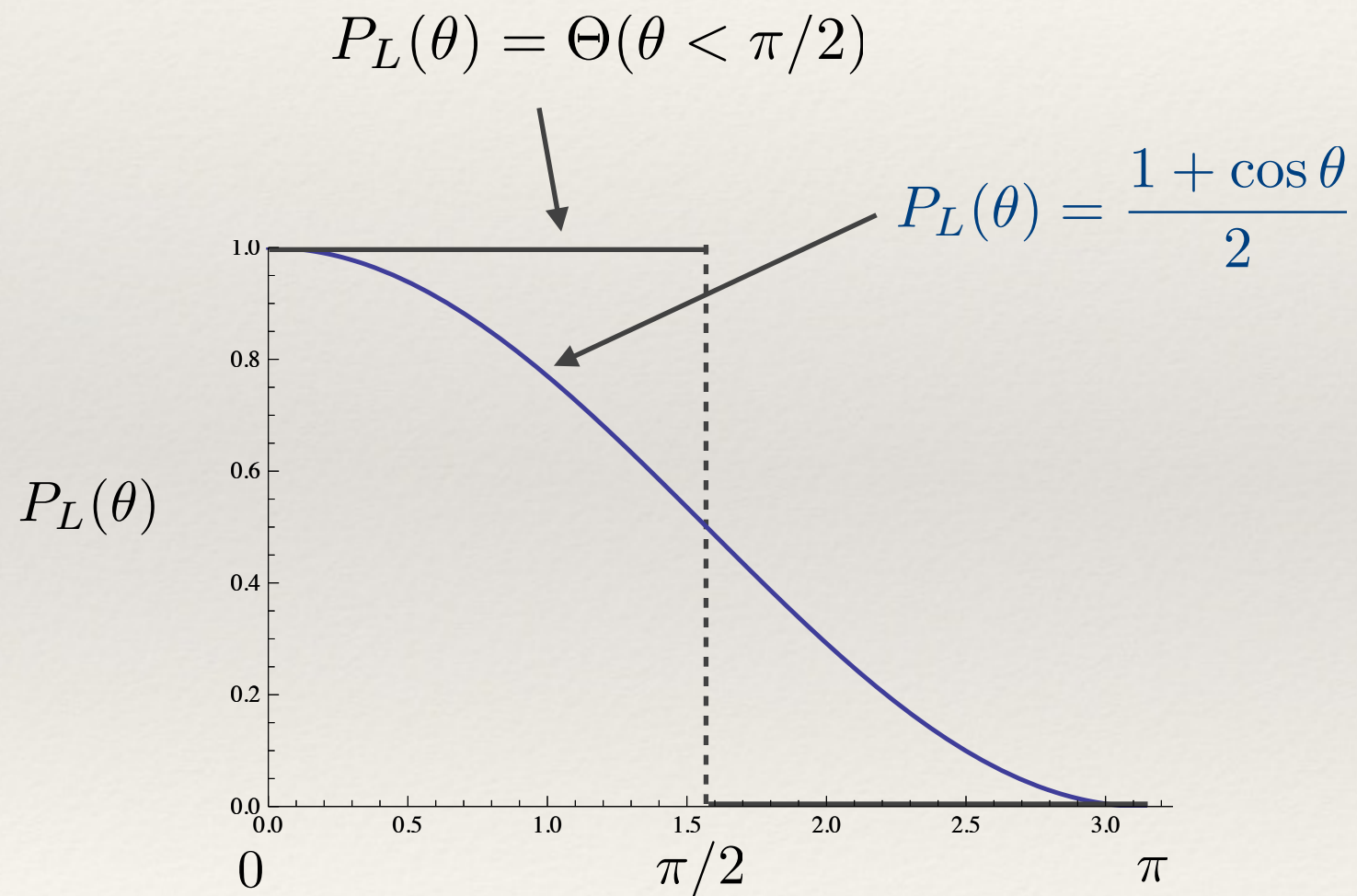
❖ Examples:

$$P_L(\theta) = \Theta(\theta < \pi/2)$$



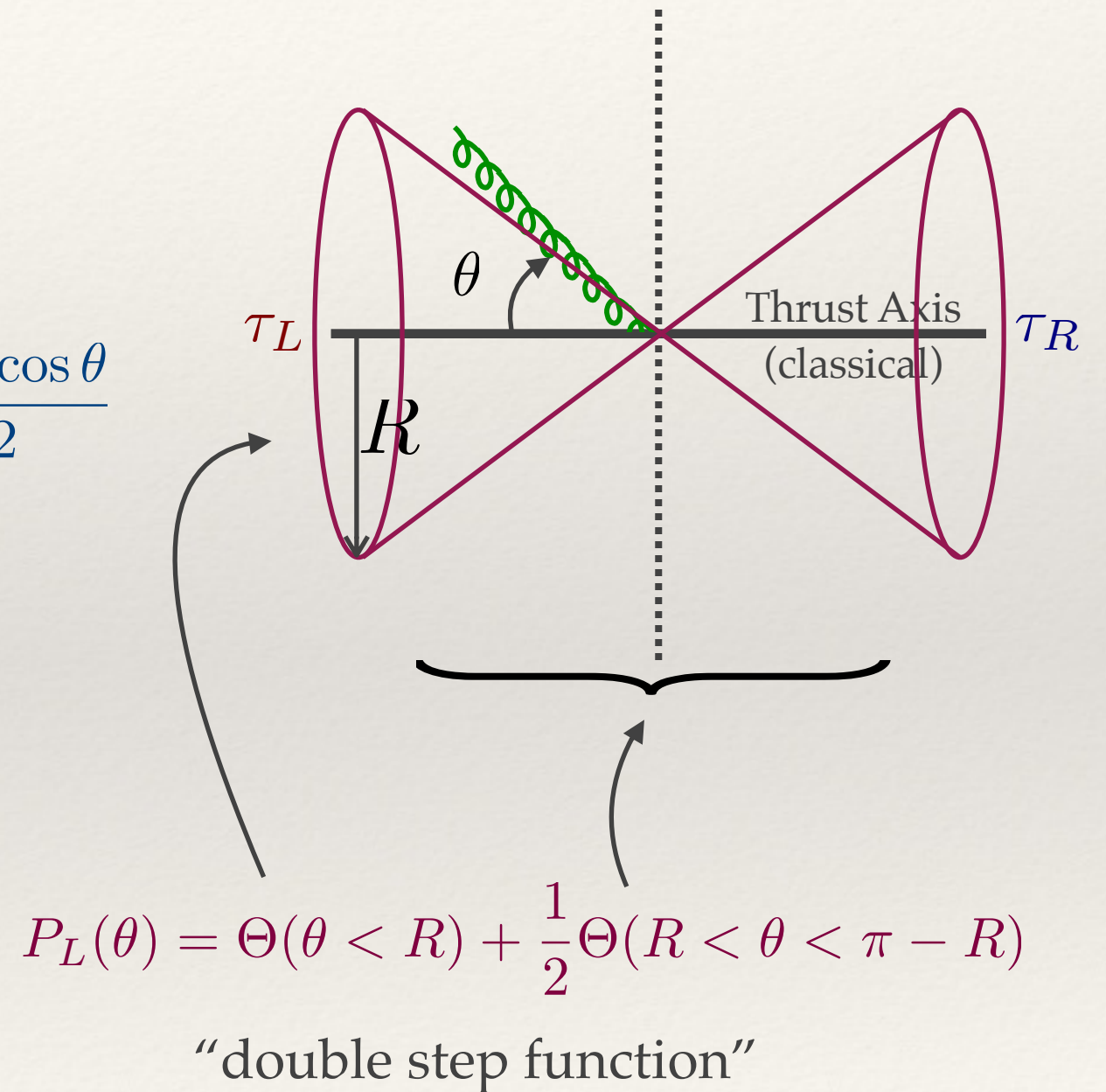
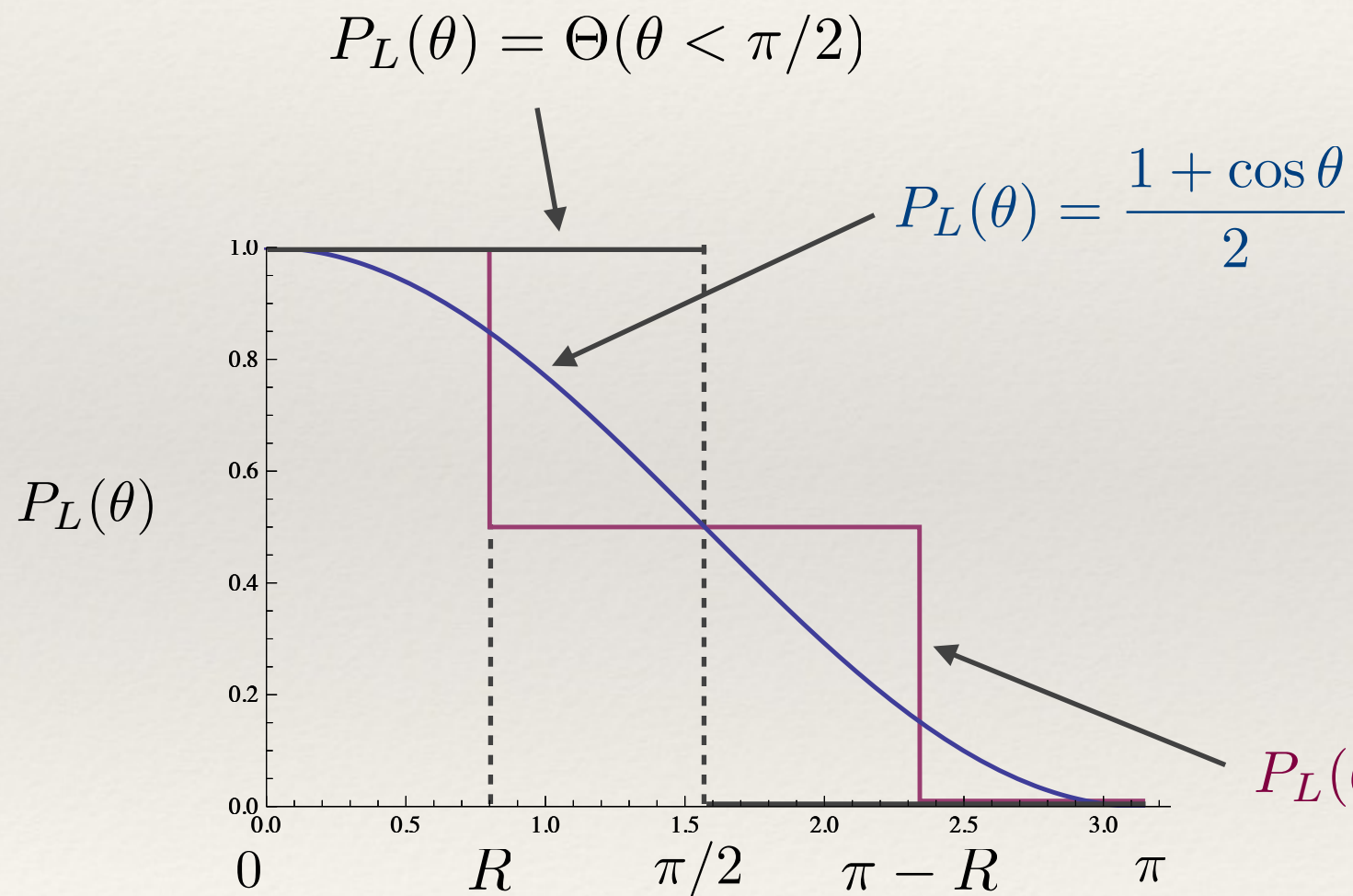
Q-Thrust

❖ Examples:



Q-Thrust

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Q-Thrust

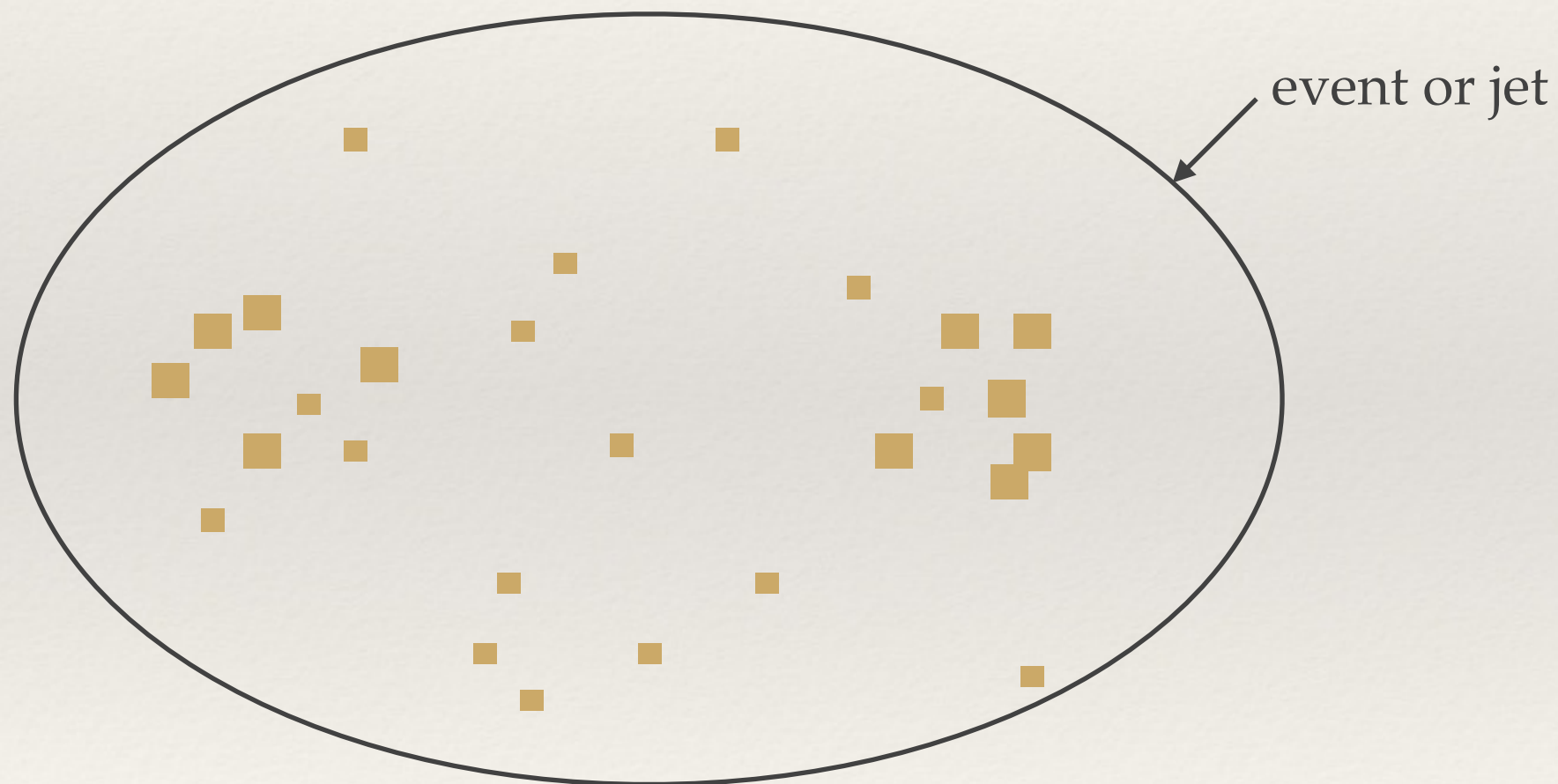
- ❖ definition for higher orders - IR safety:
 - ❖ must use clustering!
 - ❖ IR safety requires (in collinear limit):
 - ❖ before splitting:
 - ❖ after splitting:

$$P_L(\text{diagram 1}) \xrightarrow[\text{limit}]{\text{collinear}} P_L(\text{diagram 2})$$

The diagram shows a transition in the collinear limit. On the left, a green line with a series of small loops (representing a gluon) is shown. On the right, the same green line is shown, but it has split into two green lines, each with its own series of small loops, representing a gluon splitting into two gluons.

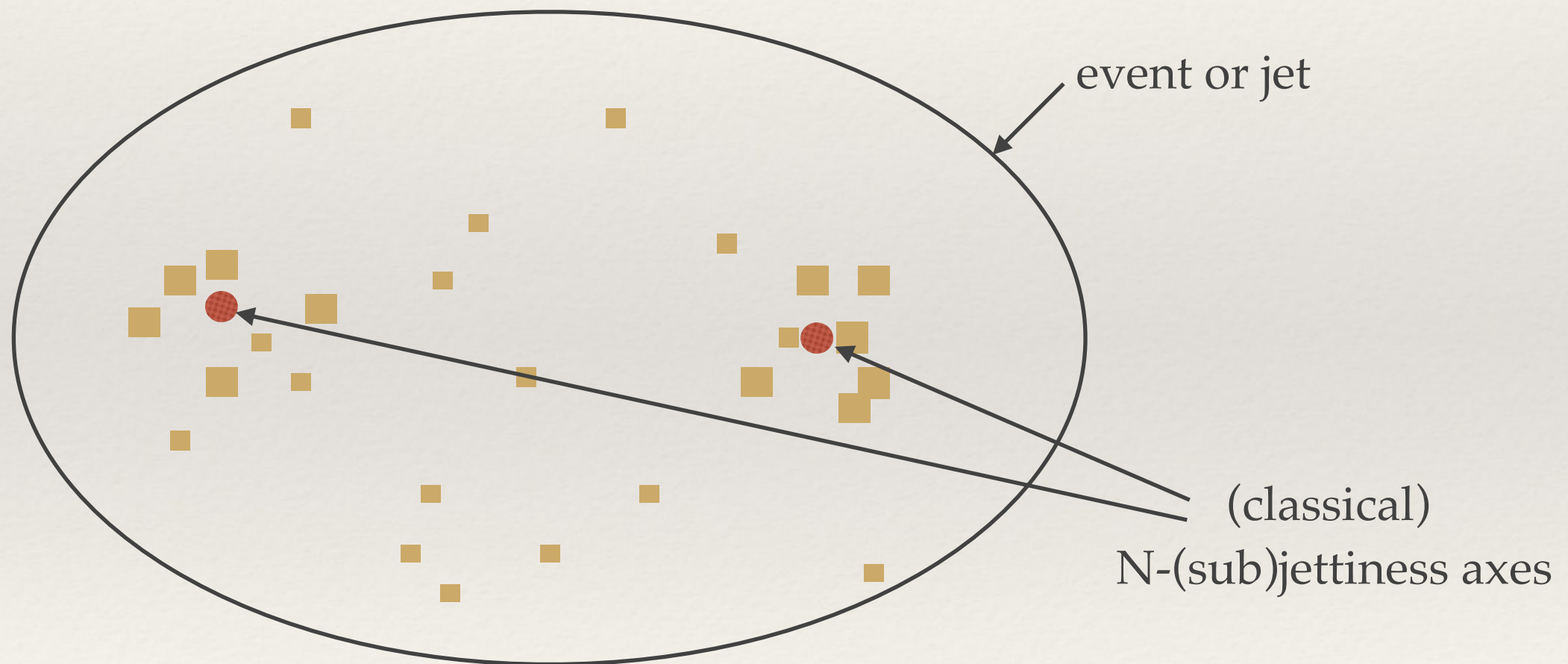
Q-Thrust to Q-(sub)jettiness

- ❖ definition for higher orders



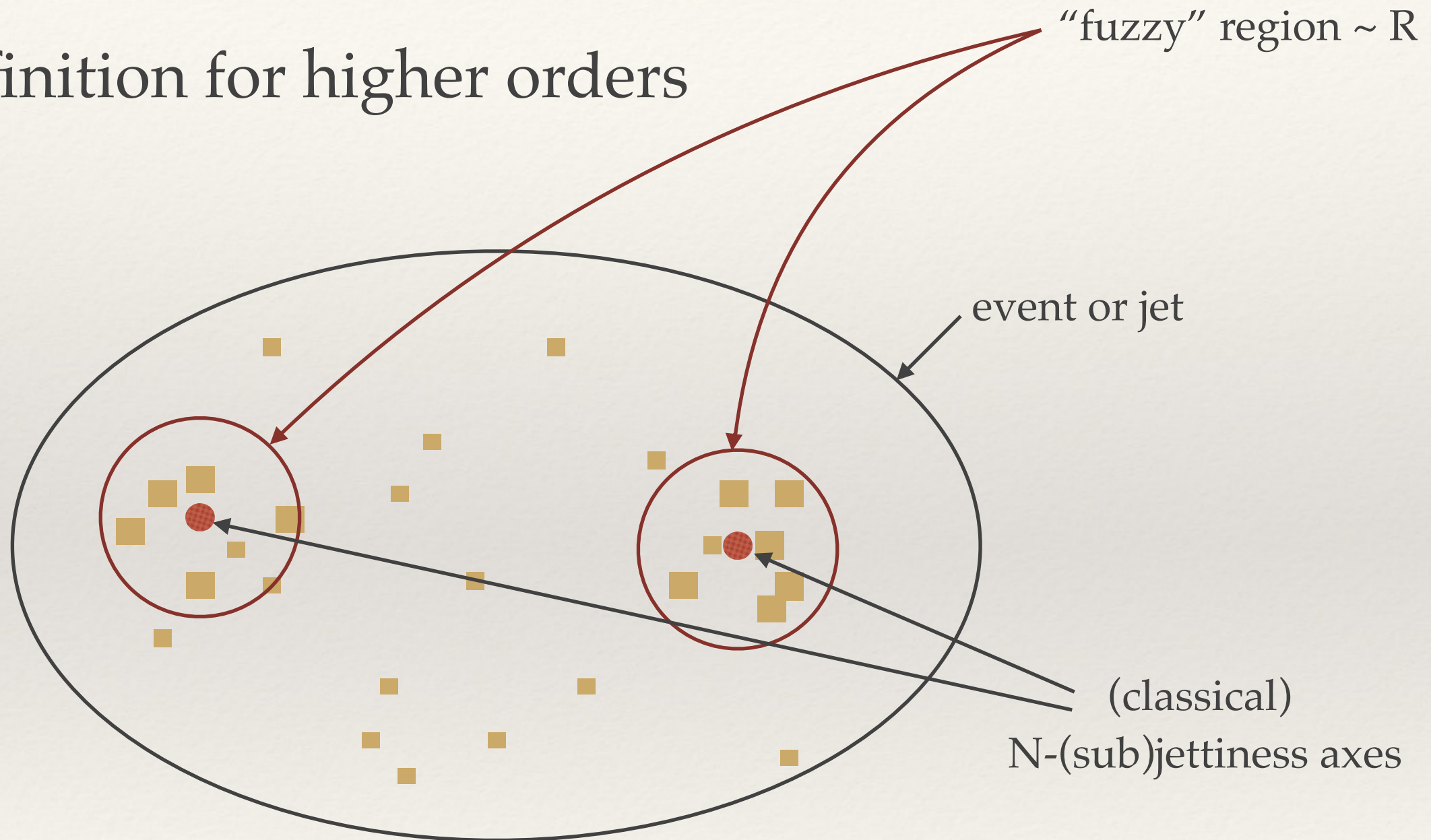
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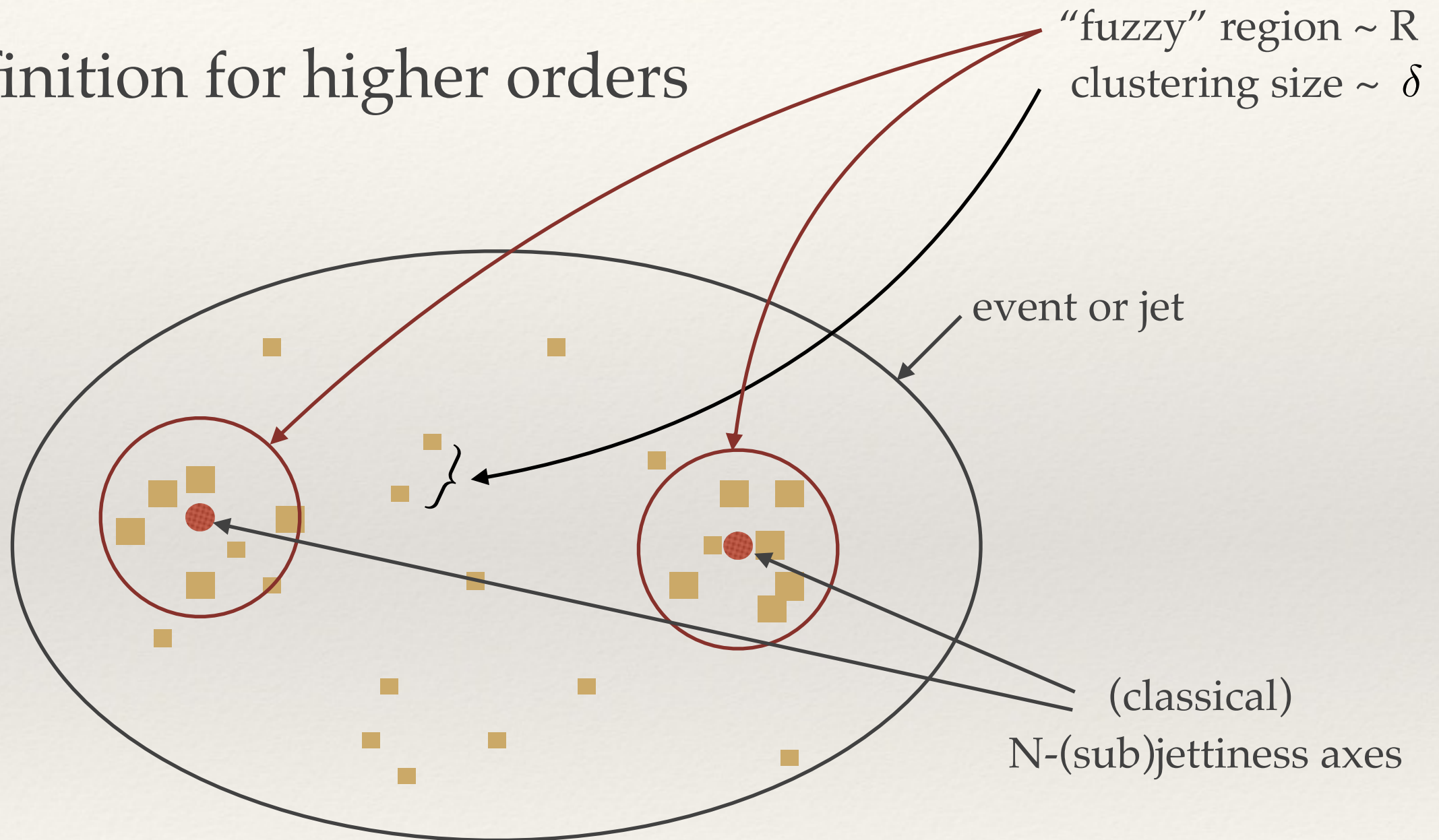
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Q-Thrust to Q-(sub)jettiness

❖ definition for higher orders



Q-Thrust

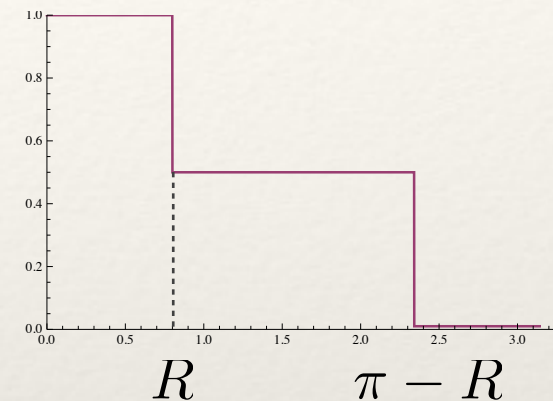
- ❖ factorization (for $R \sim 1$):

$$\frac{d\sigma}{d\tau_Q} = \int d\tau_L d\tau_R \delta(\tau_Q - \tau_L - \tau_R) \frac{d\sigma}{d\tau_L d\tau_R}$$

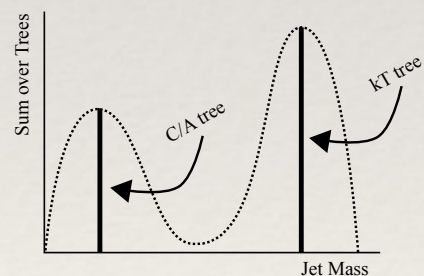
$$\frac{d\sigma}{d\tau_L d\tau_R} = H(E_{cm}) J(\tau_L) \otimes J(\tau_R) \otimes S_Q(\tau_L, \tau_R)$$

incl. jet fnc.

$$S_Q(\tau_L, \tau_R) = \frac{1}{N_C} \text{Tr} \langle 0 | \bar{Y}_{\bar{n}}^\dagger(0) Y_n^\dagger(0) \mathcal{O}(\tau_L, \tau_R) Y_n(0) \bar{Y}_{\bar{n}}(0) | 0 \rangle$$



- ❖ Inspiration for observable from Q-Jets:



$$= \sum_i \omega(\text{tree}_i) \delta(\mathcal{O} - \hat{\mathcal{O}}(\text{tree}_i))$$

$$\Rightarrow \mathcal{O}(\tau_L, \tau_R) = \sum_i \left[P_L(k_i) \delta(\tau_L - k_i^+ / Q) \delta(\tau_R) + P_R(k_i) \delta(\tau_L) \delta(\tau_R - k_i^- / Q) \right]$$

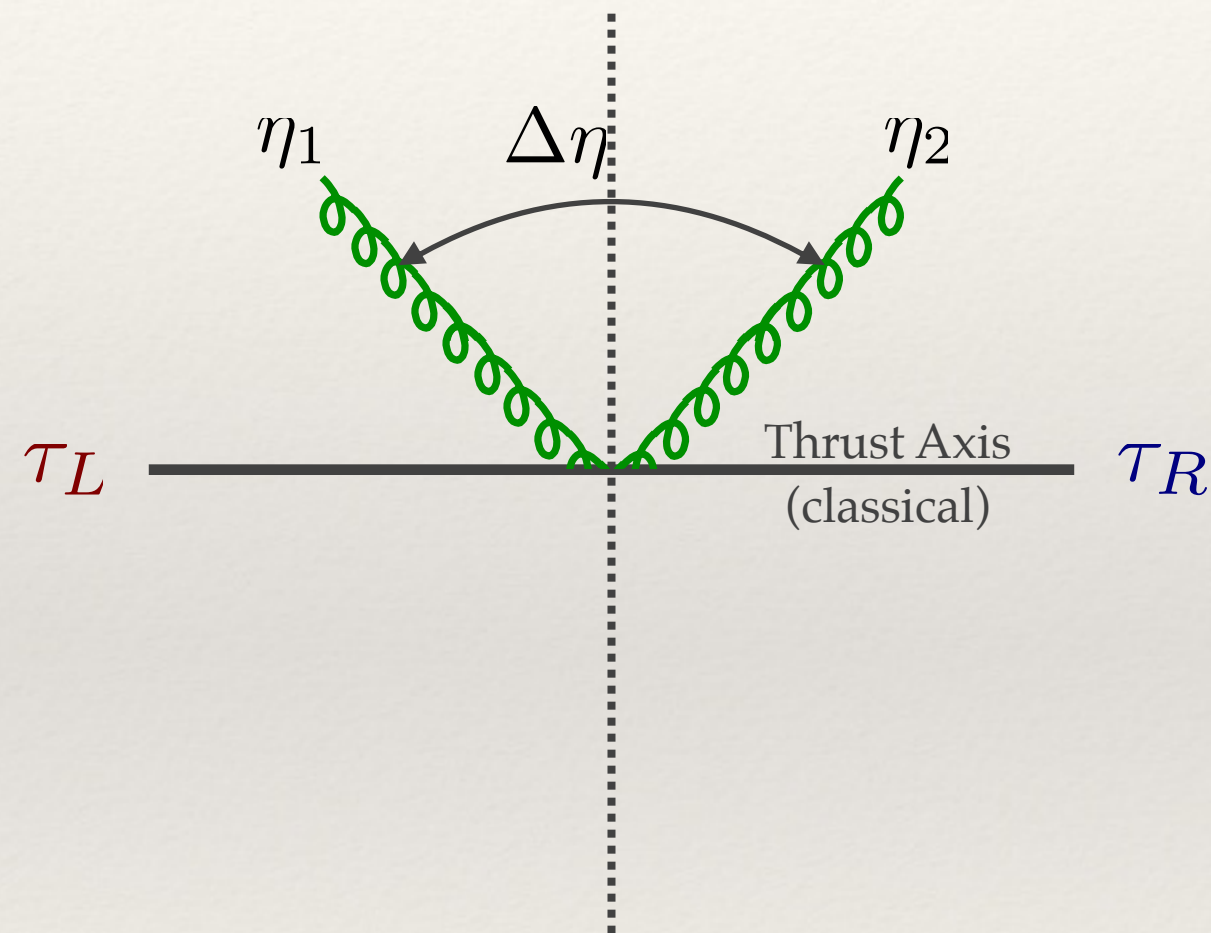
- ❖ NGLs in soft function?

Non-Global Logs (classical)

Dasgupta, Salam hep-ph/0104277

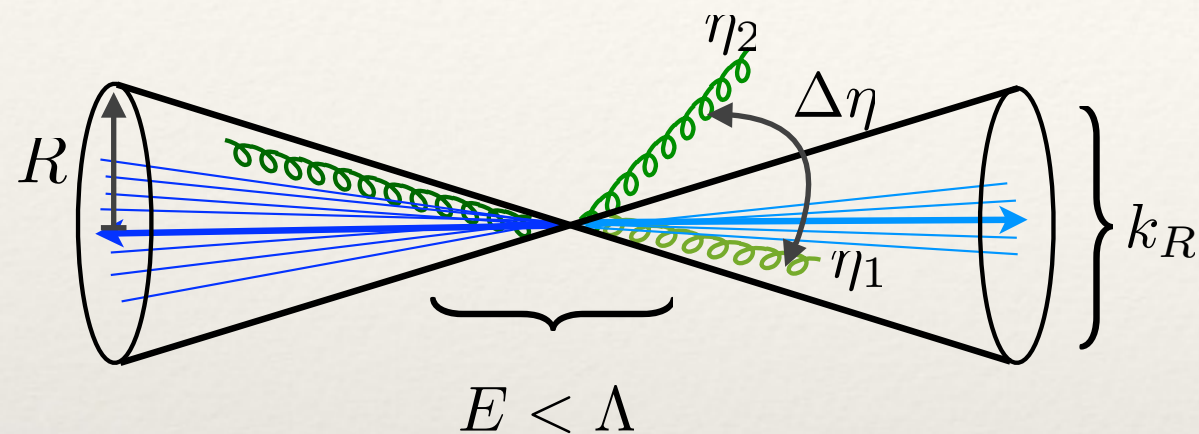
Kelley, Schwartz, Schabinger, Zhu 1105.3676

AH, Lee, Stewart, Walsh, Zuberi 1105.4628

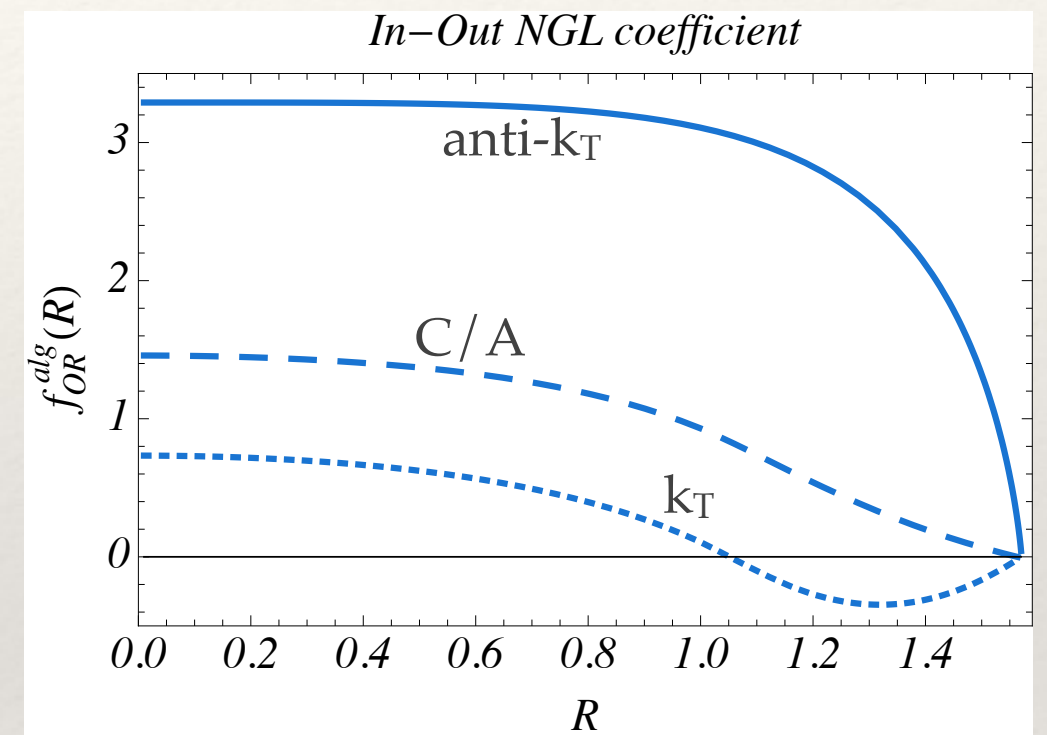


$$\alpha_s^2 C_F C_A \frac{\pi^2}{3} \ln^2 \frac{\tau_L}{\tau_R}$$

Non-Local Globs (classical)



$$-f_{\text{OR}}(R_R, R_L) \ln^2 \frac{\mu^2 \tan(R_R/2)}{2\Lambda k_R}$$



$$f_{\text{OR}}^{\text{cone}}(R_R, R_L) = \underbrace{\int_{\eta_R}^{\infty} d\eta_1 \int_{-\eta_L}^{\eta_R} d\eta_2}_{\text{1 in R, 2 outside}} \underbrace{\frac{8}{e^{2(\eta_1 - \eta_2)} - 1}}_{\text{largest when 1,2 both approach boundary}}$$

(Soft) Clustering NGLs (classical)

- ❖ anti- $k_T \rightarrow$ only clusters soft when $\Delta\theta \sim E \sim \lambda^2$
 \Rightarrow cone and anti- k_T don't have clustering logs



<i>total</i> NGLs (including clustering)	C_F^2 (pure clustering) : Kelley, Walsh, Zuberi 1202.2361	$C_F C_A$: AH, Lee, Walsh, Zuberi 1110.0004
anti- k_T : (soft last)	0	<p>The graph shows the sum of NGL coefficients as a function of the parameter R. The y-axis is labeled $\frac{1}{2} f_R^{alg}(R)$ and ranges from 0.0 to 3.5. The x-axis is labeled R and ranges from 0.0 to 1.4. Three curves are shown: a solid orange line for anti-k_T which is constant at approximately 3.3; a dashed orange line for C/A which starts at 1.5 and decreases to about 0.8; and a dotted orange line for k_T which starts at 0.7, decreases to a minimum of about 0.1 at $R \approx 1.1$, and then increases to about 0.4 at $R = 1.4$.</p>
C/A : (democratic in angle)	non-zero	
k_T : (soft first)	non-zero and bigger	

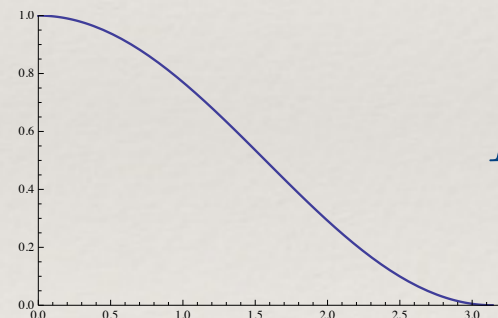
- ❖ Note: while C/A and k_T induce C_F^2 NGLs, clustering reduces $C_F C_A$ NGLs

1 Loop Results

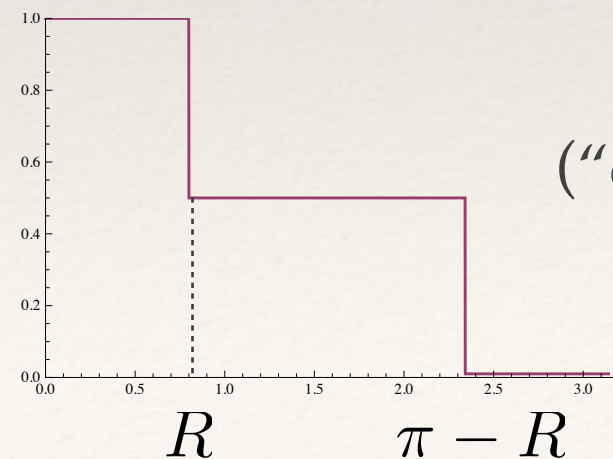
- ❖ jet function = inclusive (up to $O(\tau/R)$)
- ❖ same γ_s (soft anom dim) as thrust
- ❖ soft function has finite shift:

$$\Delta S^{(1)}(\tau) = -4\delta(\tau) \frac{\alpha_s C_F}{\pi} \int_0^1 \frac{d \cos \theta}{1 - \cos^2 \theta} \ln \tan \frac{\theta}{2} [P_L(\theta) - 1] \quad (\text{shift per hemi})$$

$$= -\delta(\tau) \times \left\{ \begin{array}{l} \frac{\pi^2}{12} \\ \ln^2 \tan \frac{R}{2} \end{array} \right.$$



$$P_L(\theta) = \frac{1 + \cos \theta}{2}$$



(“double step function”)

2 Loop Results

- ❖ after adding clustering, same as thrust $\gamma_s^{(2)}$

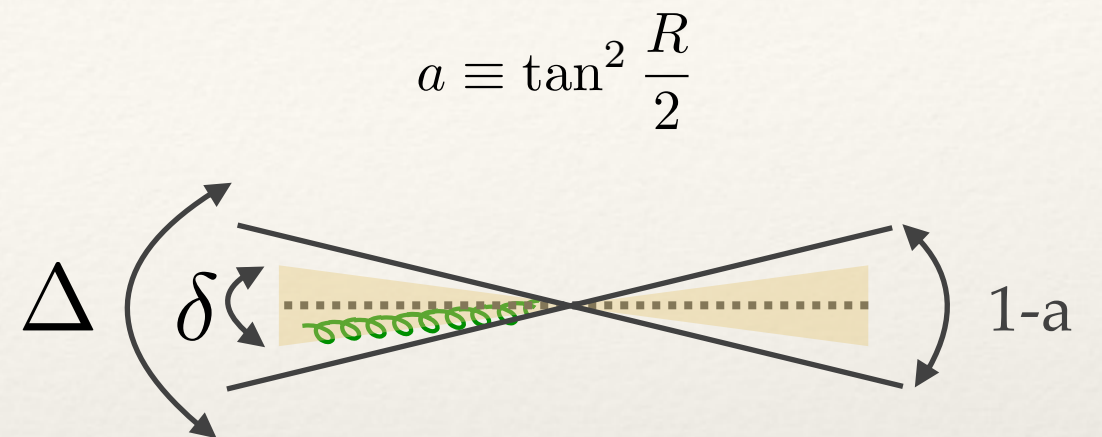
$$\mathcal{M} = \delta_L^{12} \delta^R f_1^L f_2^L + \delta_1^L \delta_2^R f_1^L f_2^R + (L \leftrightarrow R)$$

$$\rightarrow \delta_L^{12} \delta^R f_1^L f_2^L + \delta_1^L \delta_2^R f_1^L f_2^R (1 - f_{1,2}^{\text{clu}}) + \delta_{12}^L \delta^R f_{1+2}^L f^R f_{1,2}^{\text{clu}} + (L \leftrightarrow R)$$

- ❖ C_F^2 (can separate soft clustering)
 - ❖ $S^{(2)}(x, y) = 1/2(S^{(1)}(x, y))^2 + \text{clustering}$
- ❖ $C_F C_A$ (IR safety \Rightarrow don't separate clustering effect)
 - ❖ w/ clustering, all divergences cancel \rightarrow left with different (finite) NGLs

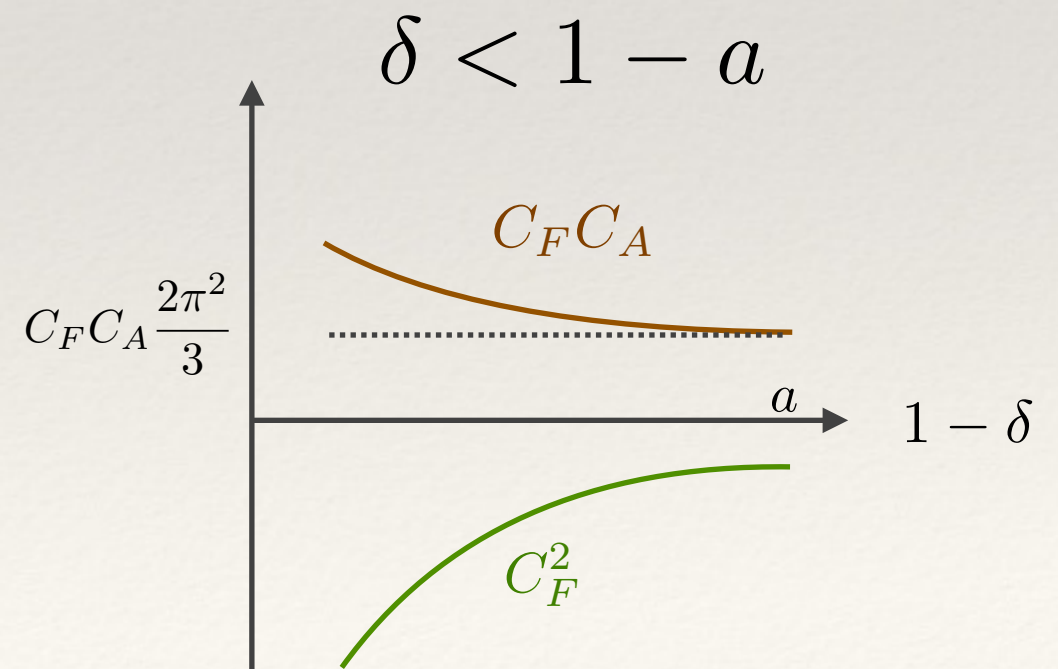
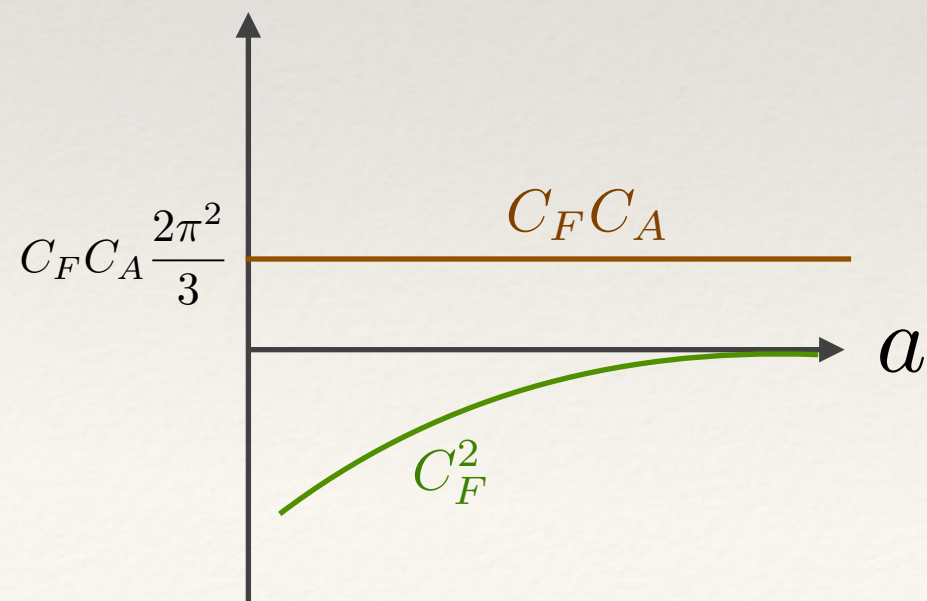
2 Loop Results

“fuzzy” region $\sim 1 - a$
 clustering size $\sim \delta < 1 - a$
 clustering region $\sim \Delta > 1 - a$



❖ Cancellation of NGLs:

$\delta = 1 - a$
 (cluster everything in fuzzy region)



Conclusions and Outlook

- ❖ Q-thrust:
 - ❖ non-deterministic *but* energy-flow variable
 - ❖ calculable!
 - ❖ interesting (important?) effect on NGLs!
 - ❖ generalizes naturally to Q-(sub)jettiness
- ❖ Outlook:
 - ❖ performance & correlations
 - ❖ many related observables to study, should exhibit same generic properties (calculability and NGLs)