

(Mutual) Information about Quarks vs. Gluons

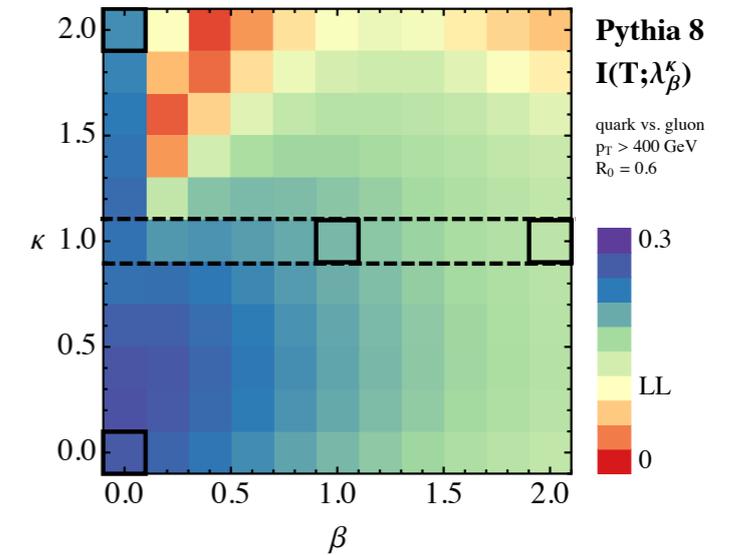
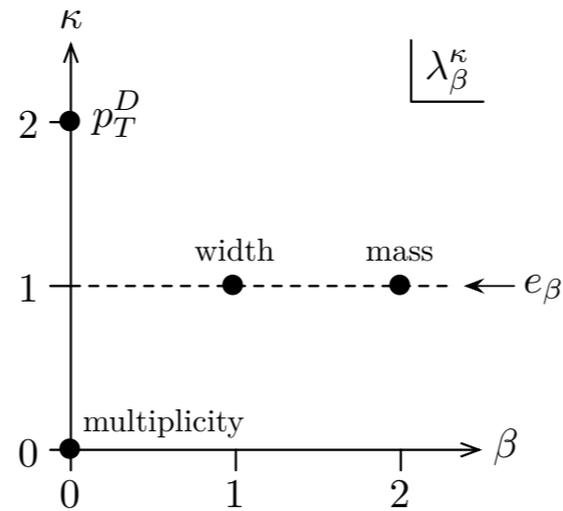
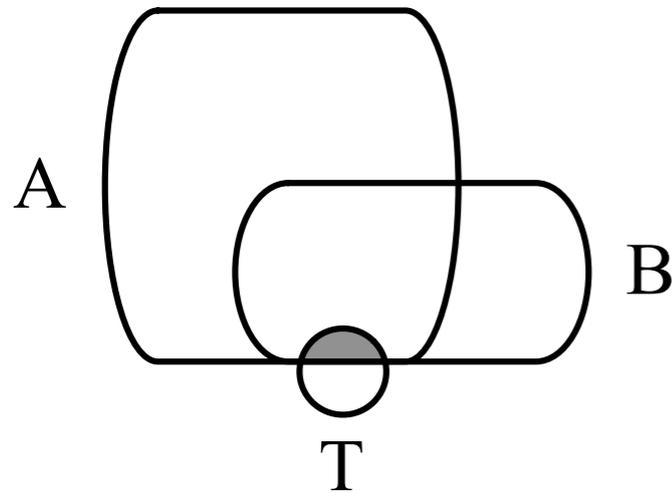
Jesse Thaler



In collaboration with Andrew Larkoski and Wouter Waalewijn (1408.3122)

Boost 2014, UCL — August 18, 2014

Outline



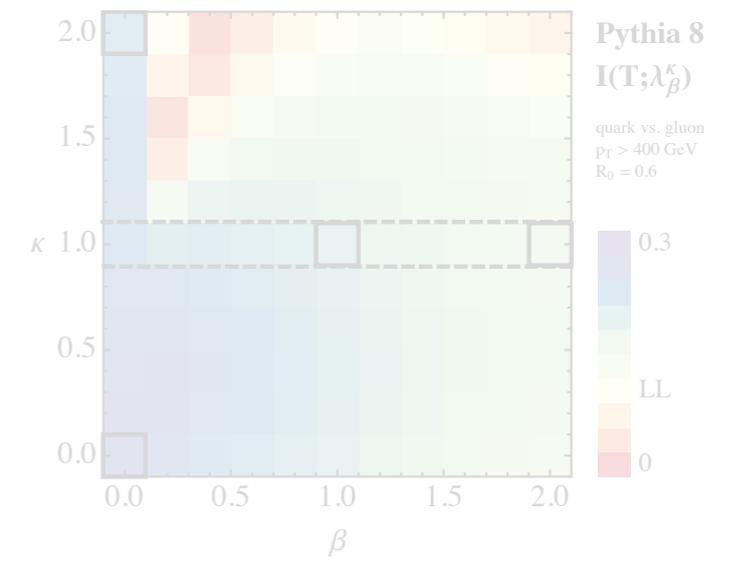
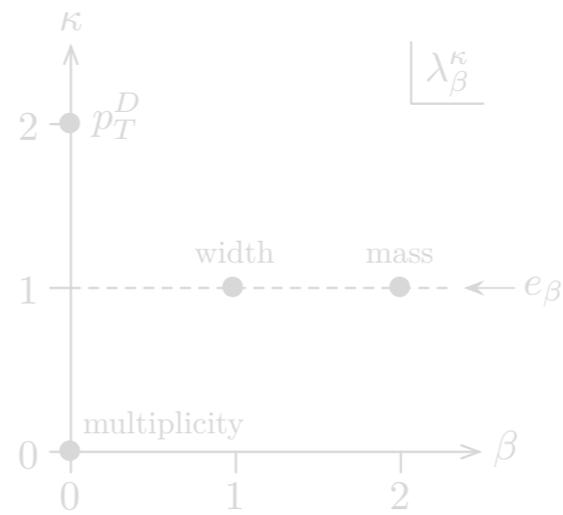
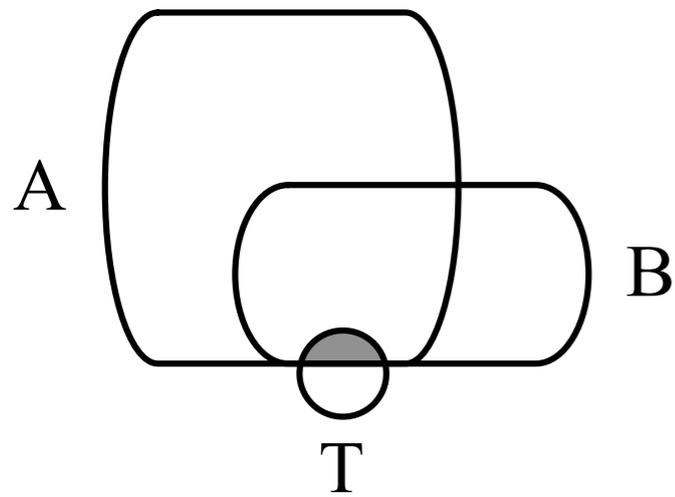
Mutual Information
with the Truth

Generalized
Angularities

LL vs. NLL vs.
Pythia vs. Herwig

*“Gaining (Mutual) Information
about Quark/Gluon Discrimination”*

[Andrew Larkoski, JDT, Wouter Waalewijn, 1408.3122]



Mutual Information with the Truth

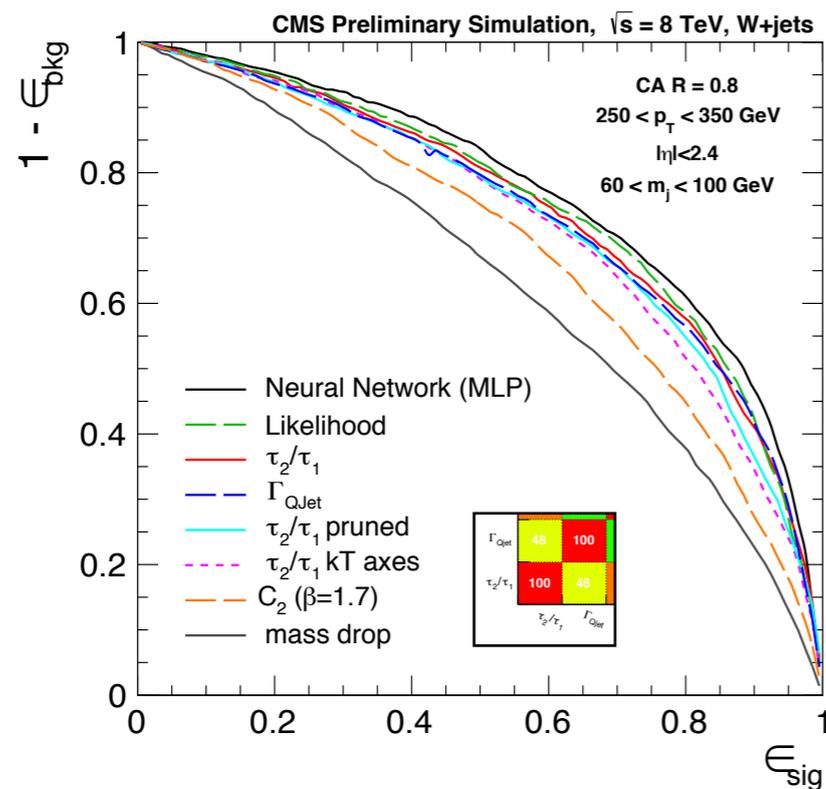
Generalized Angularities

LL vs. NLL vs. Pythia vs. Herwig

Dueling Themes at Boost 2013

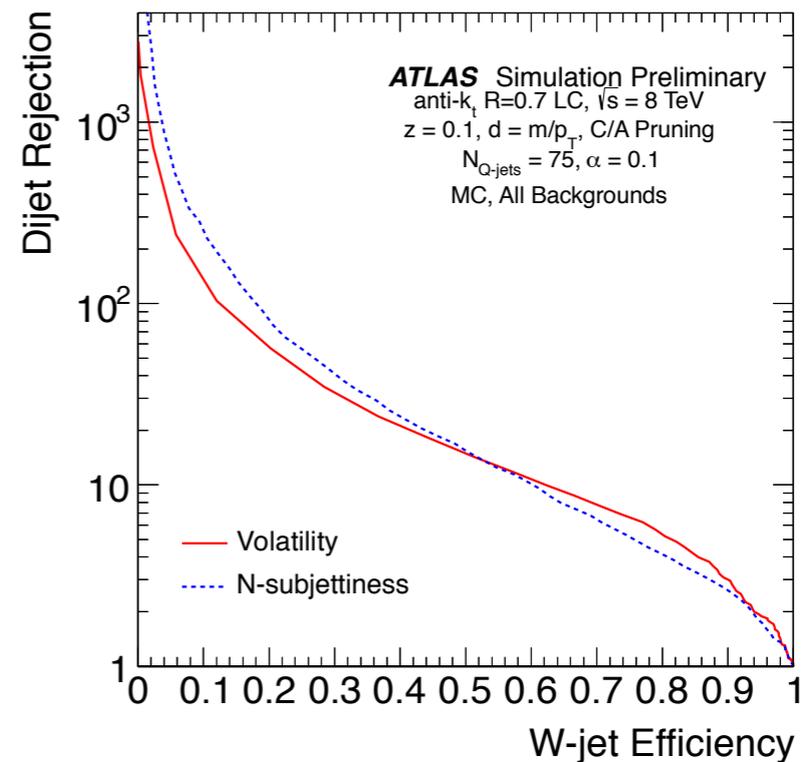
W tagging with *Q*-jet Volatility + *N*-subjettiness

“Redundant Variables”



[CMS-PAS-JME-13-006;
talk by Usai at Boost 2013]

“Complementary Variables”



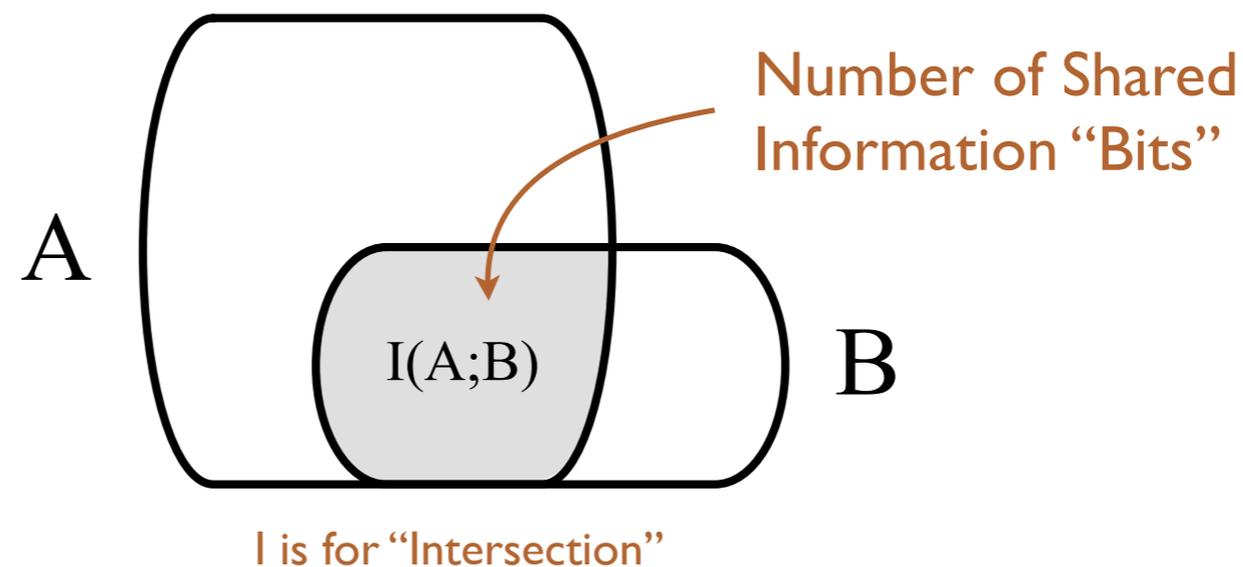
[ATLAS-CONF-2013-087;
talk by Swiatlowski at Boost 2013]

[Ellis, Hornig, Krohn, Roy, Schwartz, 1201.1914]
 [JDT, Van Tilburg, 1011.2268, 1108.2701]

Mutual Information

Robust Measure of Correlations

$$I(A; B) = \int da db p(a, b) \log_2 \frac{p(a, b)}{p(a)p(b)}$$



Can obtain directly from
double differential cross section

$$p(a, b) = \frac{1}{\sigma} \frac{d^2\sigma}{da db}$$

[see next talk by Larkoski]

Related to (binned)
Shannon entropy

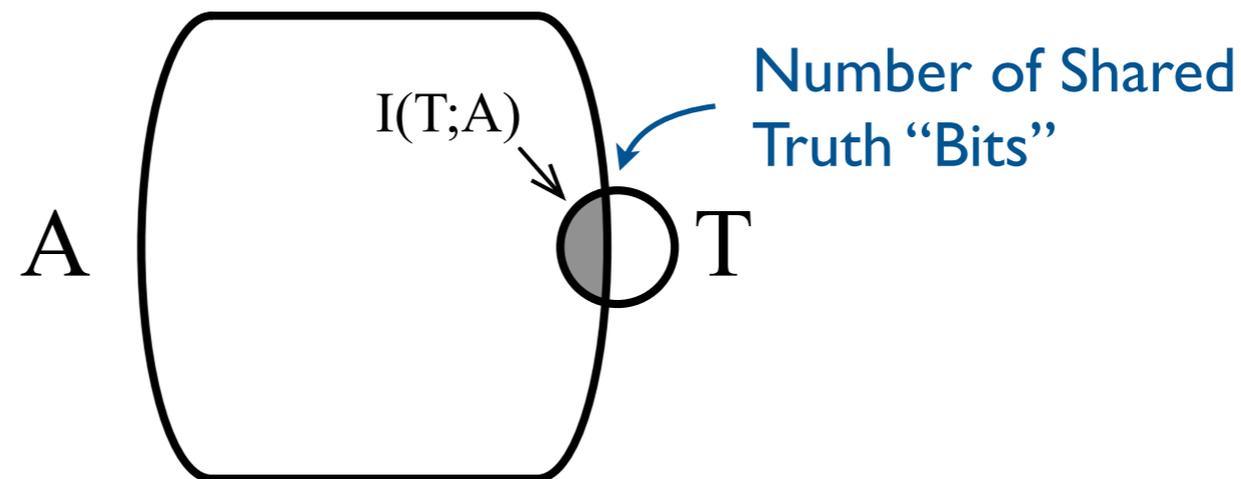
$$H(A) = - \sum_{a \in A} p(a) \log_2 p(a)$$

$$I(A; B) = H(A) + H(B) - H(A, B)$$

Mutual Information with the Truth

Robust Measure of Discrimination Power

$$I(T; A) = \int da \left(\frac{p_q(a)}{2} \log_2 \frac{p_q(a)}{p_{\text{tot}}(a)} + \frac{p_g(a)}{2} \log_2 \frac{p_g(a)}{p_{\text{tot}}(a)} \right)$$



For simplicity, equal parts
quarks and gluons

$$p_{\text{tot}}(a) = \frac{p_q(a) + p_g(a)}{2}$$

Possible range for
"Truth Overlap"

$$0 \leq I(T; A) \leq 1$$

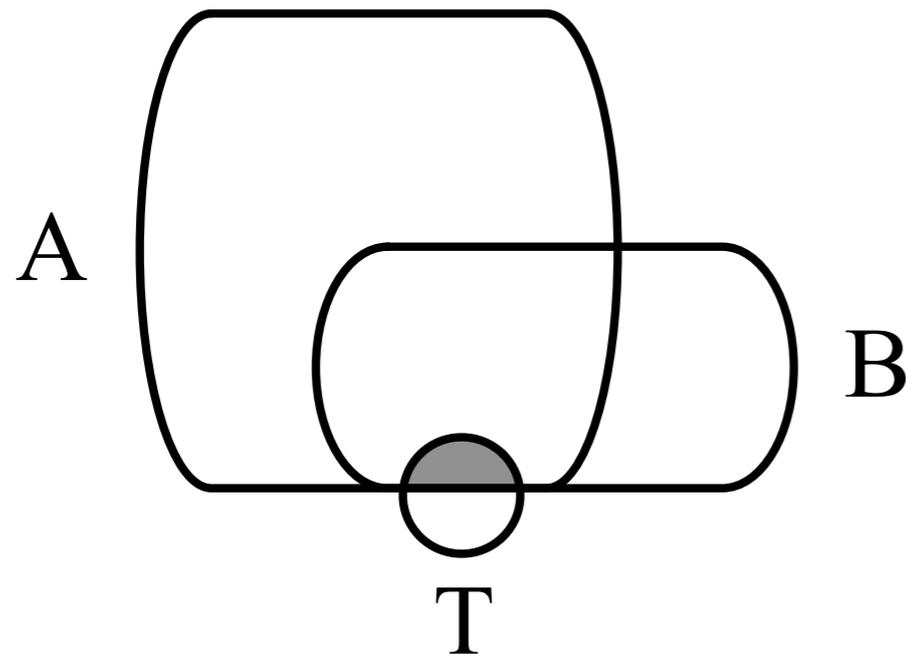
Zero Bits

One Bit
(0 = q, 1 = g)

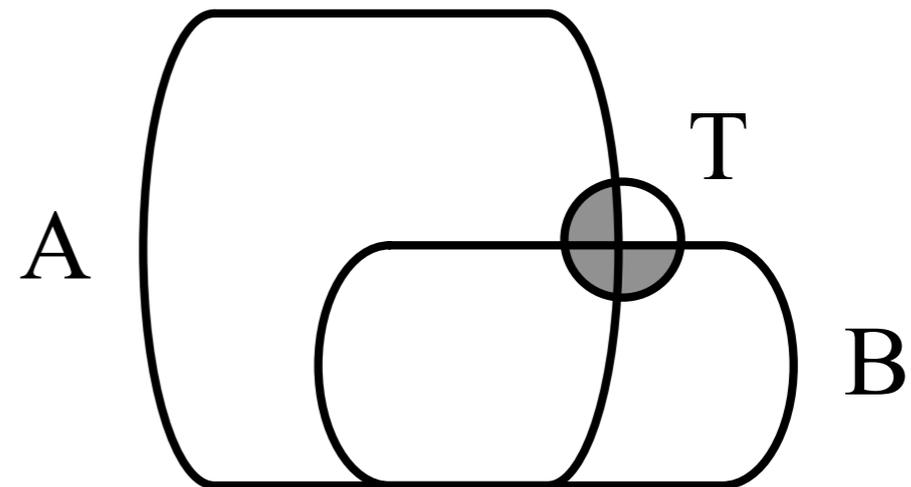
Joint Discrimination Power

Visualizing the Dueling Themes

“Redundant Variables”



“Complementary Variables”



$I(A;B)$: Same correlations

$I(T;A)$ and $I(T;B)$: Same individual discrimination power

$I(T;A,B)$: Different joint discrimination power

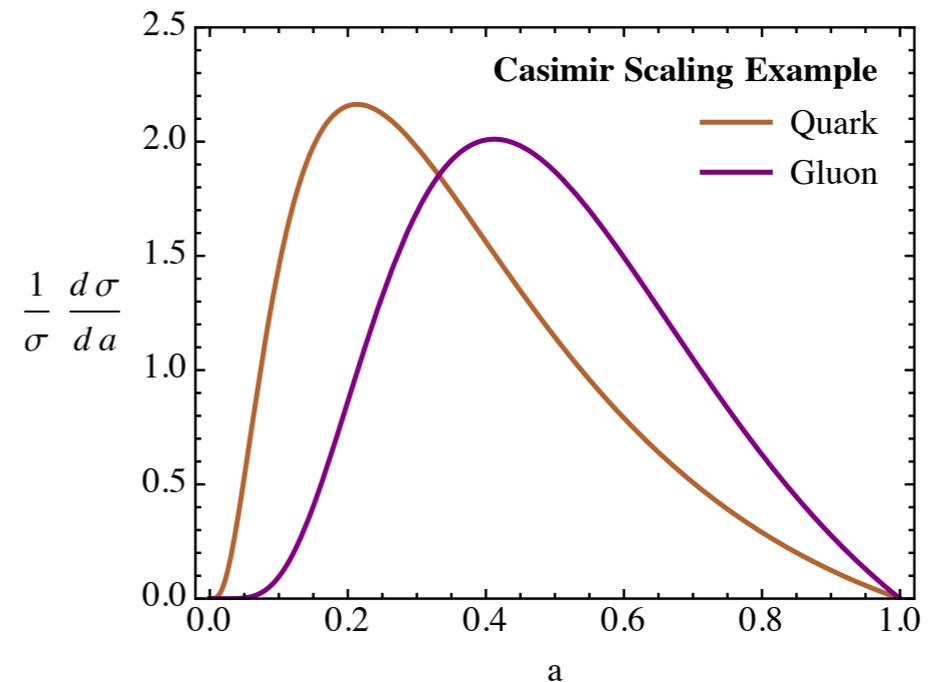
Test Case: Casimir Scaling

Most discriminants at LL:

$$p_i(a) = \frac{\partial}{\partial a} e^{-C_i f(a)}$$

$$C_F = 4/3 \text{ (quarks)}$$

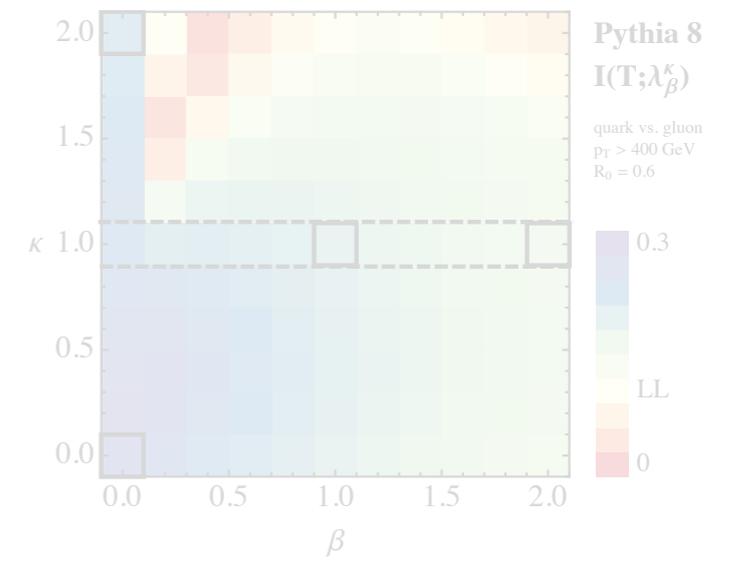
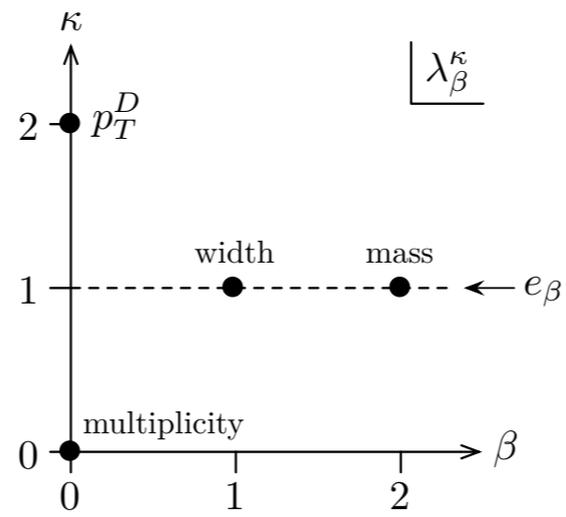
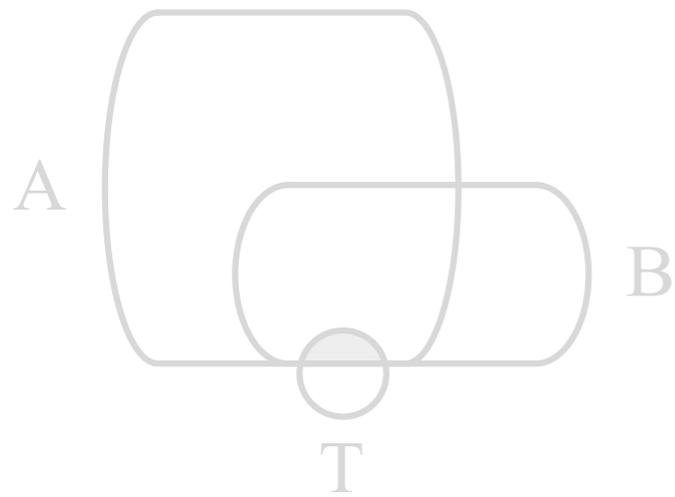
$$C_A = 3 \text{ (gluons)}$$



$$I(T; A) = \frac{1}{\ln 4} \left[\frac{(C_A - C_F)^2}{C_F C_A} \left(1 - {}_2F_1 \left(1, \frac{C_F}{C_A - C_F}; \frac{C_A}{C_A - C_F}; \frac{-C_A}{C_F} \right) \right) - \ln \left(\frac{(C_A + C_F)^2}{4C_F C_A} \right) \right]$$

$$I(T; A) \simeq 0.103$$

Much less than
a full “truth bit”!



Mutual Information
with the Truth

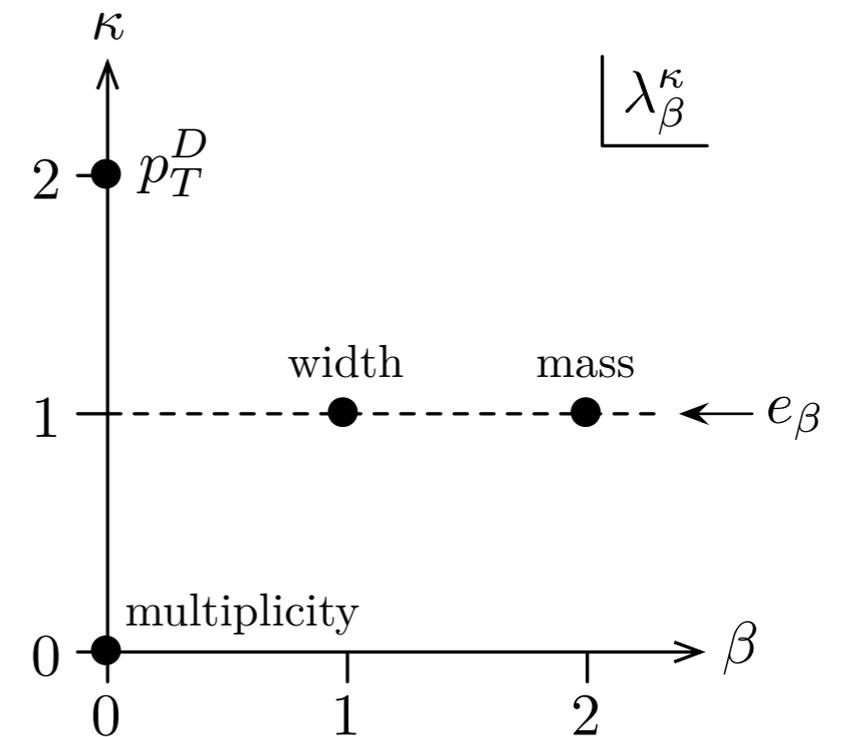
Generalized
Angularities

LL vs. NLL vs.
Pythia vs. Herwig

Generalized Angularities

$$\lambda_{\beta}^{\kappa} = \sum_{i \in \text{jet}} z_i^{\kappa} \theta_i^{\beta}$$

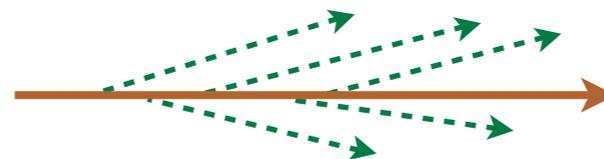
↑ momentum fraction ↑ angle to recoil-free axis



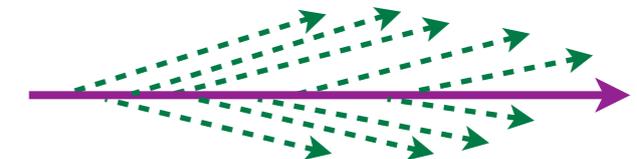
Measure of
gluon radiation
about hard jet core

$\mathbf{z}_i = p_{Ti}/p_{T\text{jet}}$
 $\theta_i = R_i/R_0$

Quark Jet



Gluon Jet

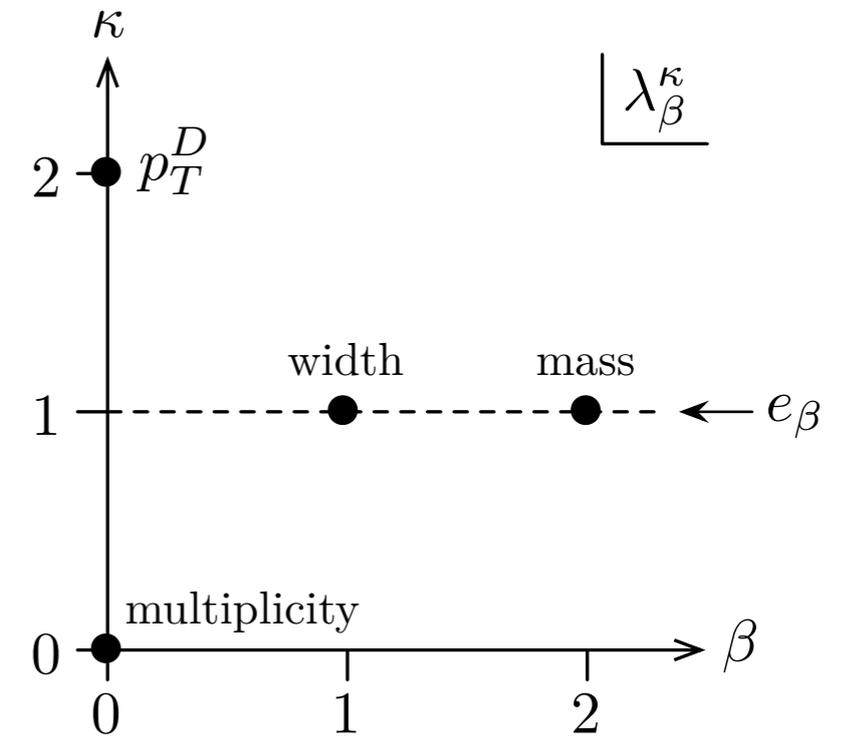


[For a more complete catalog, see Gallicchio, Schwartz, 1106.3076, 1211.7038]

Generalized Angularities

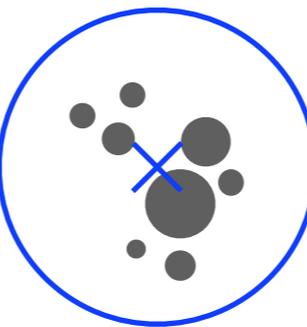
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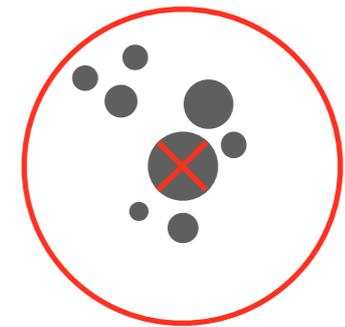
Measure of
gluon radiation
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$\mathbf{z}_i = p_{Ti}/p_{T\text{jet}}$
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Jet Axis (Mean)

Recoil-Free:
Measurement Axis \approx Hard Parton



e.g. WTA Axis (Median)

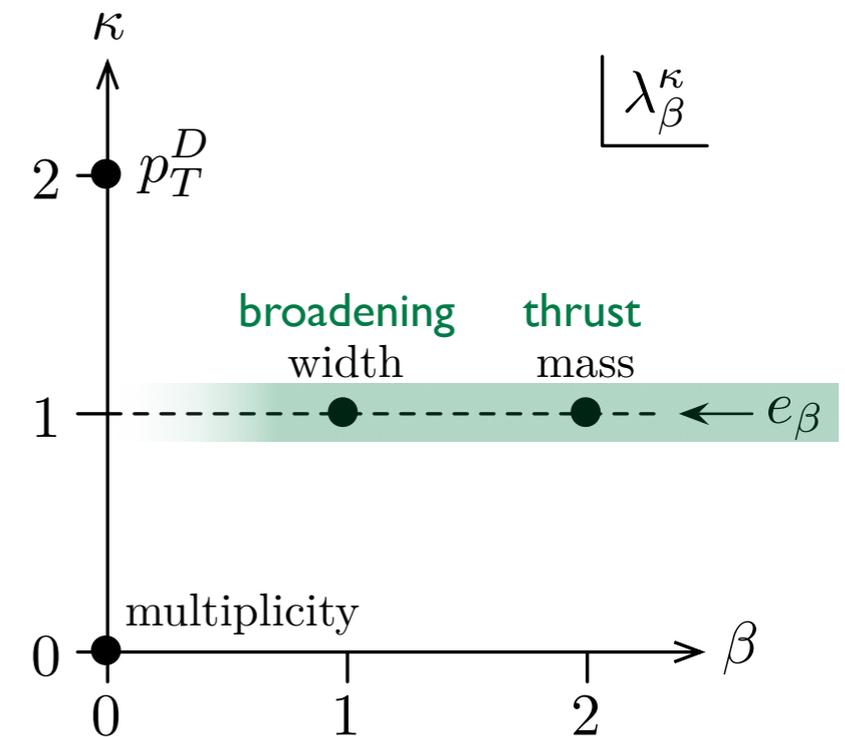
[For a more complete catalog, see Gallicchio, Schwartz, 1106.3076, 1211.7038]

[Bertolini, Chan, JDT, 1310.7584;
Larkoski, Neill, JDT, 1401.2158;
Salam, unpublished]

IRC Safe Angularities ($\kappa = 1$)

$$e_\beta = \sum_{i \in \text{jet}} z_i \theta_i^\beta$$

\uparrow
 Linear means
 IRC Safe



Single Differential Calculations (NLL):

$$\beta > 1 \quad [\text{Berger, Kucs, Sterman, hep-ph/0303051}; \\ \text{Ellis, Vermilion, Walsh, Hornig, Lee, 1001.0014}]$$

$$\beta \leq 1 \quad [\text{Larkoski, Neill, JDT, 1401.2158}]$$

$$\simeq C_1^{(\beta)} \quad [\text{Banfi, Salam, Zanderighi, hep-ph/0407286}; \\ \text{Larkoski, Salam, JDT, 1305.0007}]$$

Double Differential Calculations:

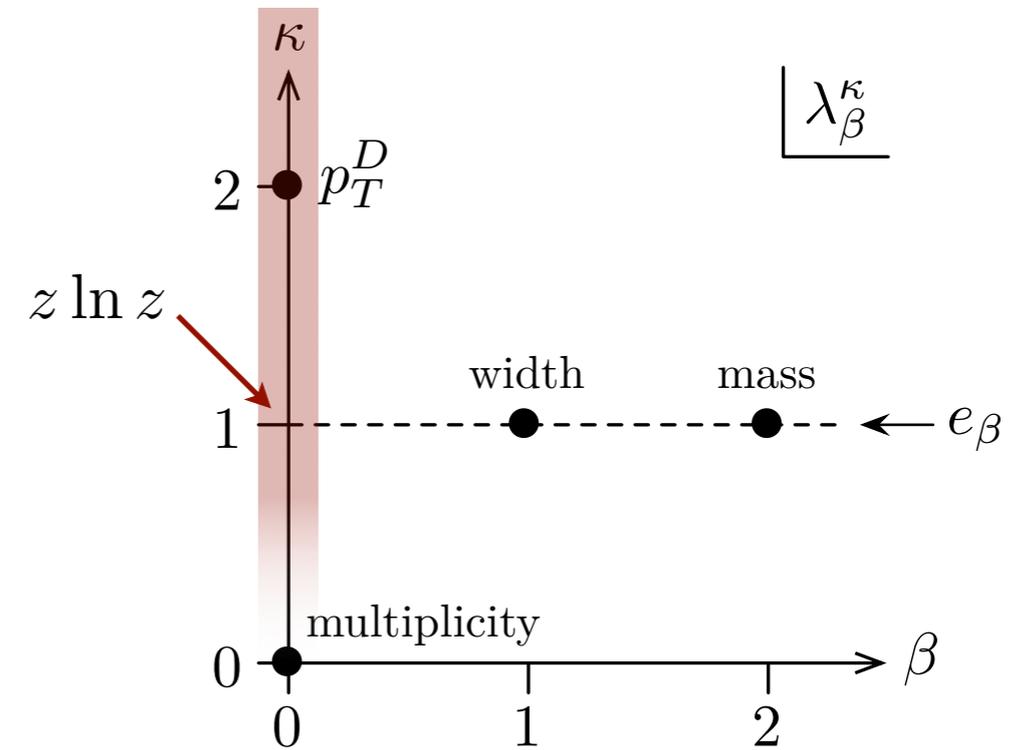
$$\text{LL} \quad [\text{Larkoski, JDT, 1307.1699}, \\ \text{see my talk at Boost 2013}]$$

$$\text{NLL} \quad [\text{Larkoski, Mout, Neill, 1401.4458}; \\ \text{see next talk by Larkoski}]$$

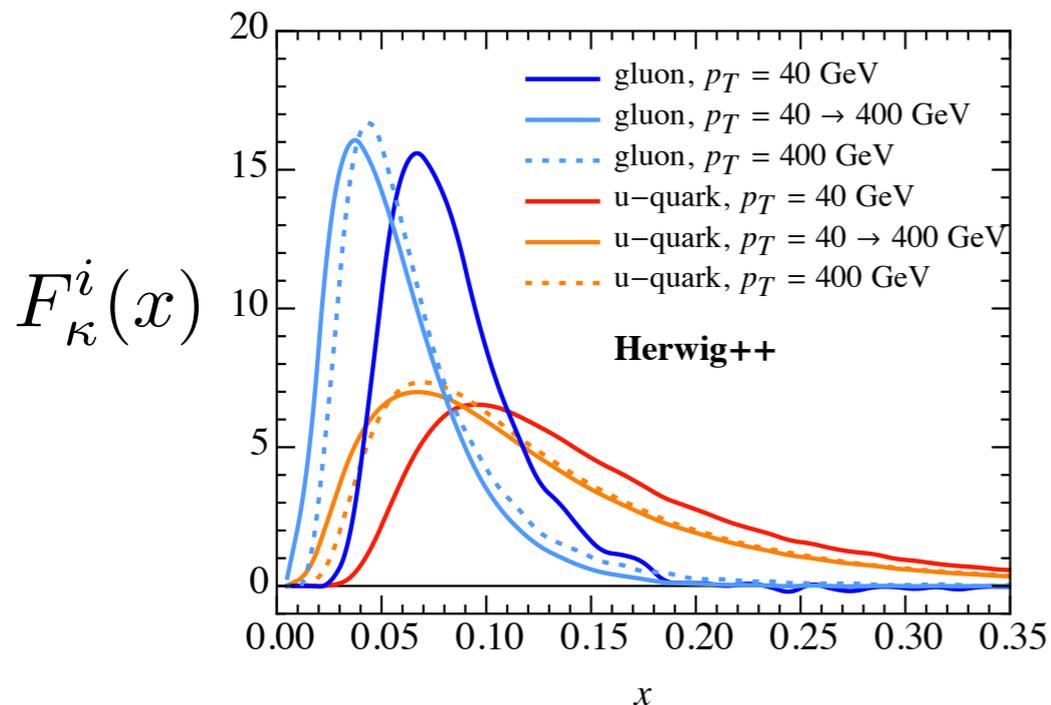
Unsafe Discriminants ($\beta = 0$)

$$\lambda_0^\kappa = \sum_{i \in \text{jet}} z_i^\kappa$$

↑
C Unsafe
IR Safe



Introducing the “Weighted-Energy Function”



Non-Perturbative, but Perturbative Evolution

$$\mu \frac{\partial}{\partial \mu} F_\kappa^i \simeq \frac{\alpha_s}{\pi} P_{i \rightarrow jk} \otimes F_\kappa^j \otimes F_\kappa^k$$

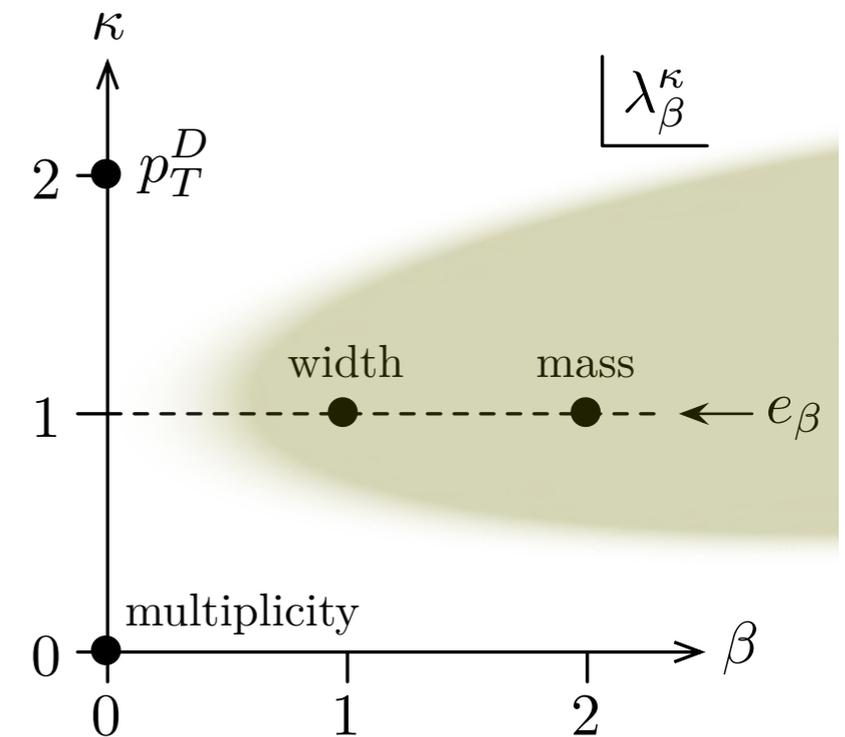
Inspired by Jet Charge & Track Function

[Krohn, Schwartz, Lin, Waalewijn, I209.2421;
 Waalewijn, I209.3019; talk by Waalewijn at Boost 2013]
 [Chang, Procura, JDT, Waalewijn, I303.6637]

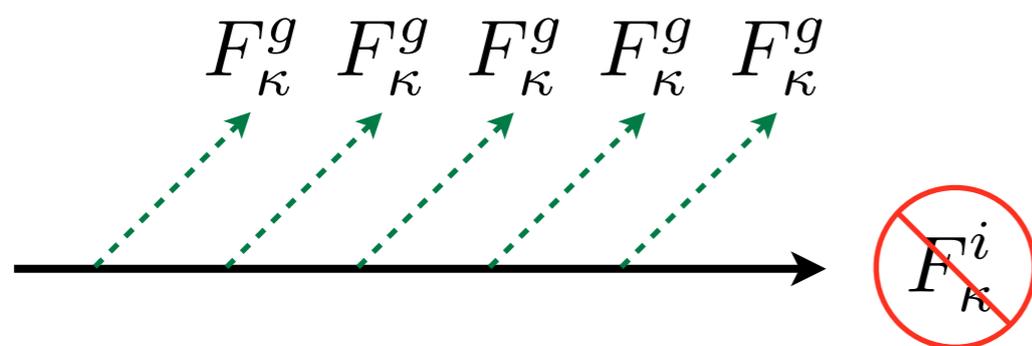
More Unsafe Discriminants

$$\lambda_{\beta}^{\kappa} = \sum_{i \in \text{jet}} z_i^{\kappa} \theta_i^{\beta}$$

↑ ↑
C Unsafe C Safe @
IR Safe $\theta = 0$



Exponentiating the Weighted-Energy Function



IRC Unsafe IRC Safe One Nonperturbative Parameter

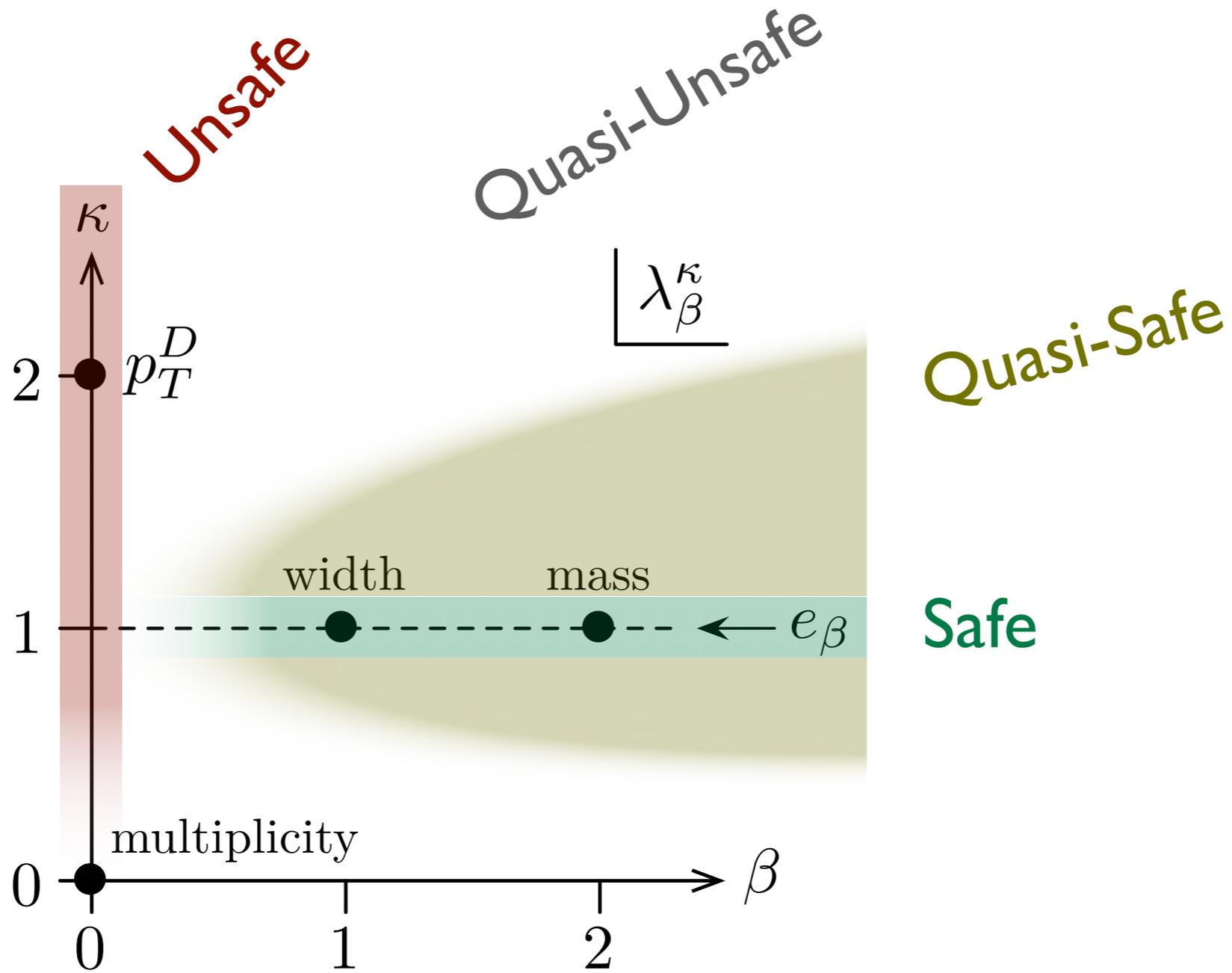
$$\lambda_{\beta}^{\kappa} \simeq \left(e_{\beta}/\kappa \right)^{\kappa} \exp \left[\langle \ln z^{\kappa} \rangle_g \right]$$

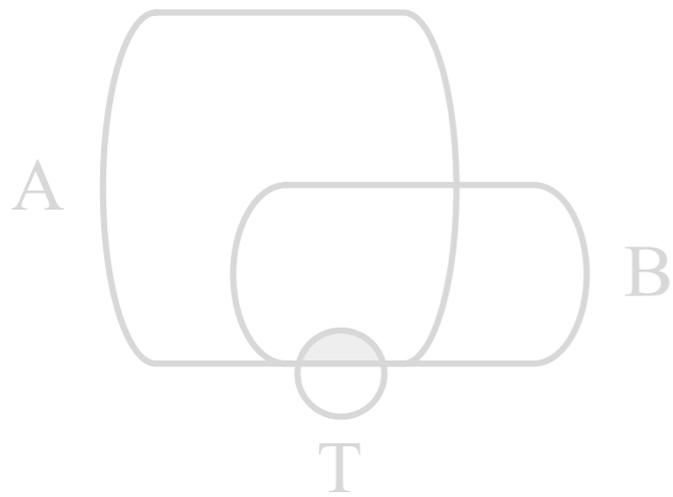
Inspired by Track-Based Thrust

[Chang, Procura, JDT, Waalewijn, 1306.6630]

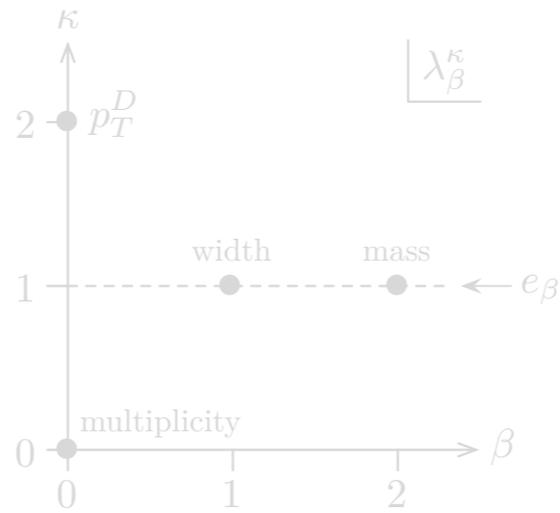
Can prove at NLL using CAESAR/SCET

[Banfi, Salam, Zanderighi, hep-ph/0407286; take 8.EFTx by Iain Stewart!]

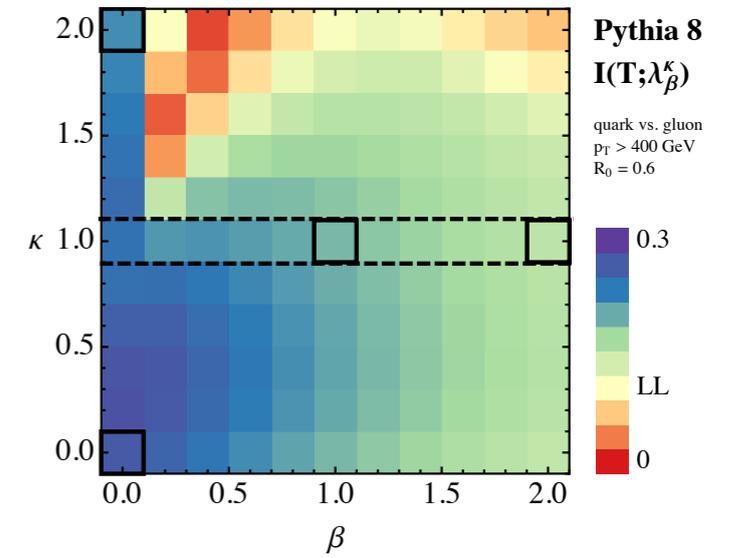




Mutual Information with the Truth

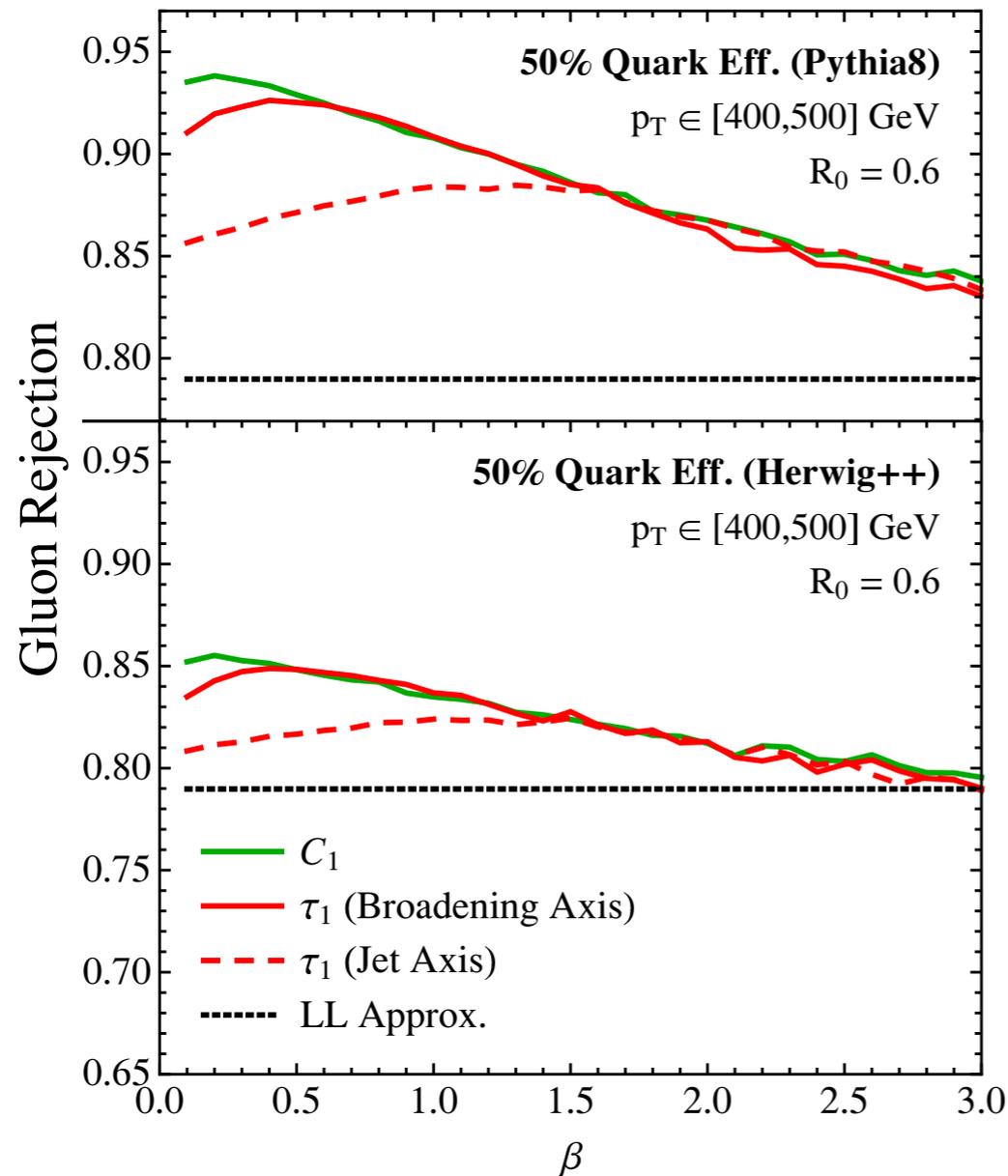


Generalized Angularities

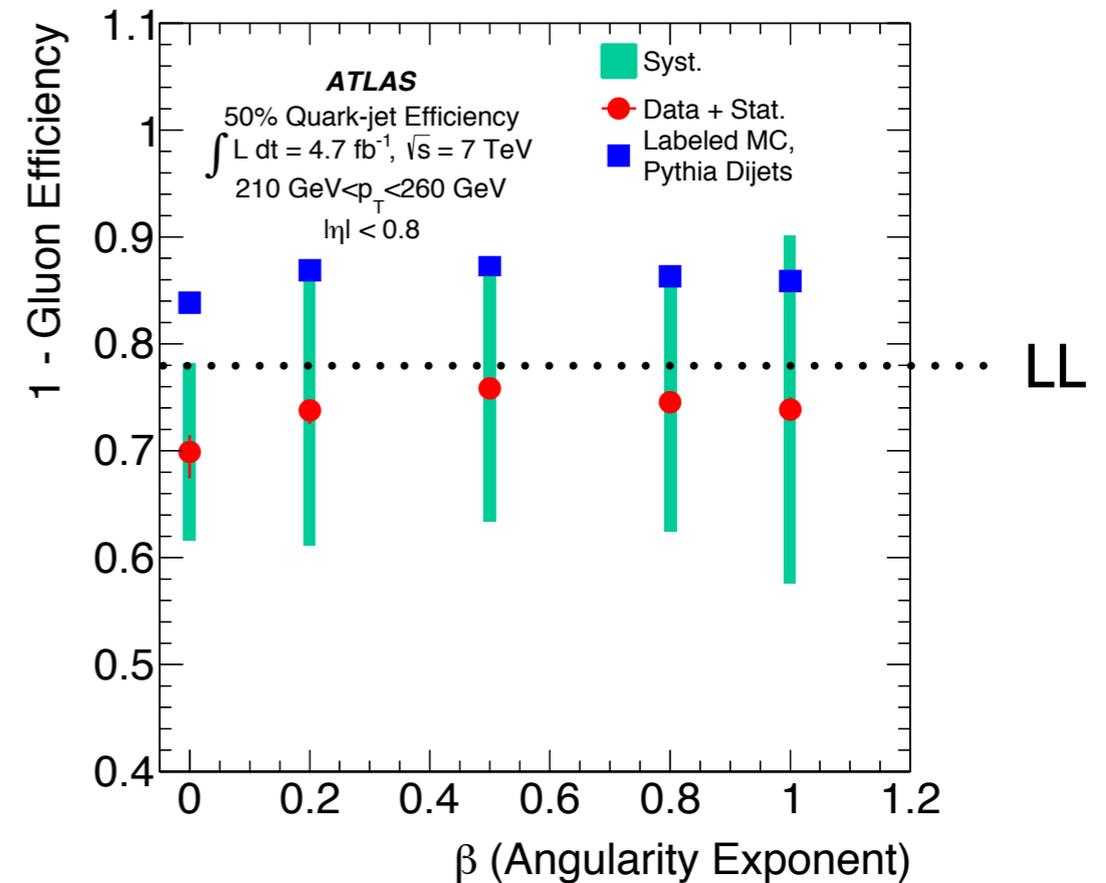


LL vs. NLL vs. Pythia vs. Herwig

(Conflicting) Information on Quarks vs. Gluons

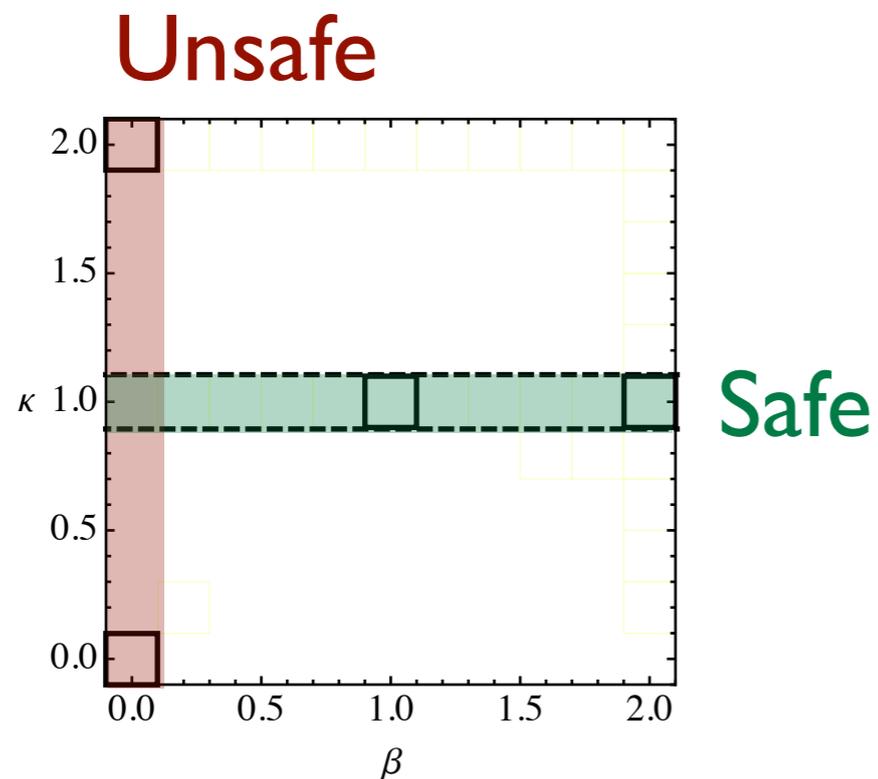


[Larkoski, Salam, JDT, 1305.0007;
talk by Larkoski at Boost 2013]

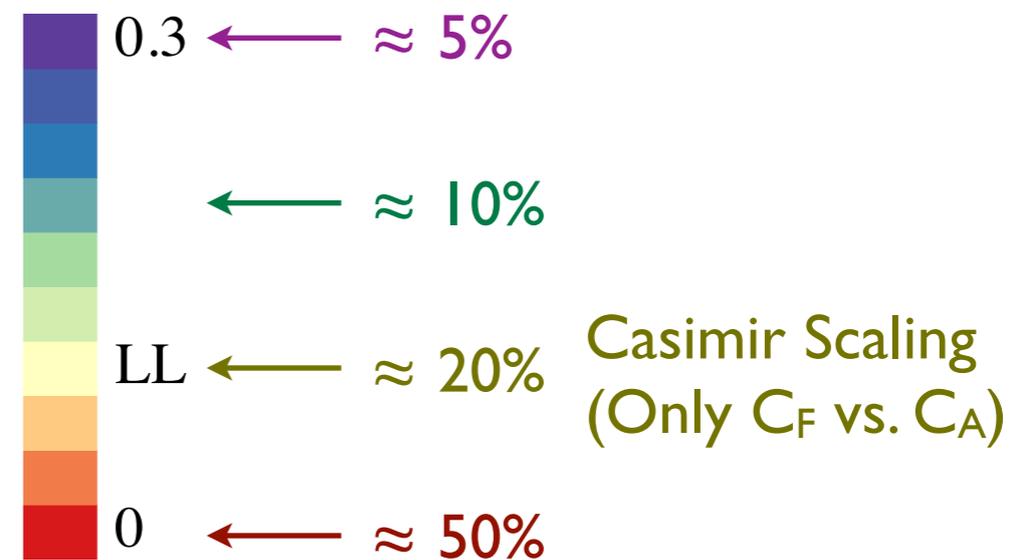


[ATLAS, 1405.6583]

Legend



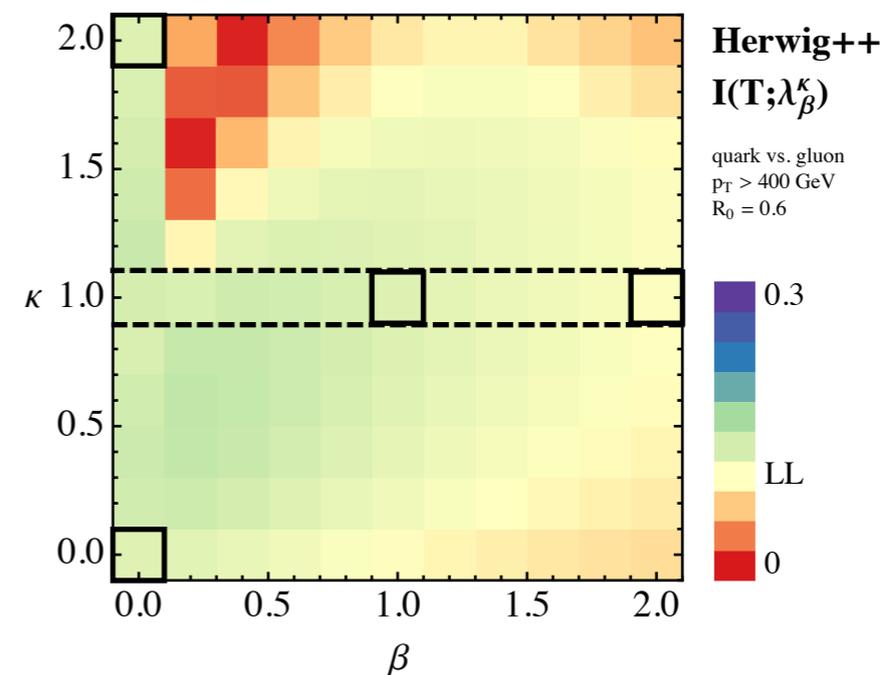
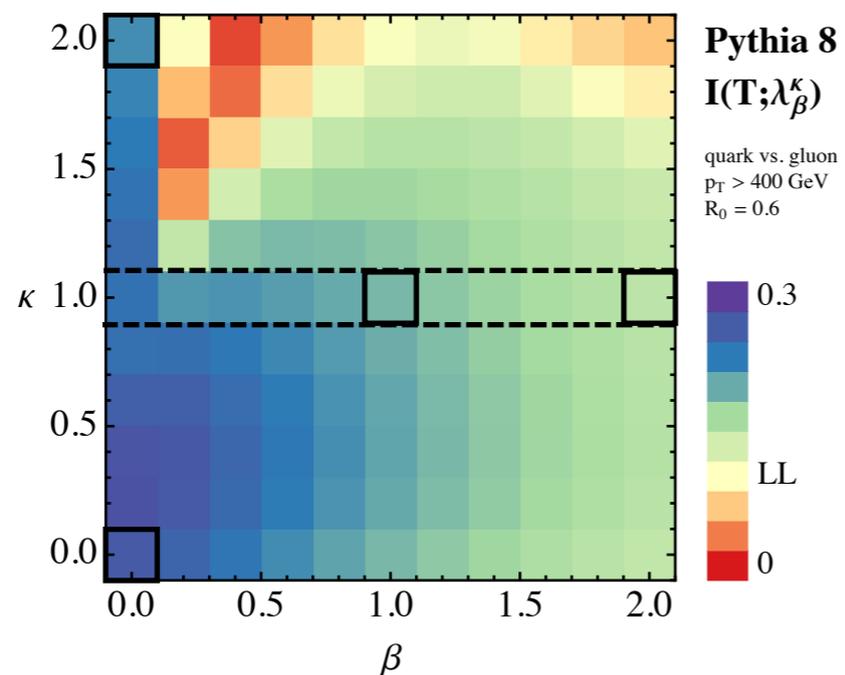
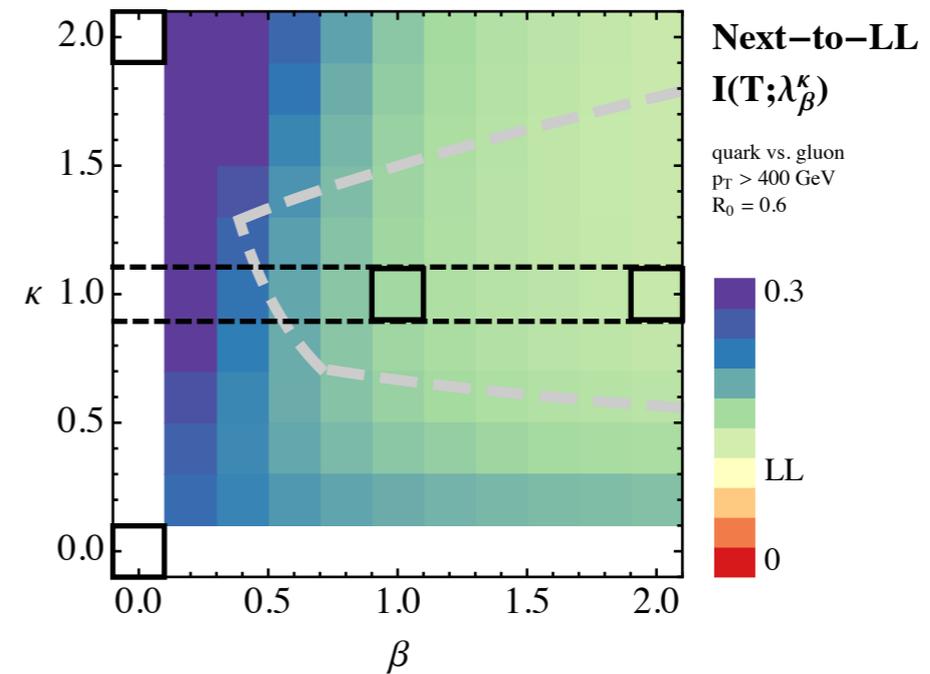
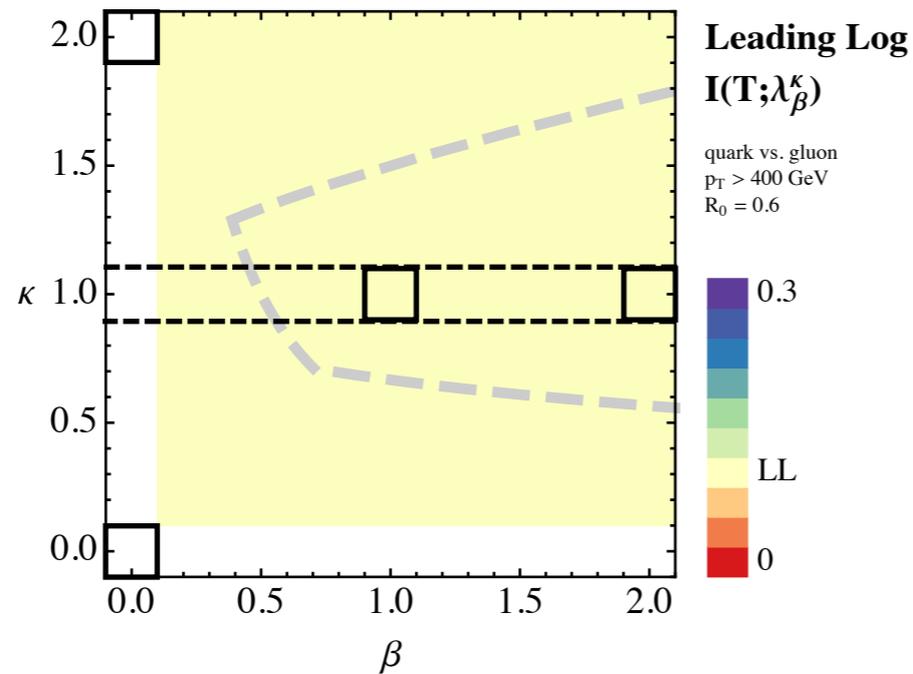
gluon mistag @
50% quark efficiency



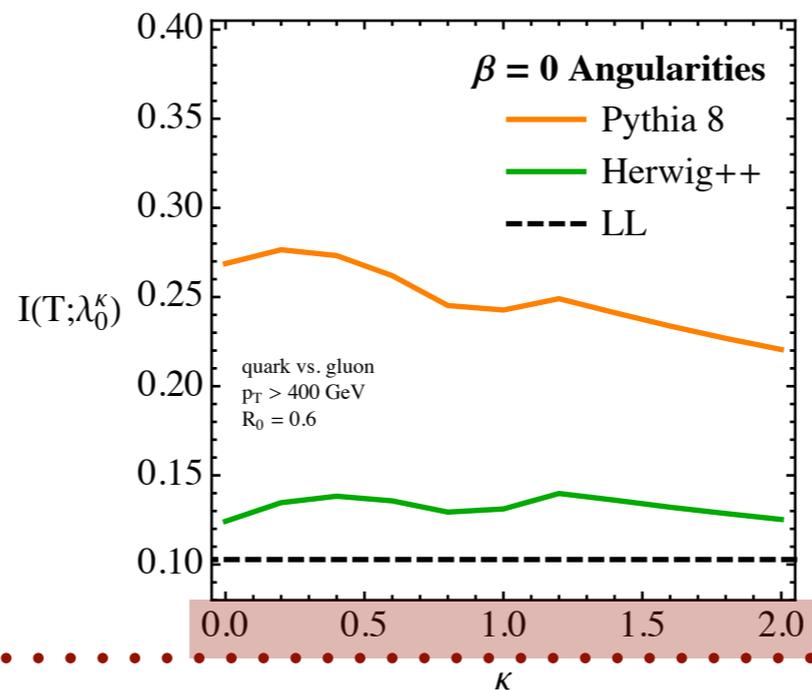
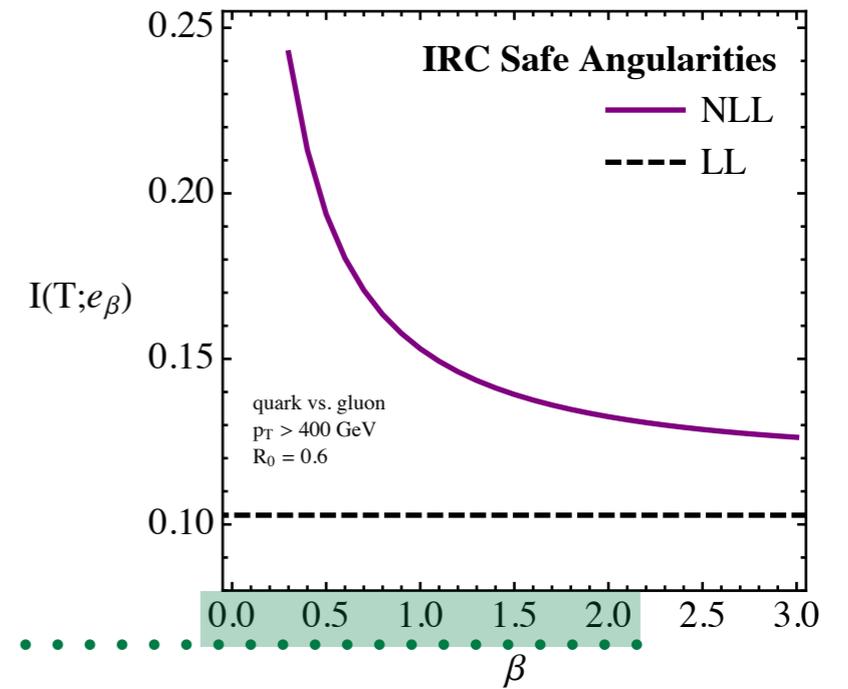
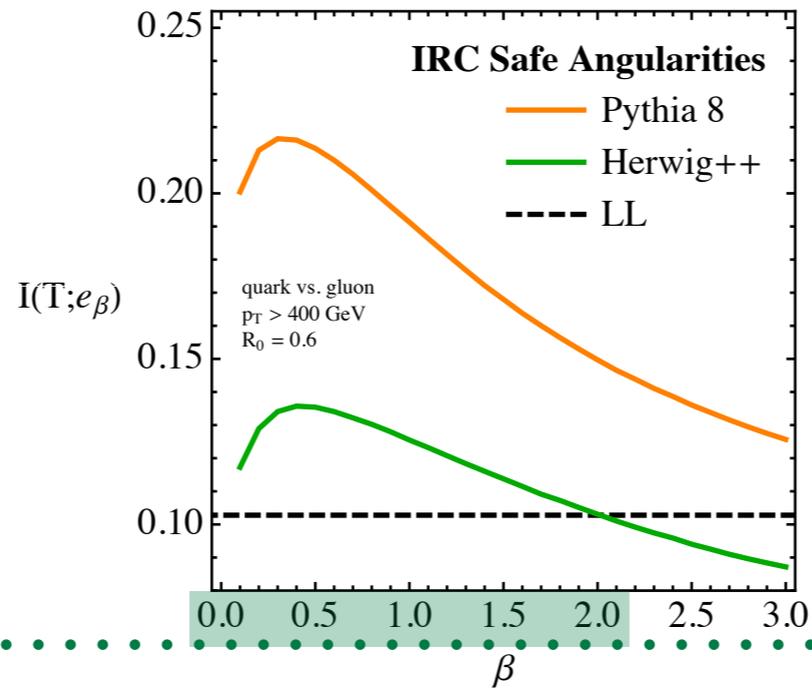
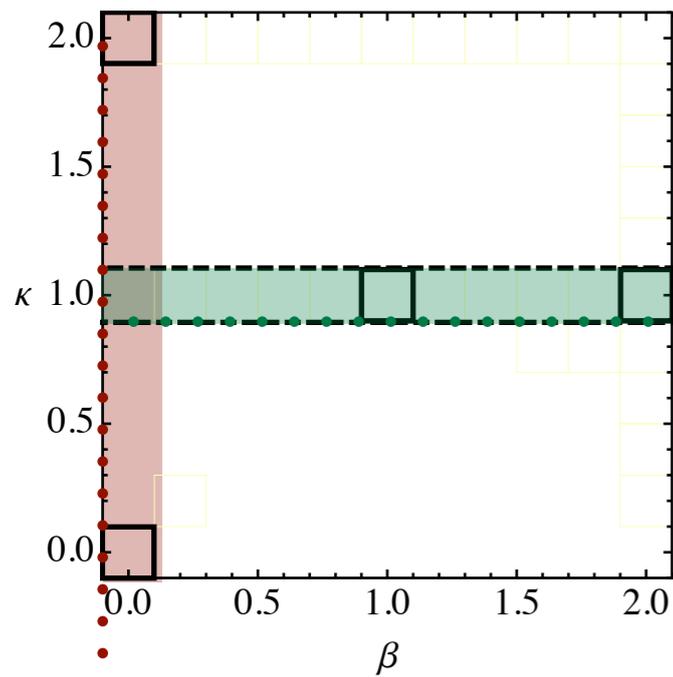
8 TeV pp
 $p_T > 400$ GeV
 $R_0 = 0.6$

“Quark” : $q q \rightarrow q q$
 “Gluon” : $g g \rightarrow g g$

Truth Overlap for One Angularity

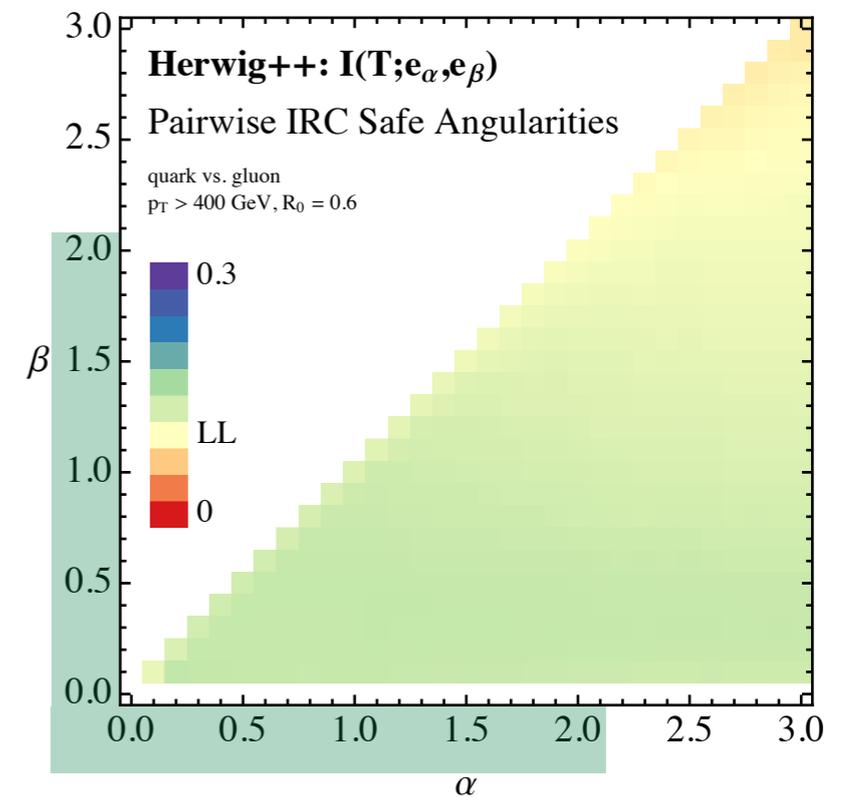
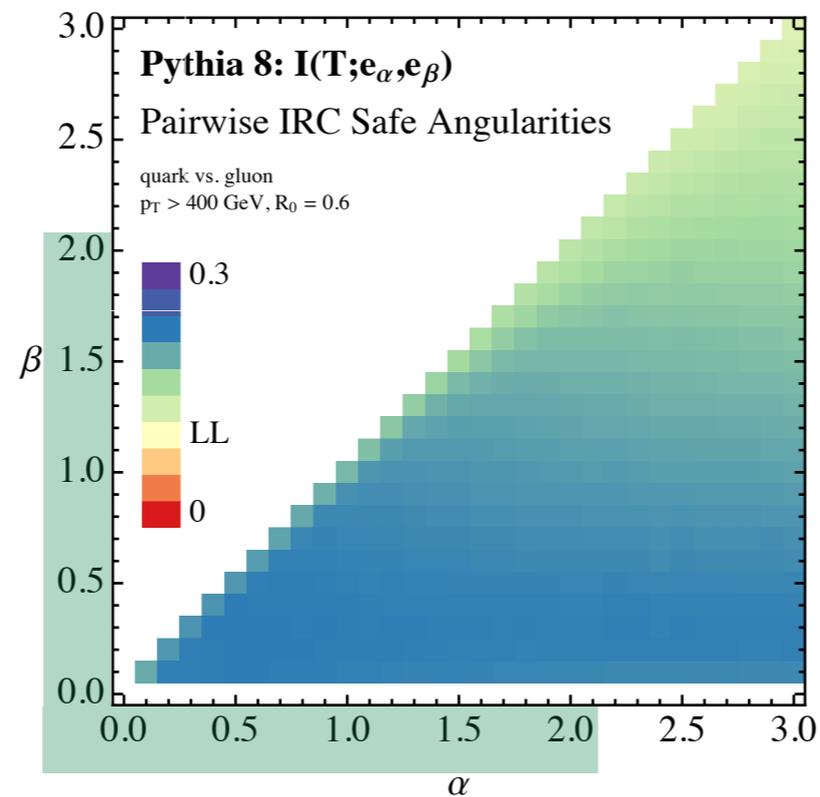
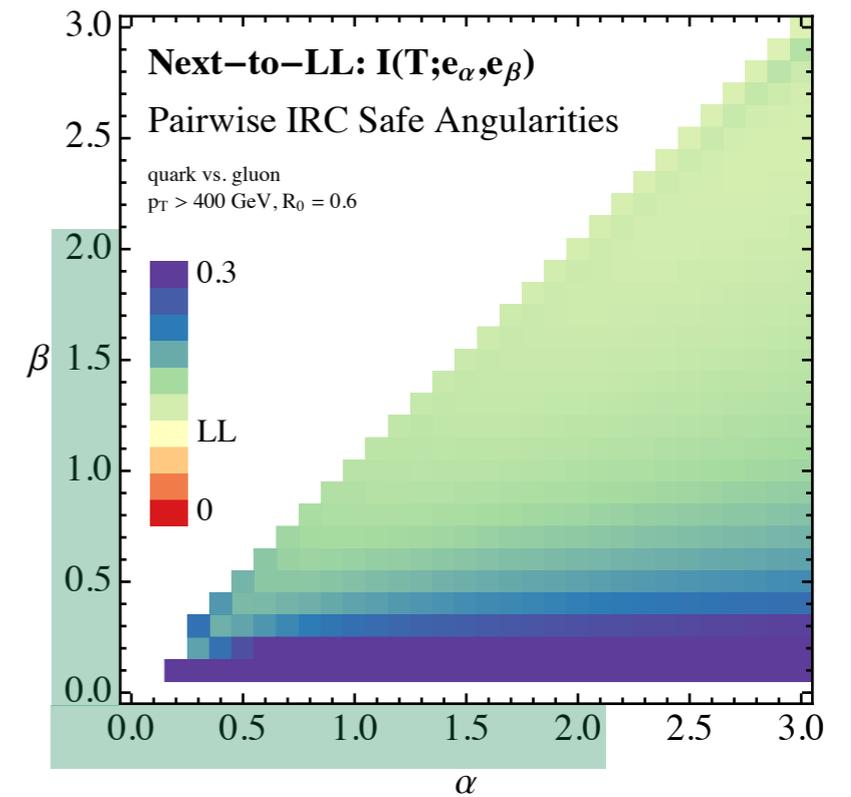
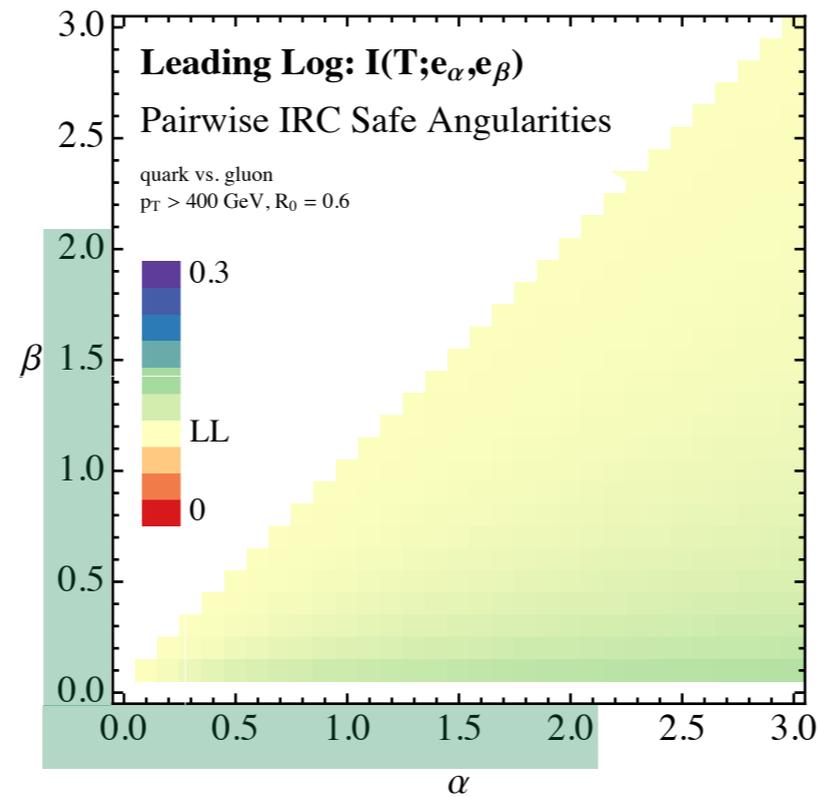
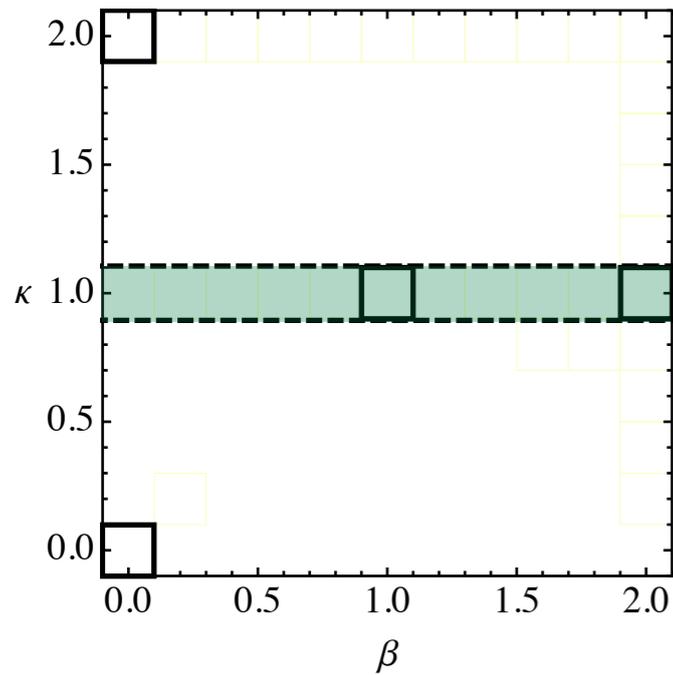


Truth Overlap for One Angularity

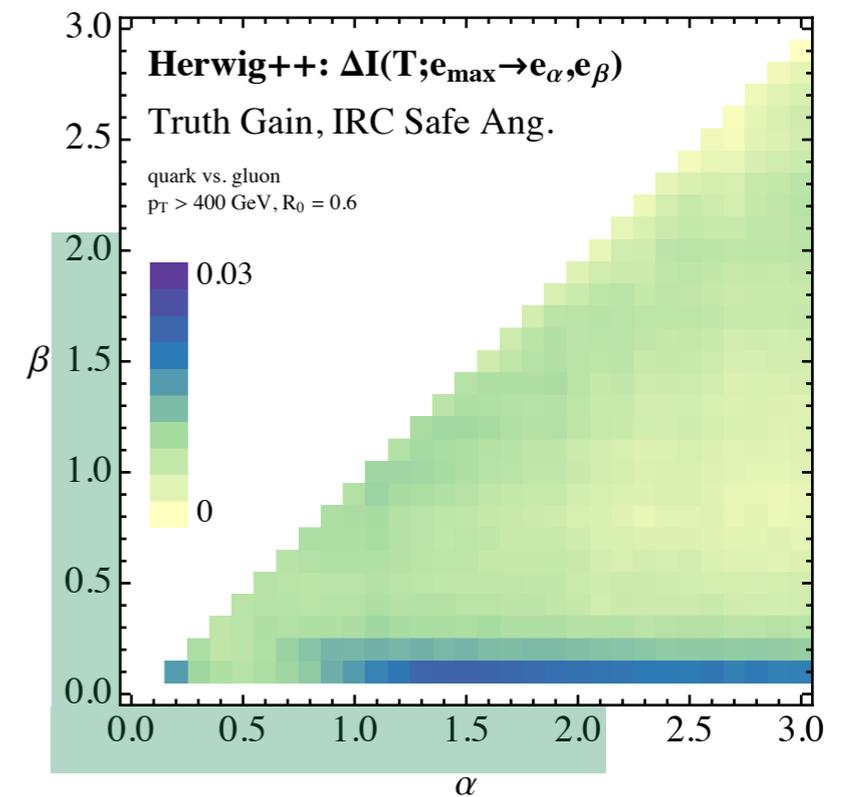
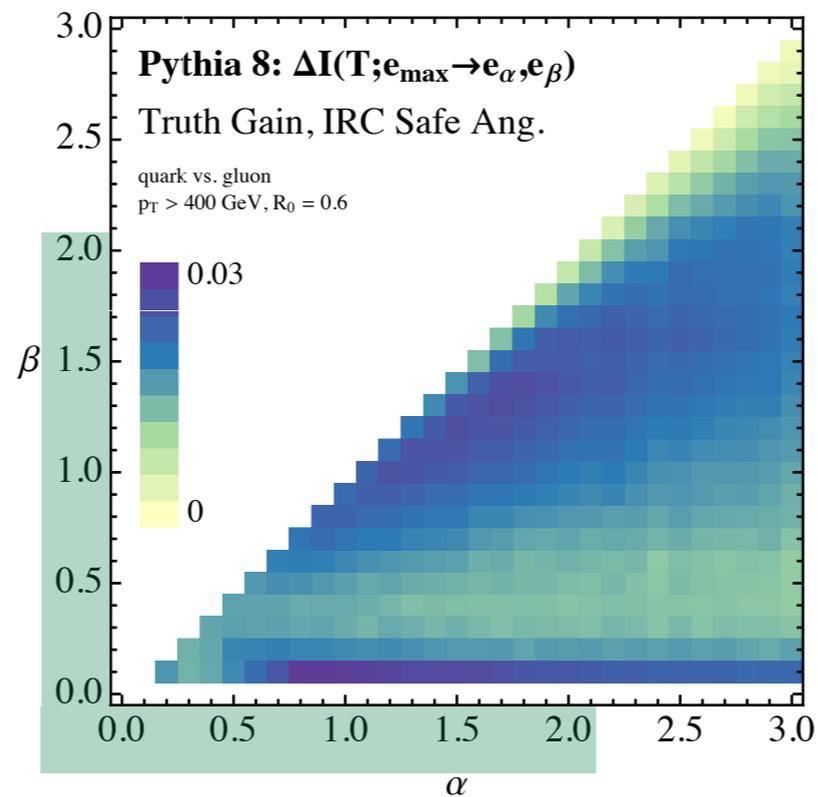
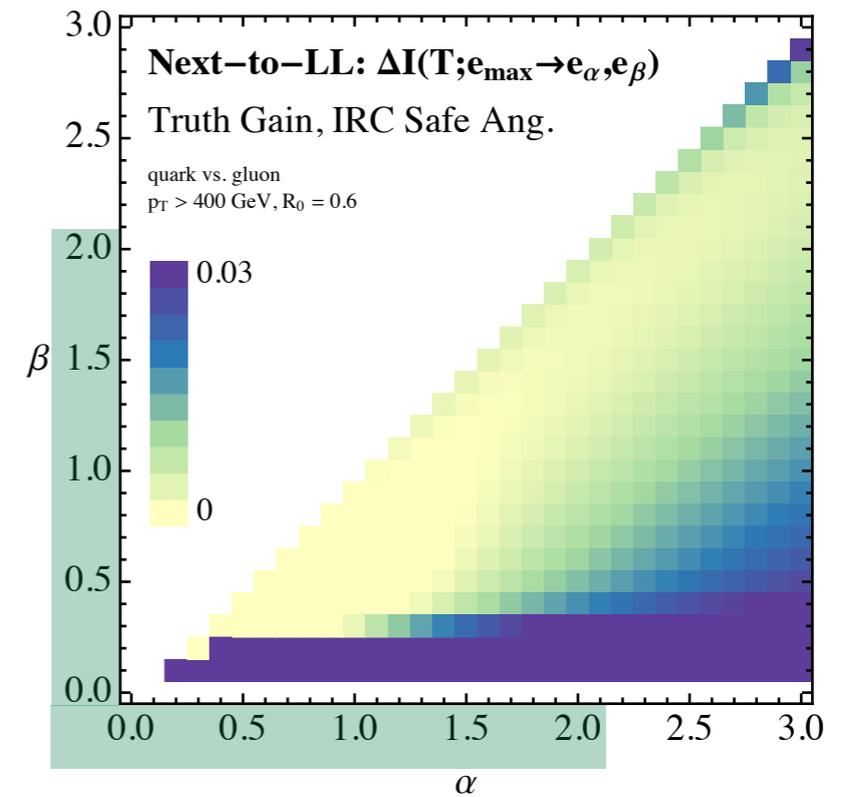
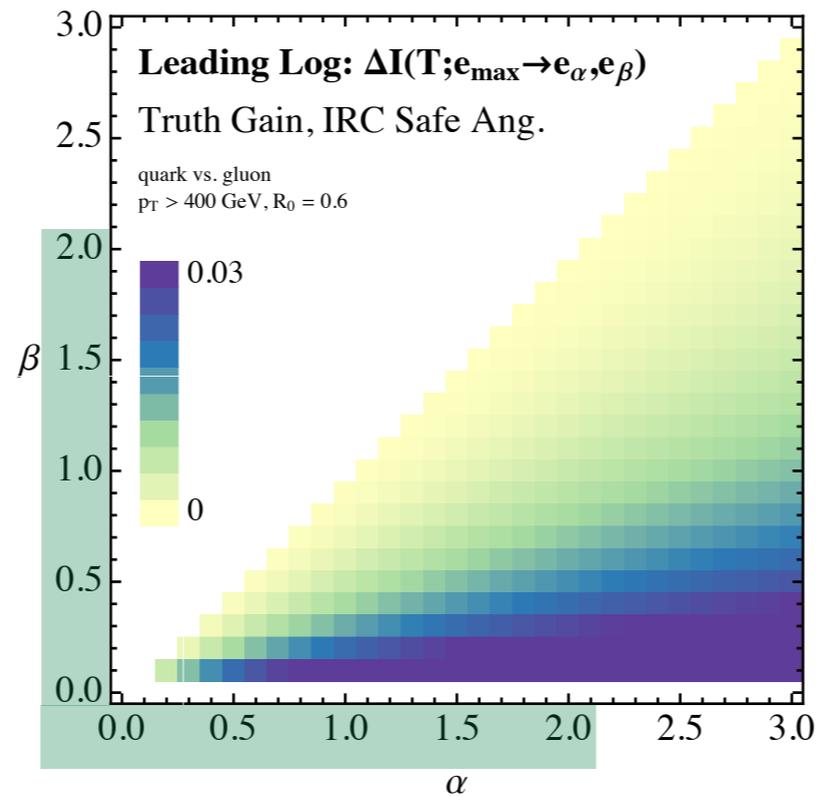
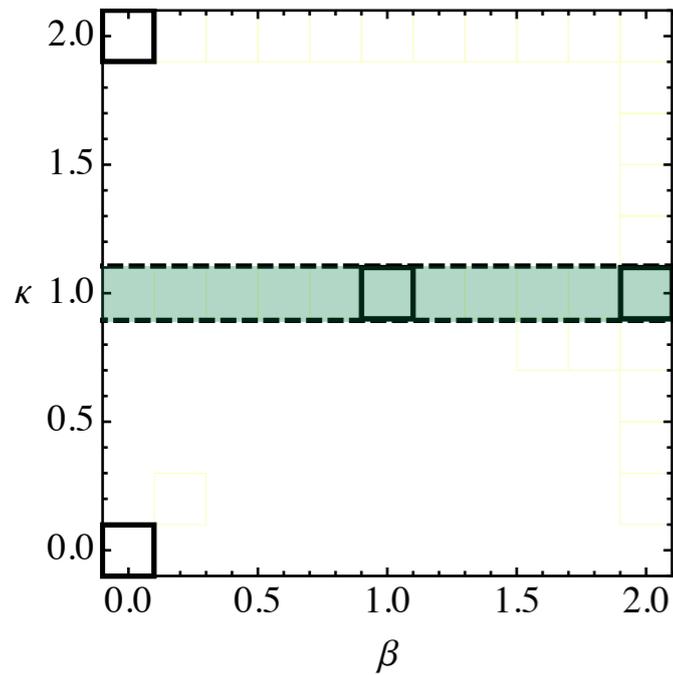


No Theory Prediction (Yet)

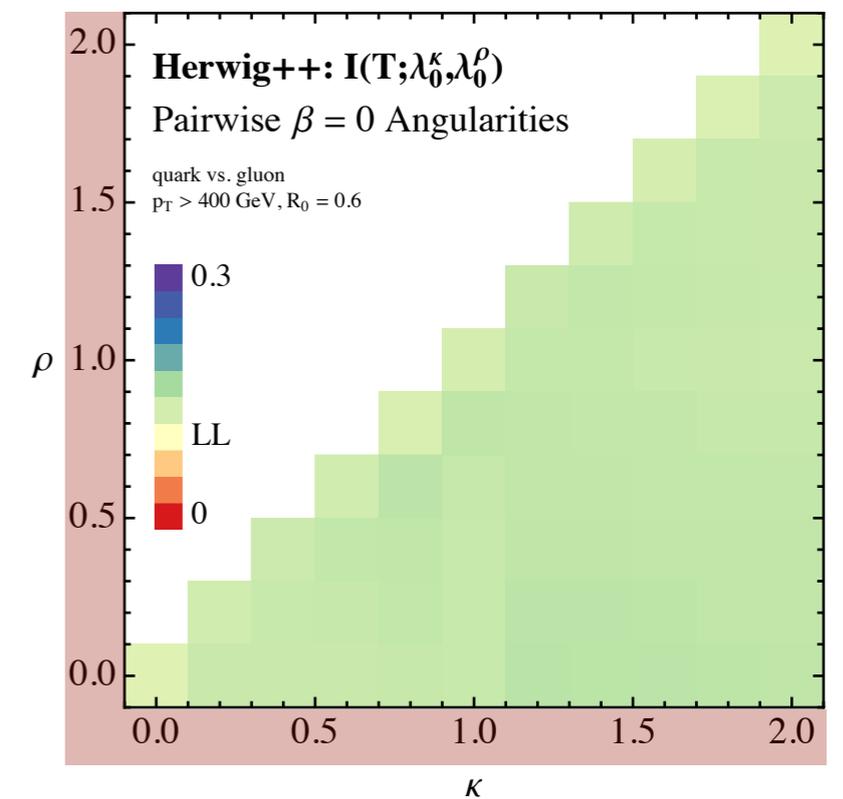
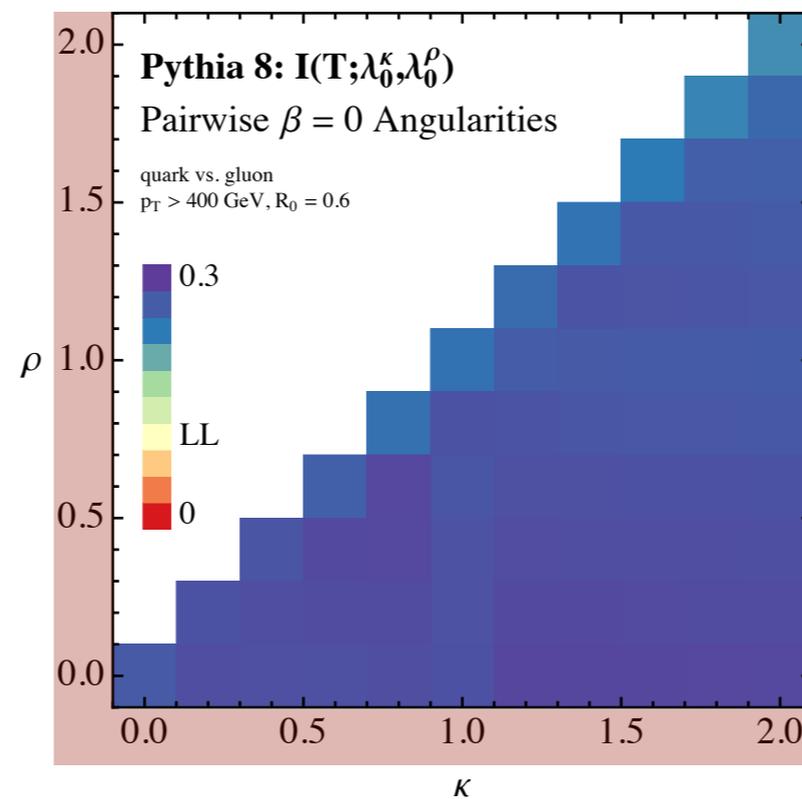
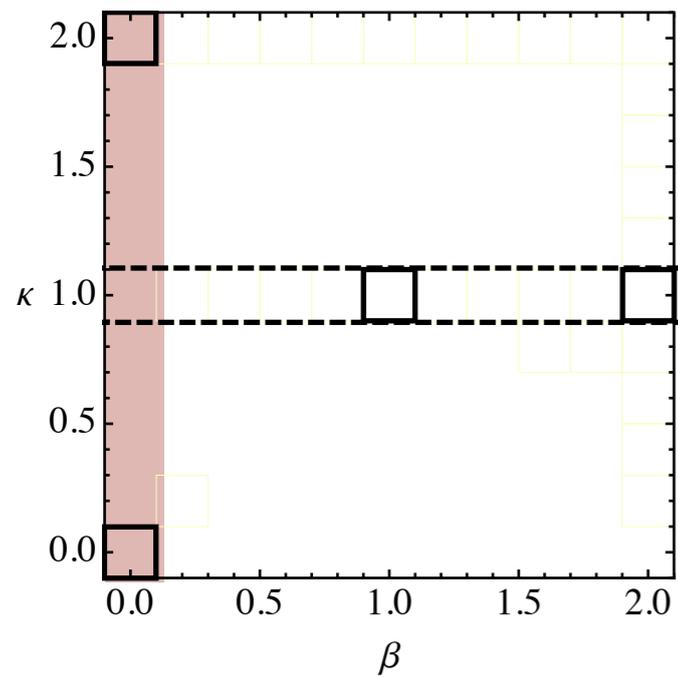
Truth Overlap for Two Angularities



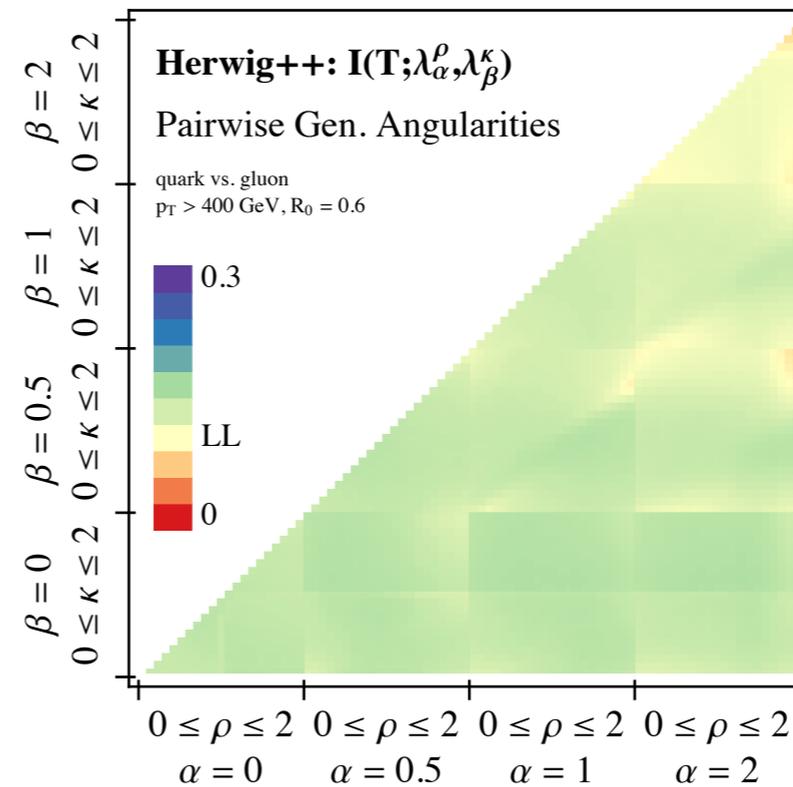
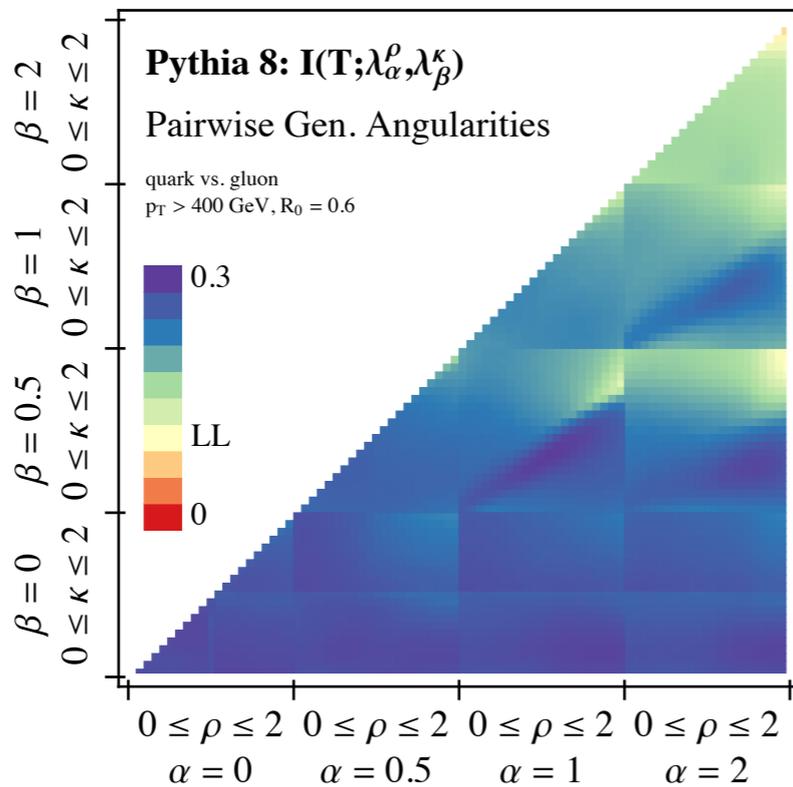
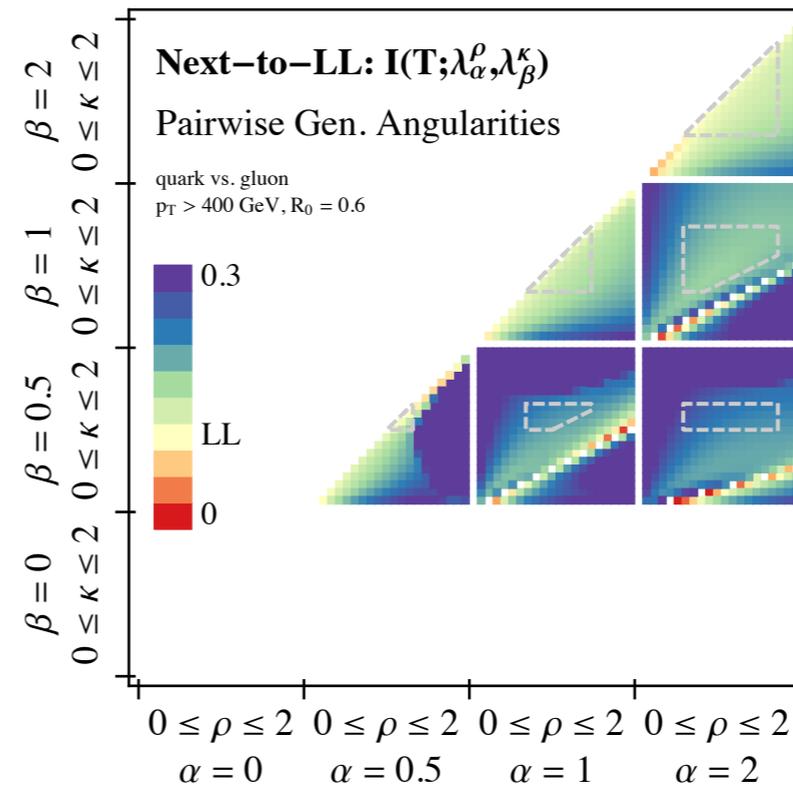
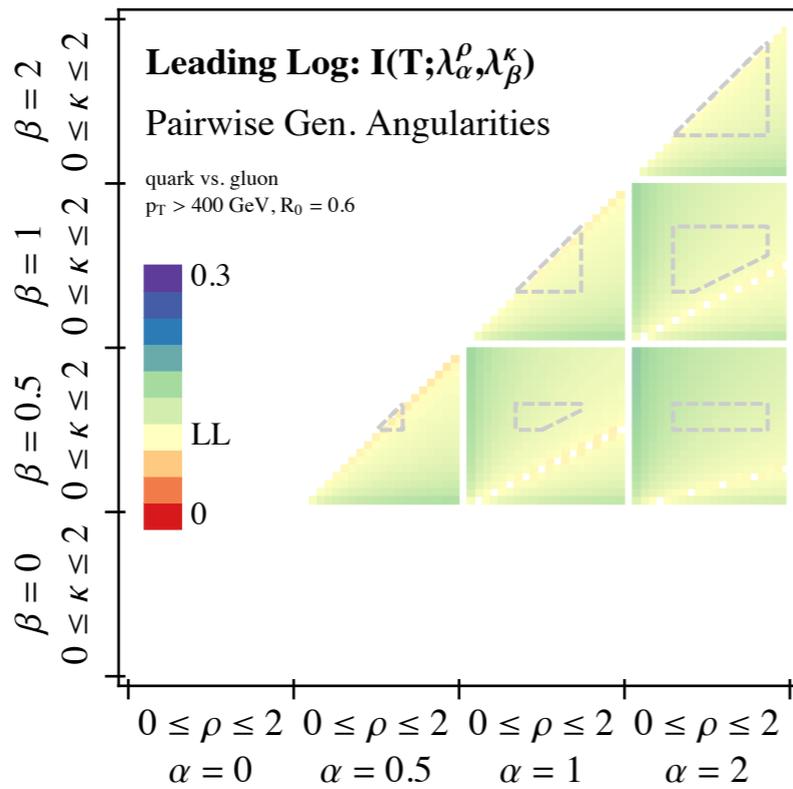
Truth Gain from One \Rightarrow Two Angularities



Truth Overlap for Two Angularities

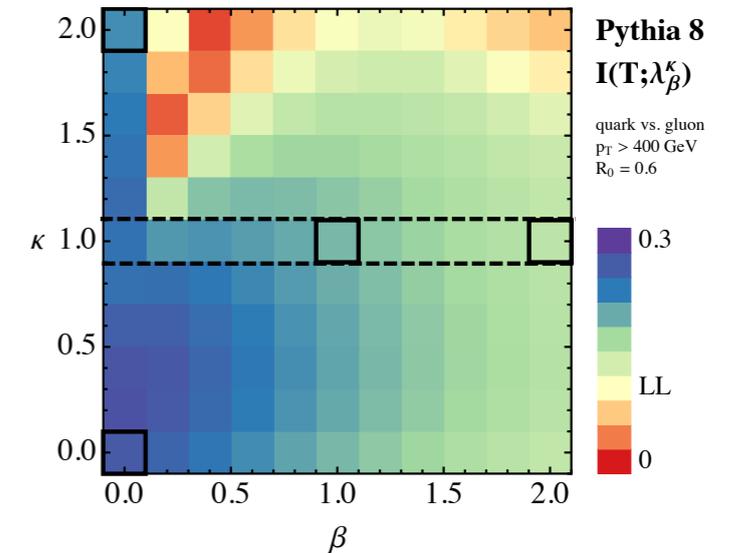
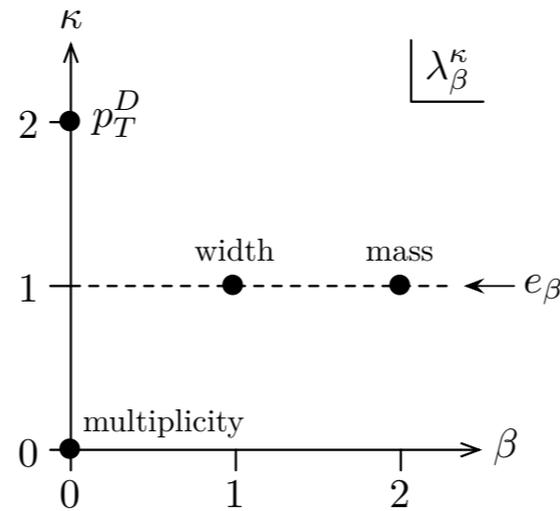
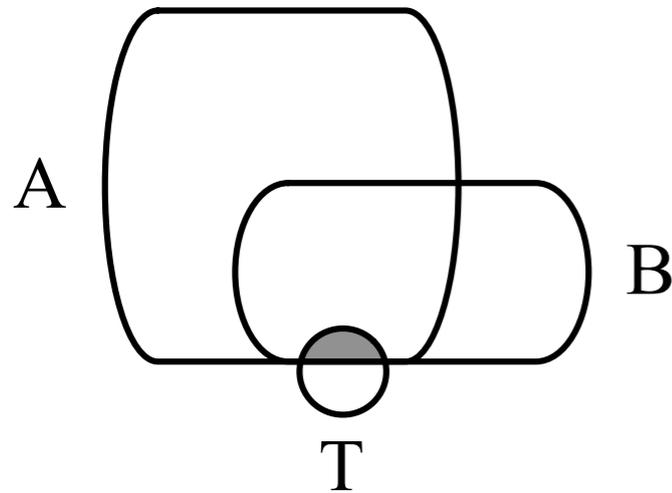


Truth Overlap for Two Angularities



Summary

[Larkoski, JDT, Waalewijn, 1408.3122]



Mutual Information with the Truth

Generalized Angularities

LL vs. NLL vs. Pythia vs. Herwig

An intuitive measure of discrimination power

Interpolates between safe and unsafe observables

Truth overlap subject to large theory uncertainties

Extra information may not be useful information

Ideal playground for analytic calculations

Higher-order calculations and measurements essential

The Future

What is a “Quark/Gluon” Jet?

[see e.g. Banfi, Salam, Zanderighi, hep-ph/0601139]

NLL' + NLO? NNLL? NNLO?

[...]

Non-Global Logarithms?

[Dasgupta, Salam, hep-ph/0104277]

Underlying Event?

[see e.g. Stewart, Tackmann, Waalewijn, 1405.6722]

Hadronization?

[see e.g. Mateu, Stewart, JDT, 1209.3781]

Soft Dropped Observables?

[Larkoski, Marzani, Soyez, JDT, 1402.2657;
see talk by Marzani]

Multiplicity?

[?]

Backup Slides

The Weighted-Energy Function

Definition:

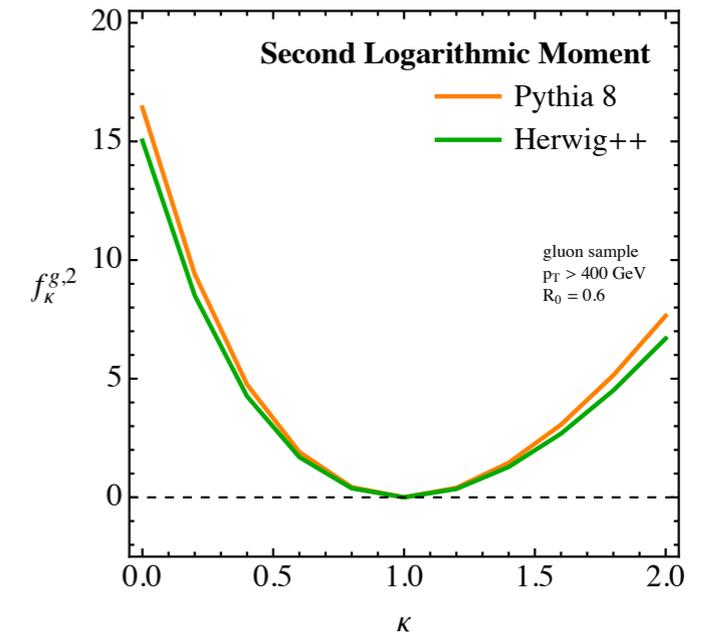
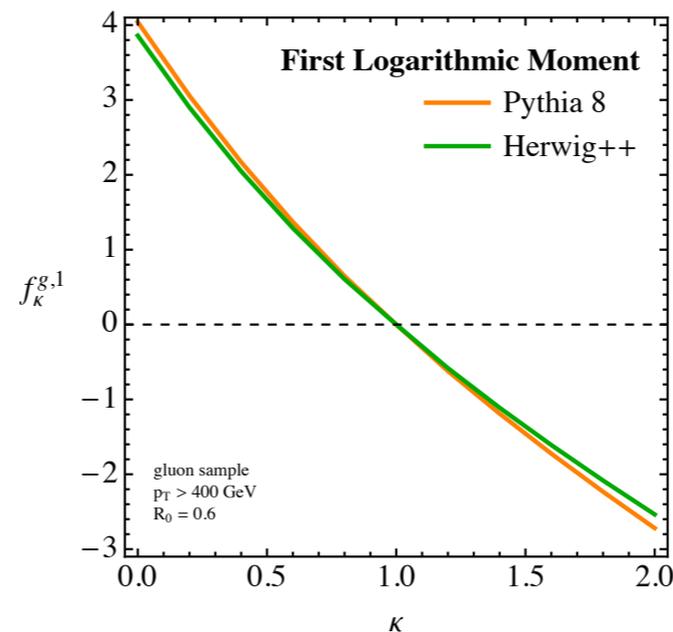
$$F_{\kappa}^g(x, \mu) = \frac{1}{2N_c} \sum_H \delta\left(x - \sum_{h \in H} (z_h)^{\kappa}\right) \times \text{tr}\left[(\gamma^0 + \gamma^3) \langle 0 | [(2\pi)^3 \delta(k^- + \hat{p}^0 + \hat{p}^3) \delta^2(\hat{p}_{\perp}) \psi] | H \rangle \langle H | \bar{\psi} | 0 \rangle\right]$$

Evolution:

$$\mu \frac{\partial}{\partial \mu} F_{\kappa}^i(x, \mu) = \frac{1}{2} \sum_{j,k} \int dz dx_1 dx_2 \frac{\alpha_s}{\pi} P_{i \rightarrow jk}(z) \times F_{\kappa}^j(x_1, \mu) F_{\kappa}^k(x_2, \mu) \delta(x - (1-z)^{\kappa} x_1 - z^{\kappa} x_2)$$

Moments:

$$f_{\kappa}^{g,n} \equiv \int_0^{\infty} dx F_{\kappa}^g(x, \mu) \ln^n x$$



CAESAR Approach to Unsafe Angularities

Master Formula:

$$\frac{1}{\sigma_i} \frac{d\sigma_i}{d\lambda_\beta^\kappa} = \frac{d}{d\lambda_\beta^\kappa} \Sigma_i(\lambda_\beta^\kappa) \quad \Sigma_i(\lambda_\beta^\kappa) = \frac{e^{-\gamma_E R'_i(\lambda_\beta^\kappa)}}{\Gamma(1 + R'_i(\lambda_\beta^\kappa))} e^{-R_i(\lambda_\beta^\kappa)} \quad R'_i \equiv -dR_i/d \ln \lambda_\beta^\kappa$$

Radiator:

$$R_i(\lambda_\beta^\kappa) = \int_0^1 \frac{d\theta}{\theta} \int_0^1 dz \frac{\alpha_s(p_T z \theta)}{\pi} P_{i \rightarrow ig}(z) \int_0^\infty dx F_\kappa^g(x, \mu) \Theta(x z^\kappa \theta^\beta - \lambda_\beta^\kappa)$$

Rescaling Trick:

$$R_i^{\text{NLL}}(\lambda_\beta^\kappa) = \hat{R}_i^{\text{NLL}} \left(e_{\beta/\kappa} = \left(\frac{\lambda_\beta^\kappa}{\exp(f_\kappa^{g,1})} \right)^{1/\kappa} \right)$$

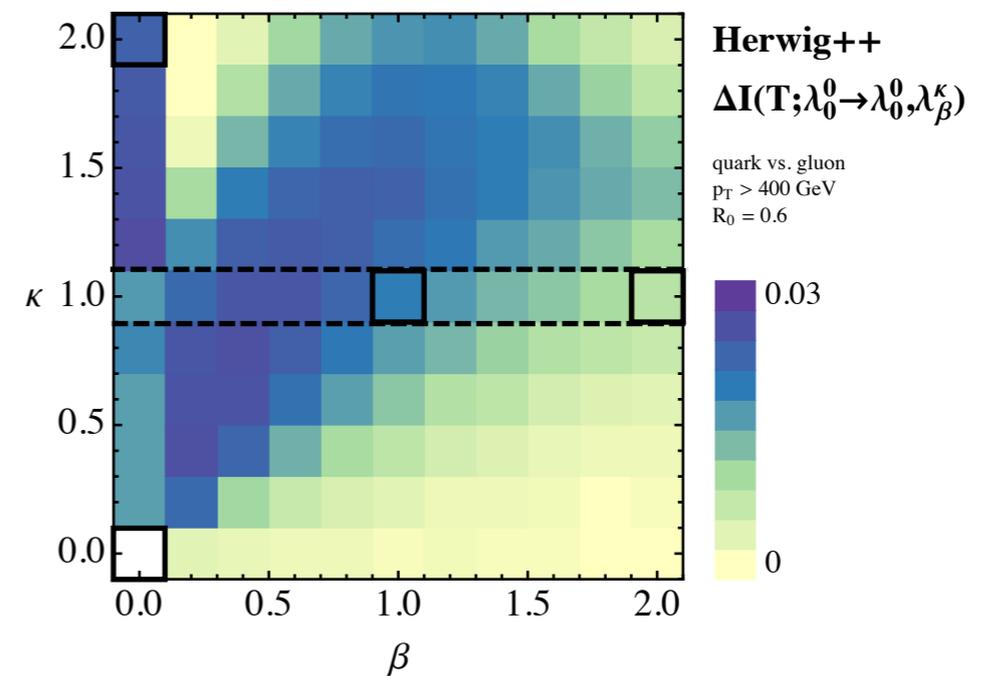
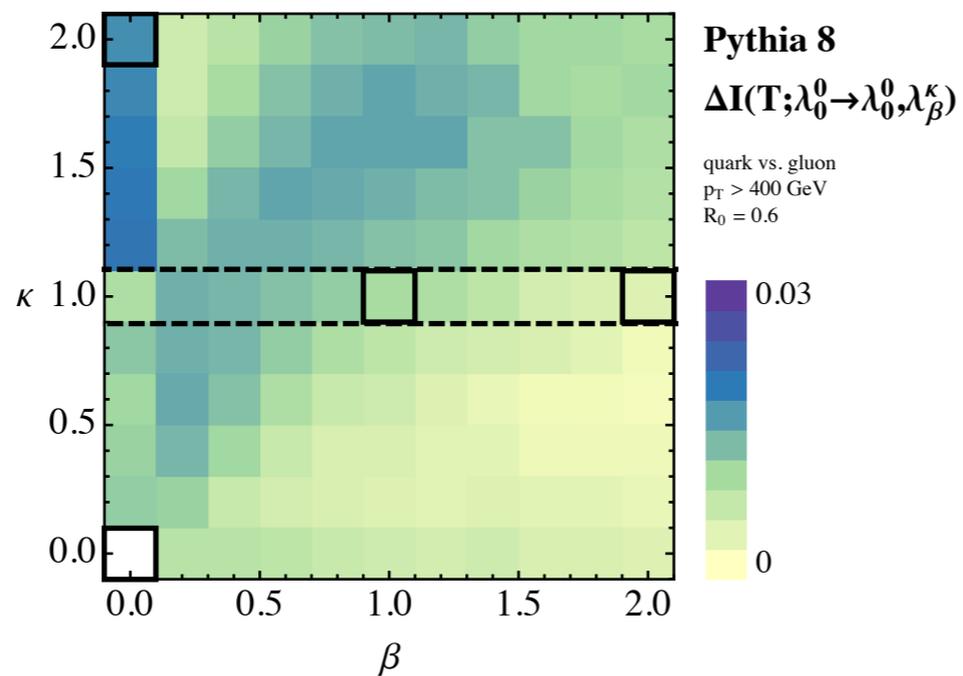
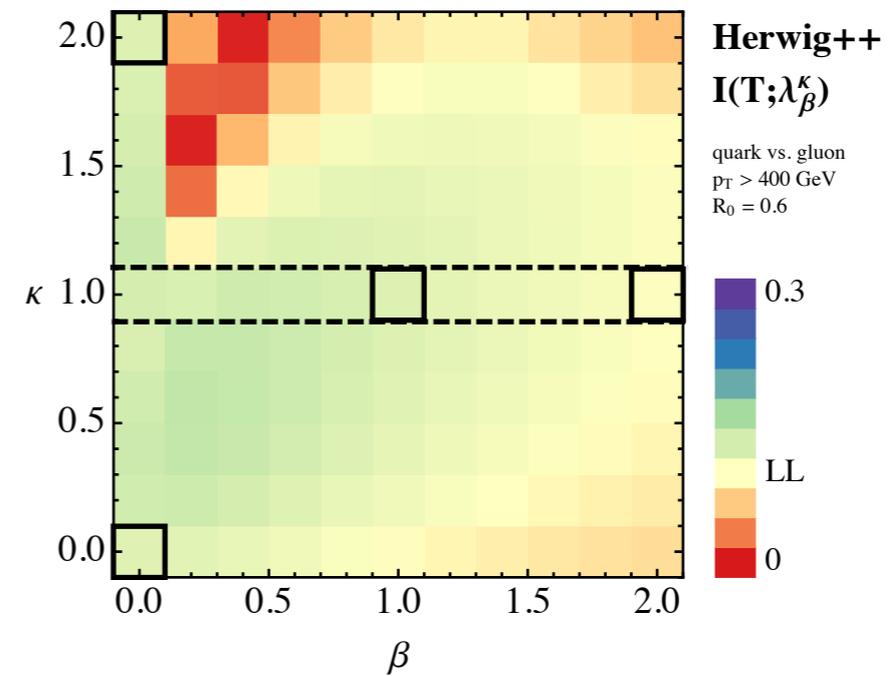
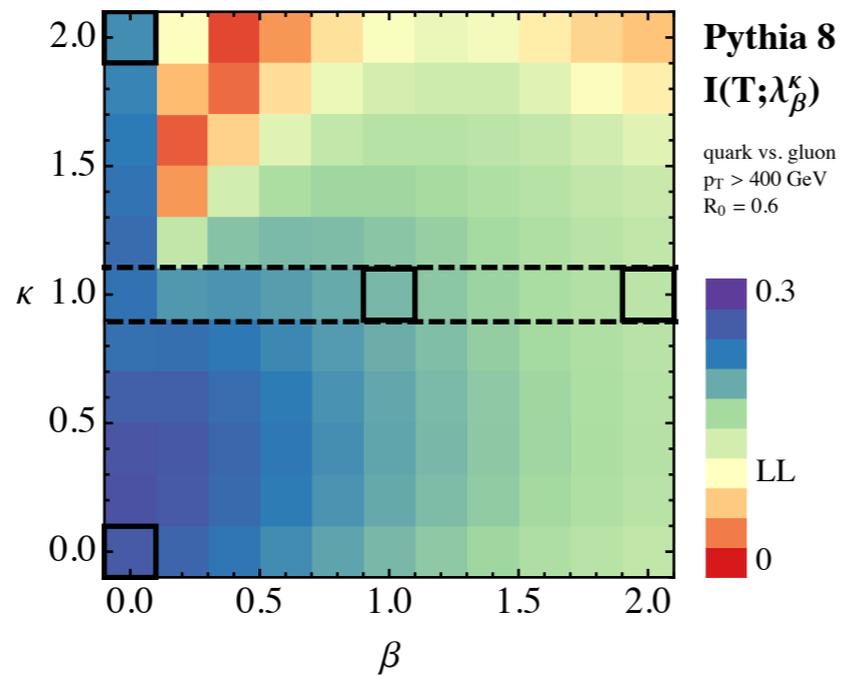
A Deluge of Discriminants

Sure
Huh?!

Gluon Efficiency % at 50% Quark Acceptance	50 GeV				200 GeV			
	Particles		Tracks		Particles		Tracks	
	P8	H++	P8	H++	P8	H++	P8	H++
2-Point Moment $\beta=1/5$	8.7*	17.8*	13.7*	22.8*	8.3	15.9	13.2	19.6
1-Subjettiness $\beta=1/2$	9.3	18.5	14.2	22.9	7.6	16.2	12.3	19.4
2-Subjettiness $\beta=1/2$	9.2	18.6	13.9	23.6	6.8	15.7*	9.8	18.7*
3-Subjettiness $\beta=1$	9.1	19.3	14.6	24.4	5.9*	16.7	8.6*	19.5
Radial Moment $\beta=1$ (Girth)	10.3	20.5	16.1	24.9	11.2	18.9	15.3	21.9
Angularity $a = +1$	10.3	20.0	15.8	24.5	12.0	19.3	14.0	21.6
Det of Covariance Matrix	11.2	21.2	18.1	27.0	9.4	20.9	13.5	24.6
Track Spread: $\sqrt{\langle p_T^2 \rangle} / p_T^{\text{jet}}$	16.5	25.3	16.5	25.3	9.3	20.1	9.3	20.1
Track Count	17.7	26.4	17.7	26.4	8.9	21.0	8.9	21.0
Decluster with $k_T, \Delta R$	15.8	24.5	20.1	28.4	13.9	20.1	16.9	23.4
Jet m/p_T for R=0.3 subjet	13.1	25.9	16.3	27.7	11.9	24.2	14.8	26.2
Planar Flow	28.7	34.4	28.7	34.4	39.6	42.9	39.6	42.9
Pull Magnitude	37.0	39.0	32.9	35.6	30.6	30.2	29.6	30.6
Track Count & Girth	9.9	20.1	13.4	23.2	7.1	17.3	7.7*	18.7
R=0.3 m/p_T & R=0.7 2-Pt $\beta=1/5$	7.9*	17.7	12.2*	22.1	5.7	14.4*	8.5	17.9
1-Subj $\beta=1/2$ & R=0.7 2-Pt $\beta=1/5$	8.5	17.3*	12.9	22.1	6.0	14.6	8.6	17.7*
Girth & R=0.7 2-pt $\beta=1/10$	12.6	21.9	12.6	21.9*	9.2	18.0	9.2	18.0
1-Subj $\beta=1/2$ & 3-Subj $\beta=1$	8.9	18.0	14.0	23.2	5.6*	15.0	8.4	18.4
Best Group of 3	7.5	17.0	11.0	20.9	4.7	14.0	6.9	16.6
Best Group of 4	7.1	16.7	10.6	20.5	4.5	13.7	6.2	16.3
Best Group of 5	6.9	16.4	10.4	20.0	4.3	13.3	6.1	15.9

[Gallicchio, Schwartz, 1106.3076, 1211.7038]

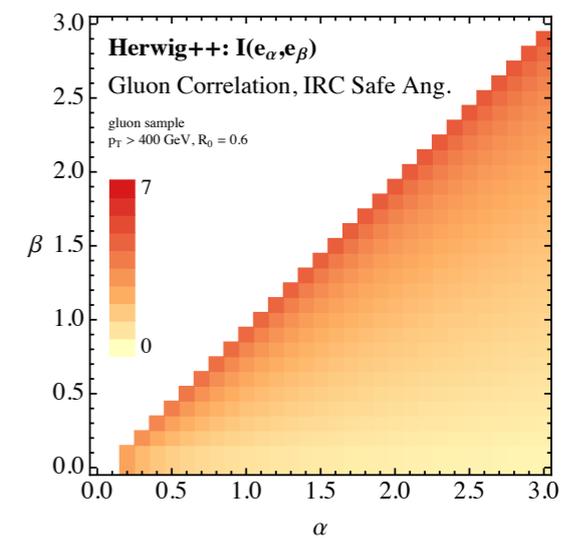
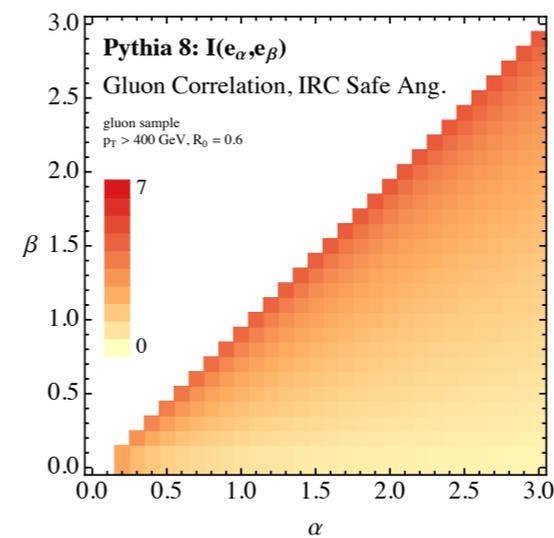
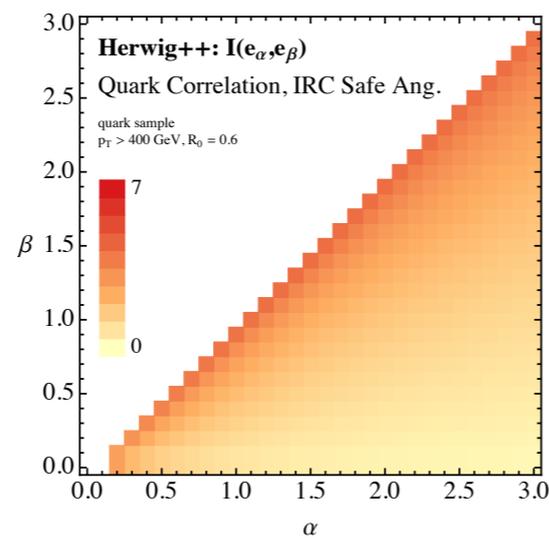
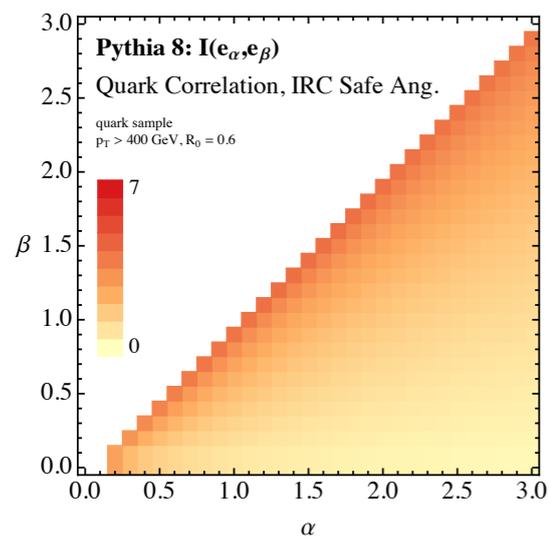
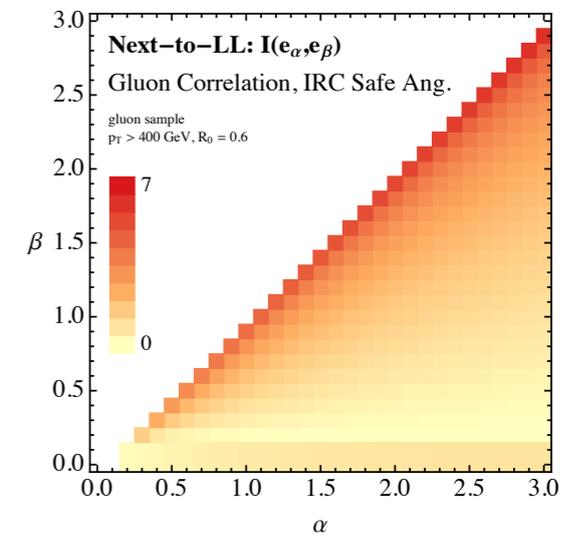
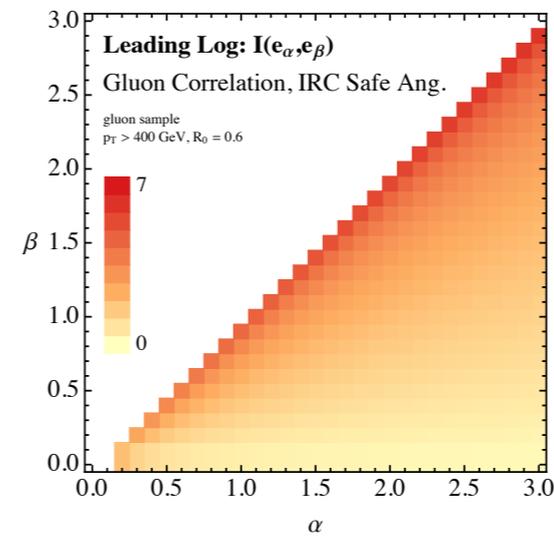
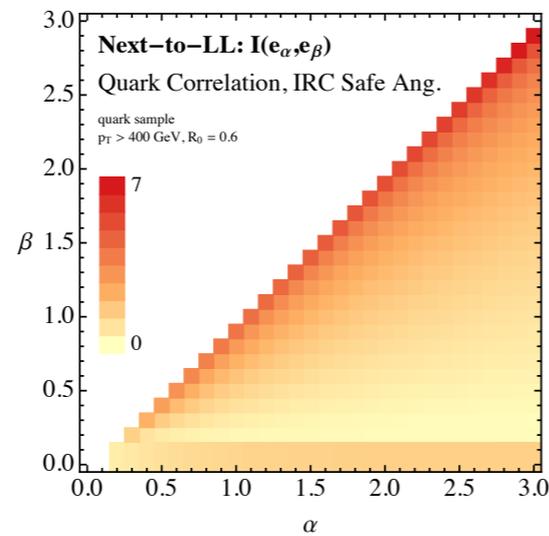
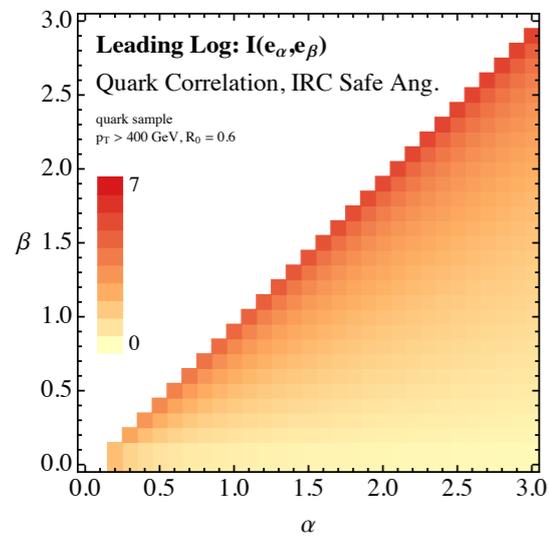
More Pythia vs. Herwig



Better Agreement on Correlations

Quark

Gluon



Raw Distributions

