

$B \rightarrow K^* \mu \mu$ in the SM and Beyond

Patricia Ball

in collaboration with W. Altmannshofer, A. Buras, D. Straub, M. Wick (TU München)
and A. Bharucha (IPPP Durham)

based on [arXiv:0811.1214](https://arxiv.org/abs/0811.1214)

Lancaster, Nov 12 2008



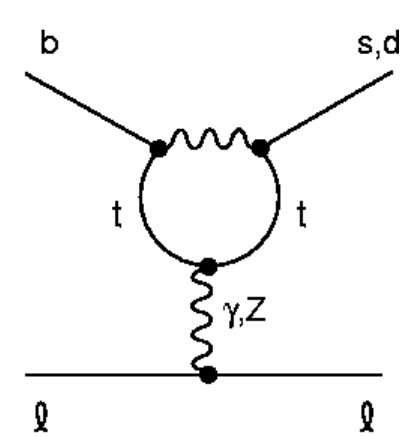
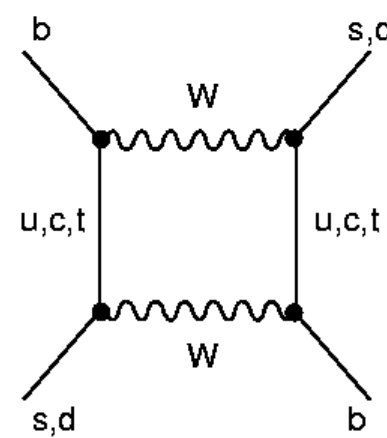
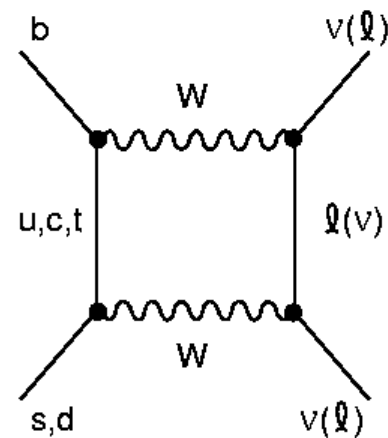
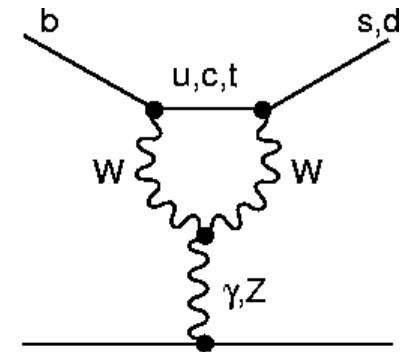
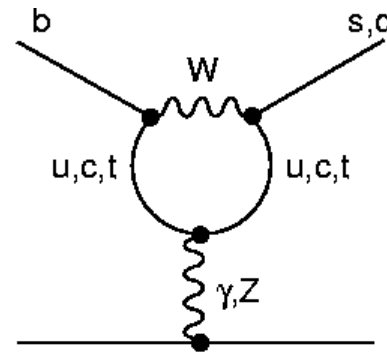
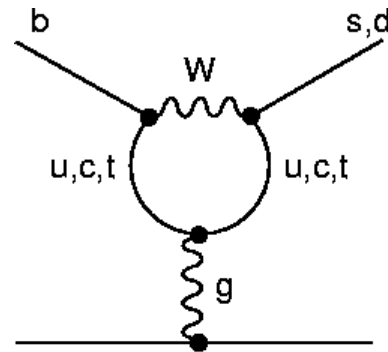
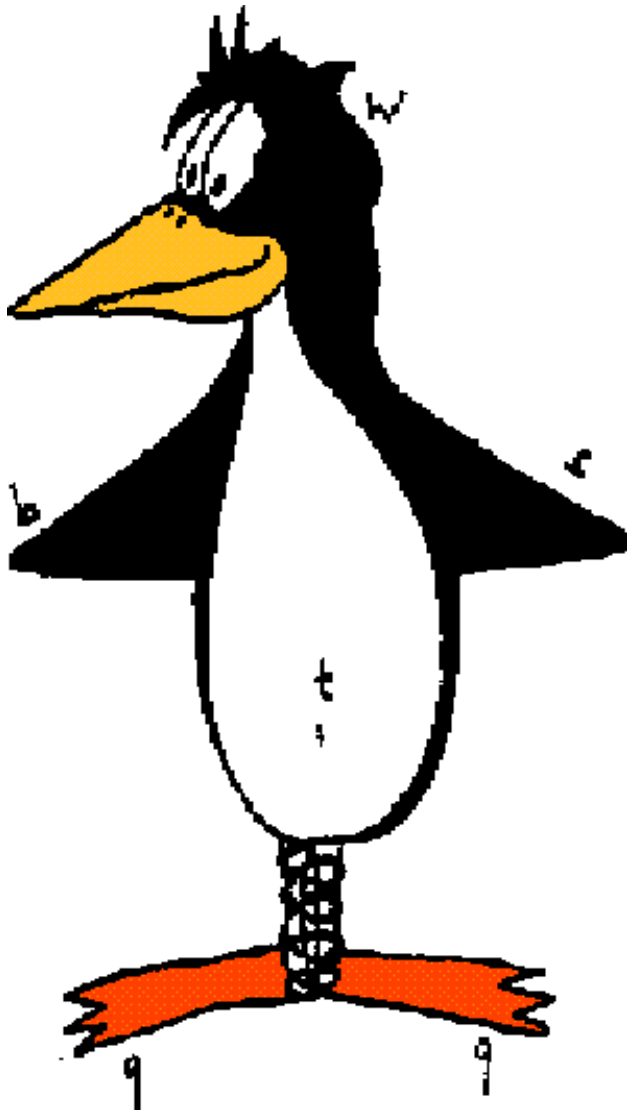
Anything wrong with $b \rightarrow s$?

- time-dependent CP asymmetry in $b \rightarrow s$: $S_{\Phi K} \neq S_{J/\psi K}$
- hints at large B_s mixing phase from Tevatron

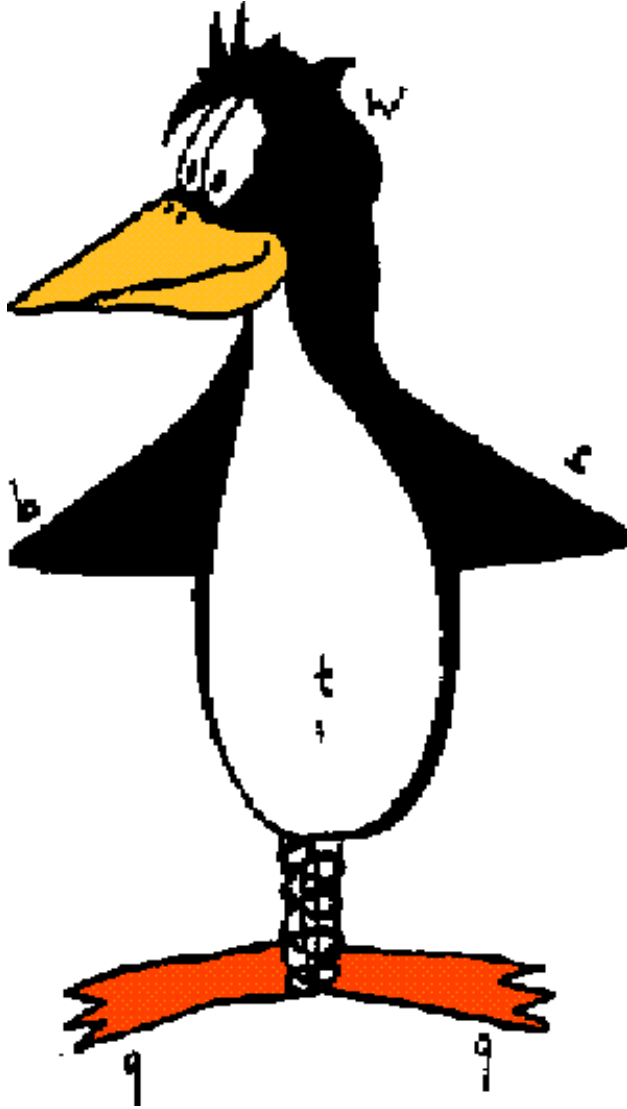
If there are BSM sources for the above, they will be found at the LHC:

- B_s mixing phase
- $B_s \rightarrow \mu^+ \mu^-$
- $B \rightarrow K^* \mu^+ \mu^-$

$B \rightarrow K^* \mu\mu$ – Diagrams



$B \rightarrow K^* \mu\mu$ – Diagrams



- $b \rightarrow sl^+l^-$ is **FCNC** (flavour-changing neutral current) – forbidden at tree-level in the SM
- in the SM induced by one-loop penguin and one-loop box diagrams
- new physics also at one loop (e.g. MSSM) or possibly tree-level (e.g. FC neutral Higgs) [heavily suppressed]
- related decay $b \rightarrow s\gamma$ extensively studied at B factories
 - also FCNC, but with less sensitivity to BSM than $b \rightarrow sl^+l^-$ (less operators in effective Hamiltonian, less observables)
 - difficult at the LHC because of γ ; $\mu^+\mu^-$ has better detection efficiency

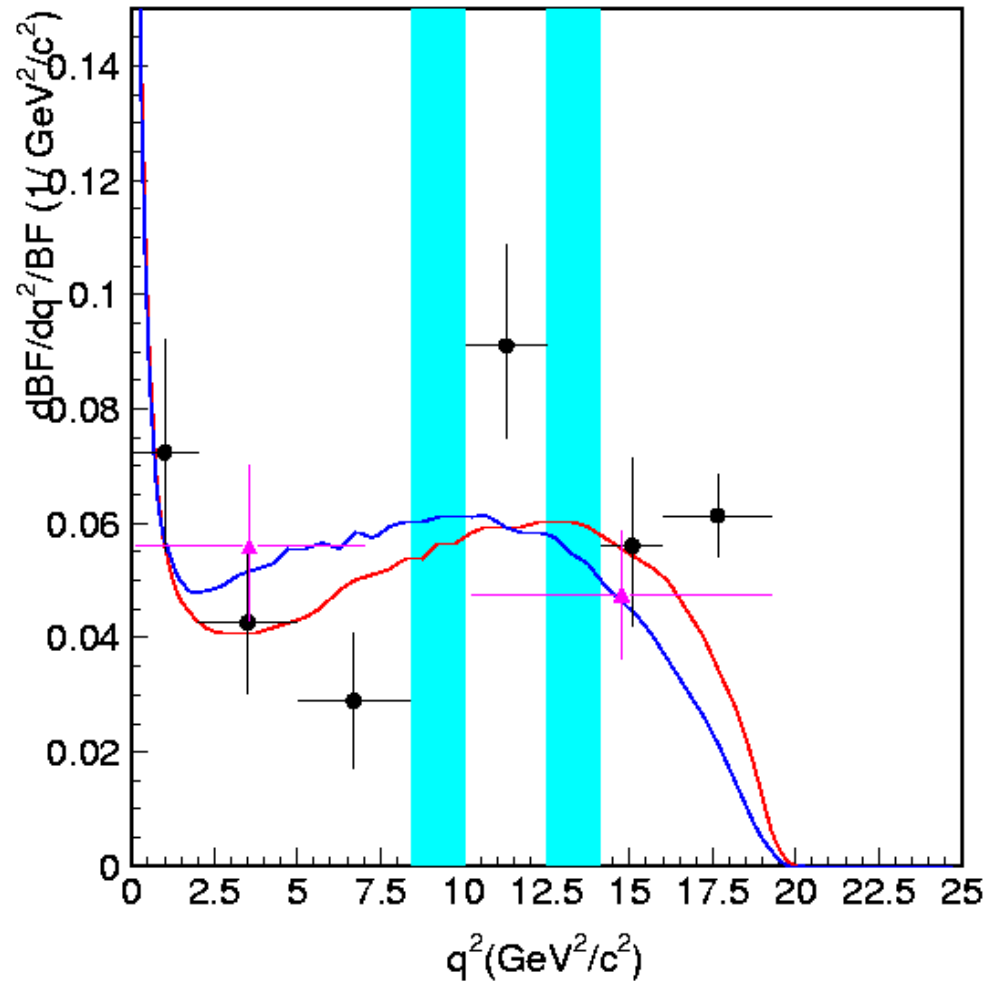
$B \rightarrow K^* \mu\mu$ – Data

Current status (ICHEP 08) at B factories and Tevatron:

Experiment	BaBar	Belle	CDF
$\mathcal{B}(B \rightarrow K^* \mu^+ \mu^-) \times 10^7$	$11.1 \pm 1.9 \pm 0.7$	$10.8_{-1.0}^{+1.0} \pm 0.9$	$8.1 \pm 3.0 \pm 1.0$
# of events	? (ca. 1/2 of Belle)	230(!)	?

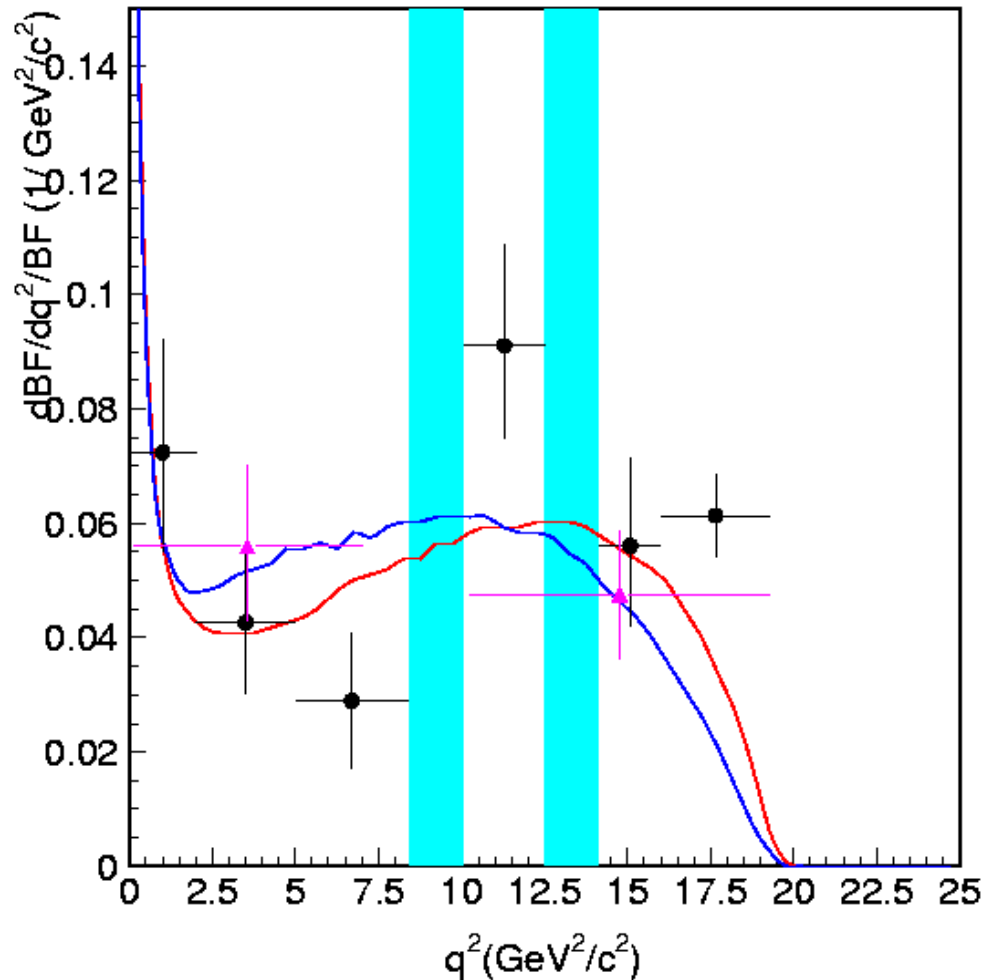
$B \rightarrow K^* \mu\mu$ – Data

Current (Belle) measurement of dilepton mass spectrum (230 events):

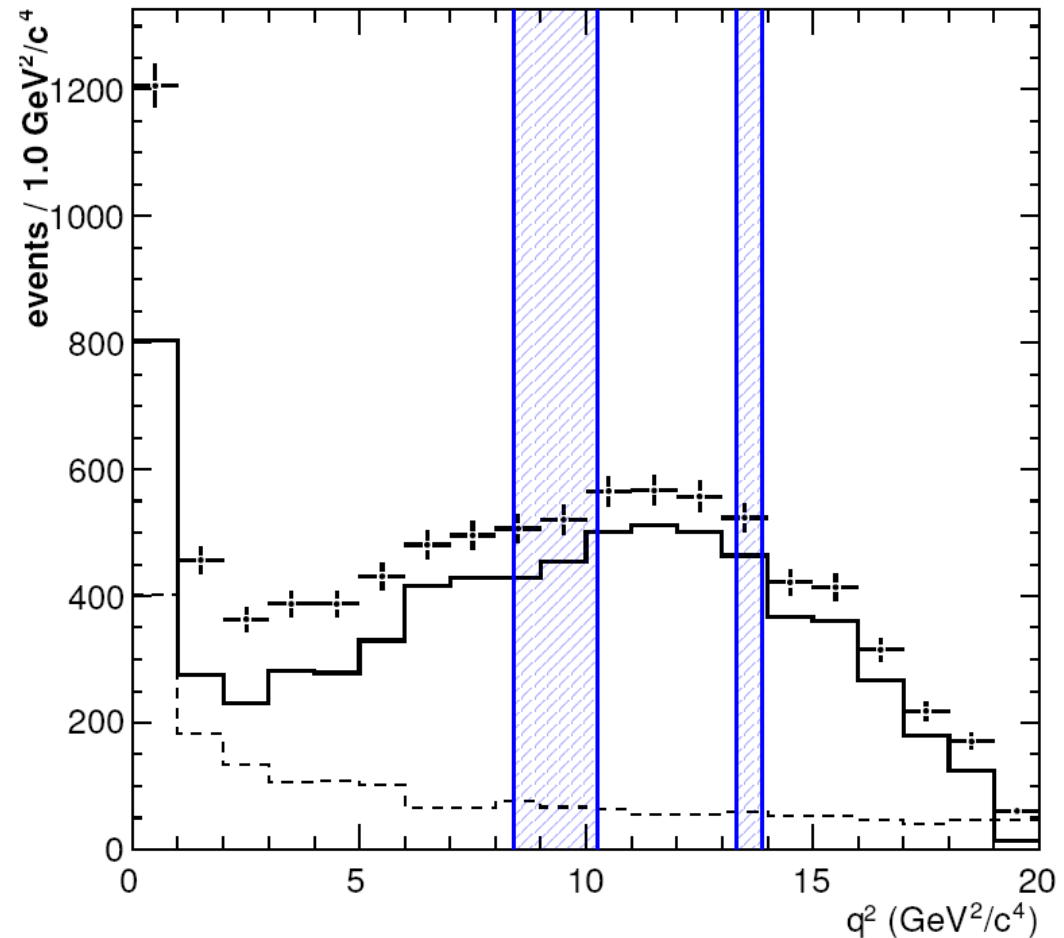


$B \rightarrow K^* \mu\mu$ – Data

Current (Belle) measurement of dilepton mass spectrum (230 events):



Projection: LHCb with 7200 events (2 fb^{-1} , i.e. 1y of running)



What are the relevant observables?

The usual suspects:

- branching ratio
- dilepton mass spectrum
- forward-backward asymmetry

Anything else?

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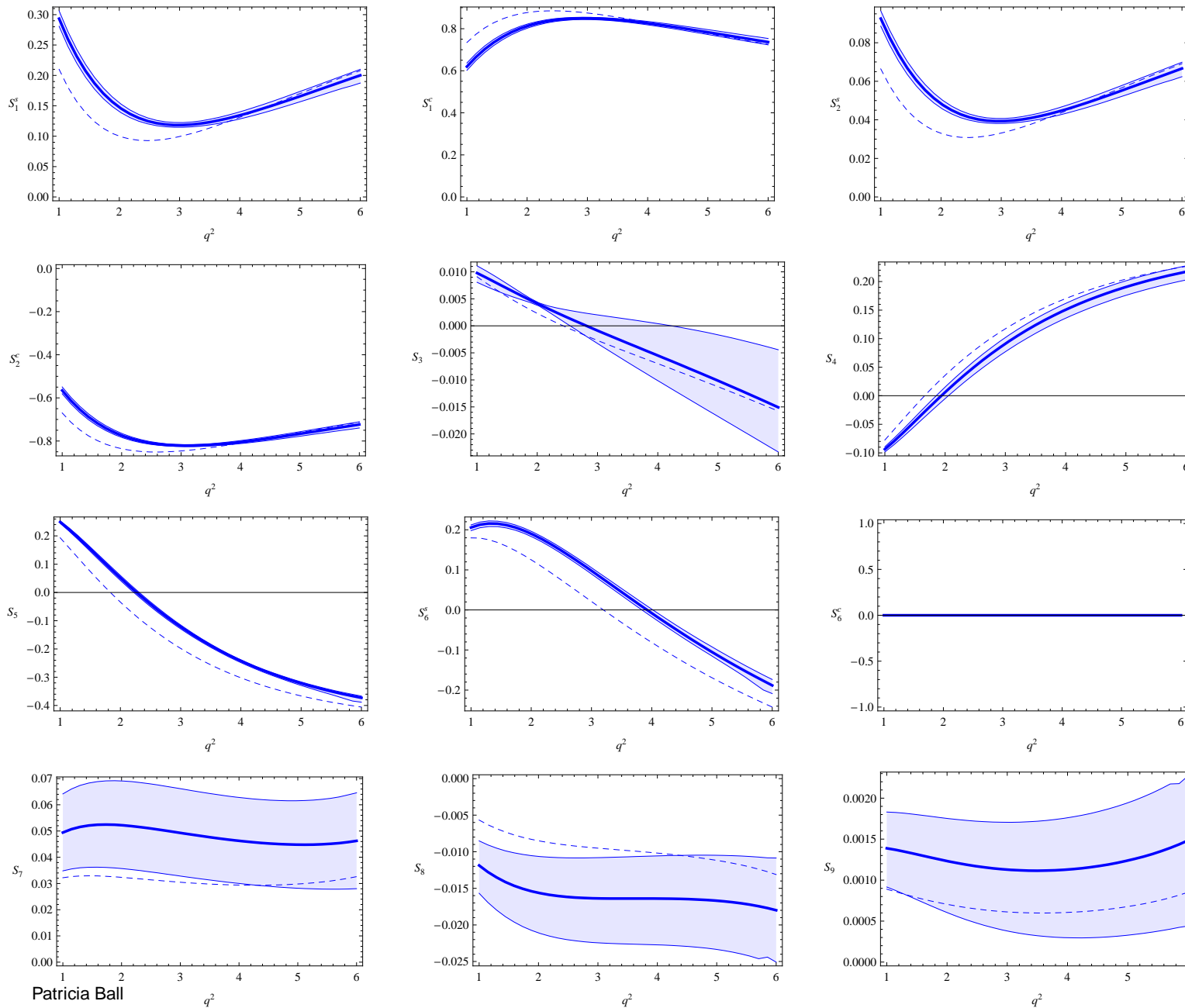
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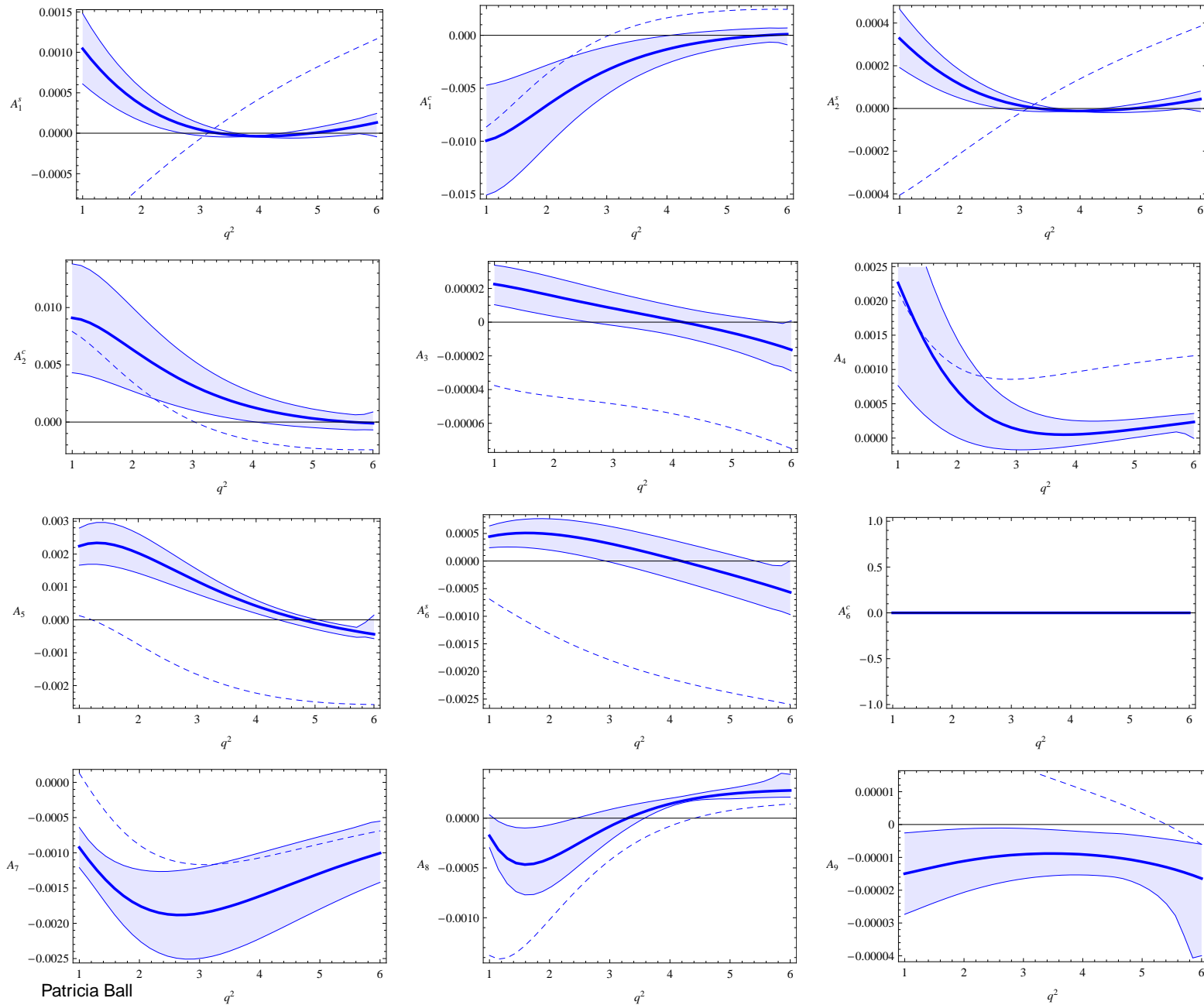
More than you think!

What are the relevant observables?



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- how to define observables
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OK, let's get serious...

The big questions:

- how to calculate $B \rightarrow K^* \mu^+ \mu^-$ decays → this talk
- how to define observables → this talk
- how to find new physics → next talk (A. Bharucha)

How to calculate

The B Physicist's Toolbox

For exclusive decays:

- effective field theories
 - disentangle physics governed by different mass scales (SM: $m_W, m_t \gg m_b \gg \Lambda_{\text{QCD}}$)
 - evolution between scales governed by renormalisation group equations (known to 3-loop accuracy for $b \rightarrow sl^+l^-$)

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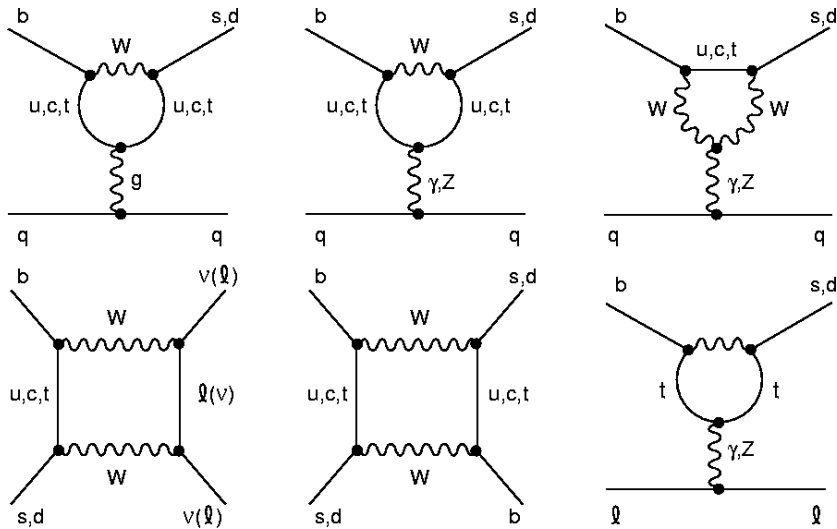
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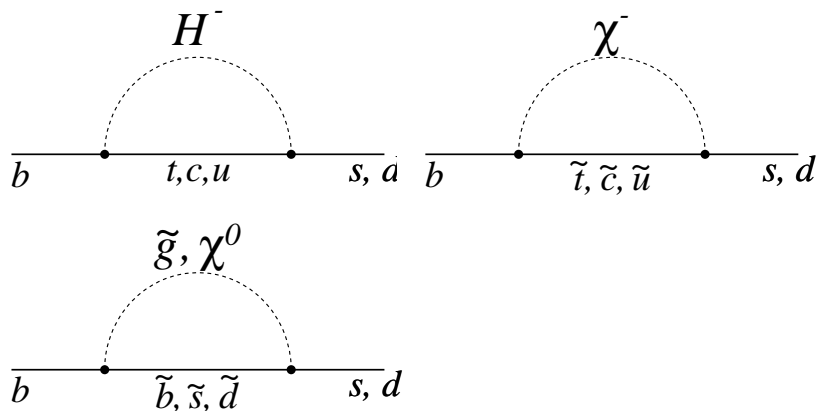
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- hadronic matrix elements (\leftrightarrow form factors): nonperturbative methods (lattice, QCD sum rules on the light-cone)
- hard spectator effects etc.: QCD factorization

The Effective Hamiltonian



Possible (MSSM): BSM diagrams



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- calculate diagrams in full theory (SM or BSM)

- match to effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \times \text{CKM} \times \sum_i C_i(\mu) \mathcal{O}_i(\mu)$$

with Wilson coefficients C_i and (e.g. 4-quark) operators \mathcal{O}_i

- μ is renormalisation scale. Matching done at $\mu \approx m_W$. For B physics, need to know C_i at $\mu \approx m_b$: solve RG equations

$$\mu \frac{d}{d\mu} \vec{C}(\mu) = \gamma^T \vec{C}(\mu)$$

- anomalous dimensions γ known to 3-loop accuracy (Gorbahn/Haisch/Misiak 04+05)
- initial conditions $C_i(m_W)$ known to 2-loop accuracy (Bobeth/Misiak/Urban 99)

Form Factors

$\langle K^* | (V - A)_\mu | B \rangle$ expressed in terms of $A_{0,1,2}$ and V .

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Want to know more?

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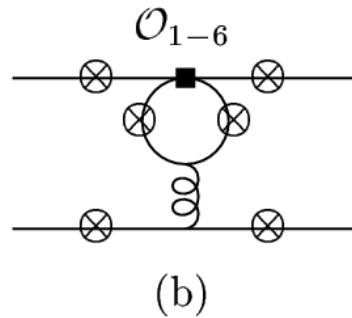
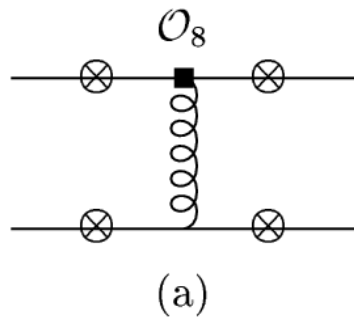
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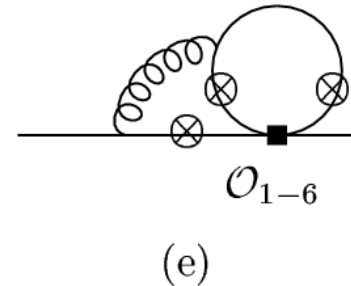
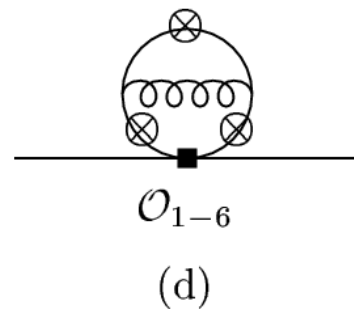
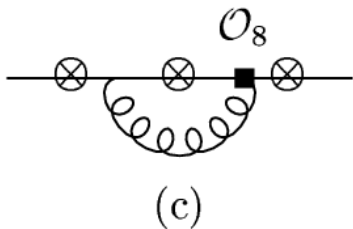


QCD Factorisation

(Beneke/Feldmann/Seidel 01+04)



⊗ = emission of photon



Include matrix elements of operators which are not related to form factors.

- mostly $O(\alpha_s)$ corrections
- plus weak annihilation: diagrams with quarks in B meson annihilating

How we differ from our competitors

Recent analyses: Bobeth/Hiller/Piranishvili 08, Egede et al. 08

We use:

- full set of 7 form factors with correlated errors
(instead of 2 form factors in heavy quark limit)
- full analysis of *all* observables
- large set of BSM models

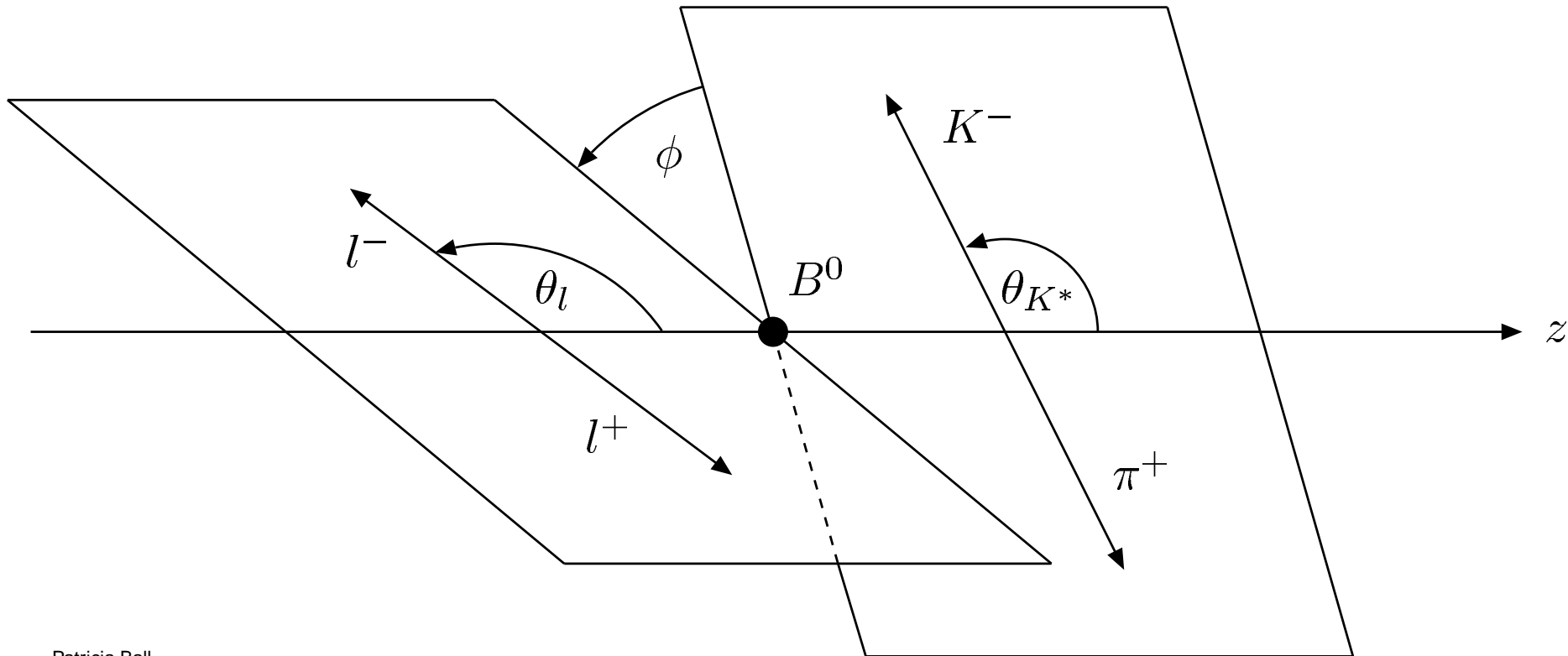
How to define observables

Kinematics

K^* decays to $K\pi$ ($\approx 100\%$). Angular distribution of $K\pi$ indicative for polarisation of K^* .

Full angular spectrum:

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_{K^*} d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_{K^*}, \phi)$$



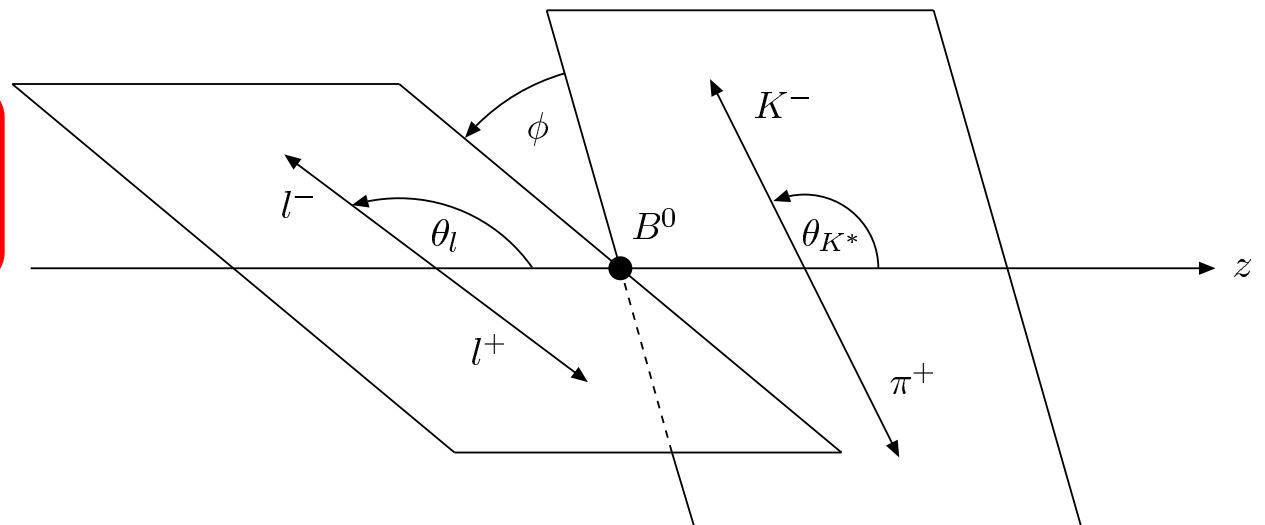
Kinematics

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_{K^*} d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_{K^*}, \phi)$$

$$\begin{aligned} I(q^2, \theta_l, \theta_{K^*}, \phi) = & I_1^s \sin^2 \theta_{K^*} + I_1^c \cos^2 \theta_{K^*} + (I_2^s \sin^2 \theta_{K^*} + I_2^c \cos^2 \theta_{K^*}) \cos 2\theta_l \\ & + I_3 \sin^2 \theta_{K^*} \sin^2 \theta_l \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos \phi \\ & + I_5 \sin 2\theta_{K^*} \sin \theta_l \cos \phi \\ & + (I_6^s \sin^2 \theta_{K^*} + I_6^c \cos^2 \theta_{K^*}) \cos \theta_l + I_7 \sin 2\theta_{K^*} \sin \theta_l \sin \phi \\ & + I_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_{K^*} \sin^2 \theta_l \sin 2\phi. \end{aligned}$$

I_j : experimental observables: **angular correlations**

→ complete information on $B \rightarrow K^* \mu \mu$



I_j and Form Factors/Wilson Coefficients

Relation between **observables** I_j and $\langle K^* \mu\mu | \mathcal{H}_{\text{eff}} | B \rangle$ given in terms of **transversity amplitudes** $A_{\parallel, \perp, 0, t}$, e.g.

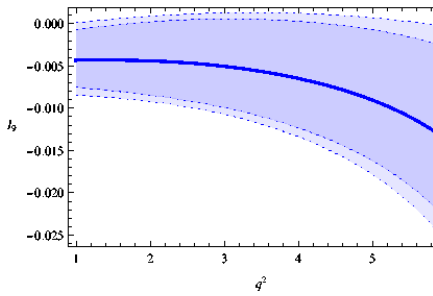
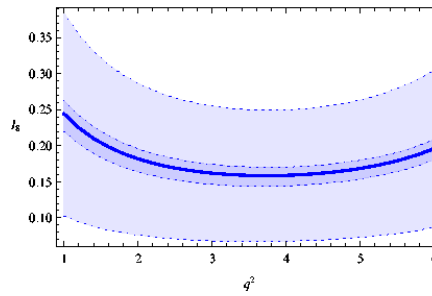
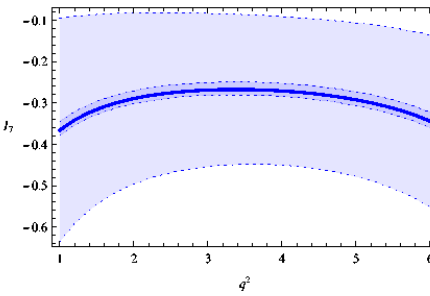
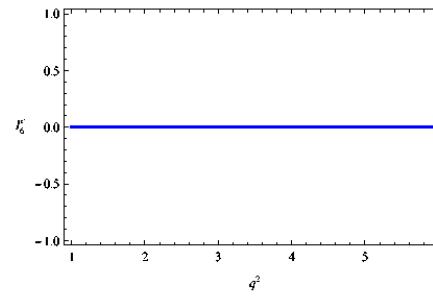
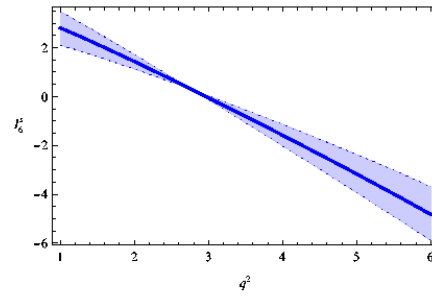
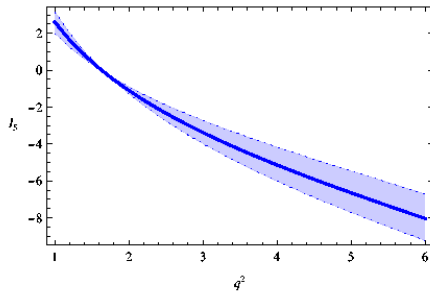
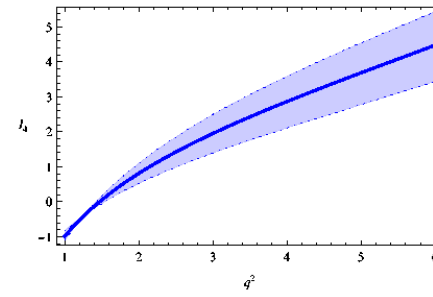
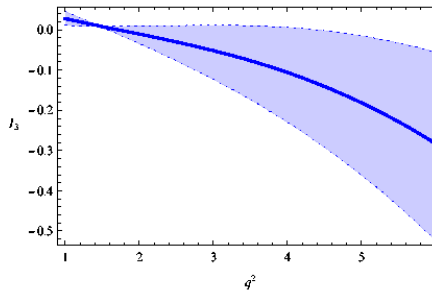
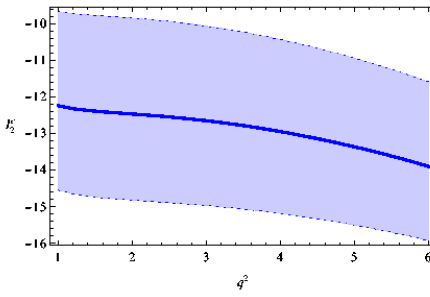
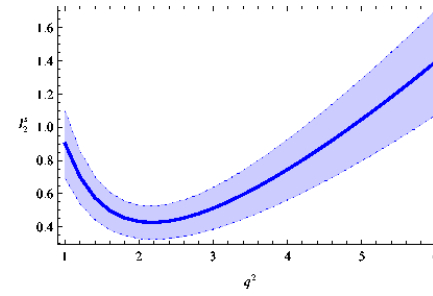
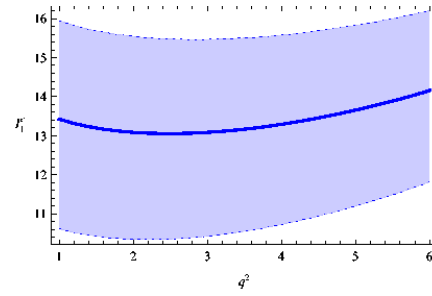
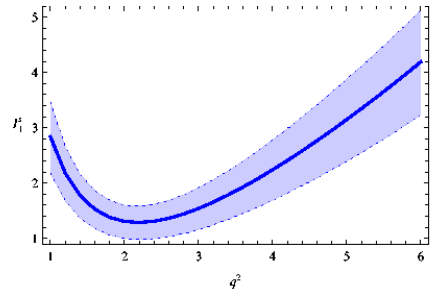
$$I_6^s = 2\beta_\ell \left[\text{Re}(A_{\parallel}^L A_{\perp}^{L*}) - (L \rightarrow R) \right],$$

with

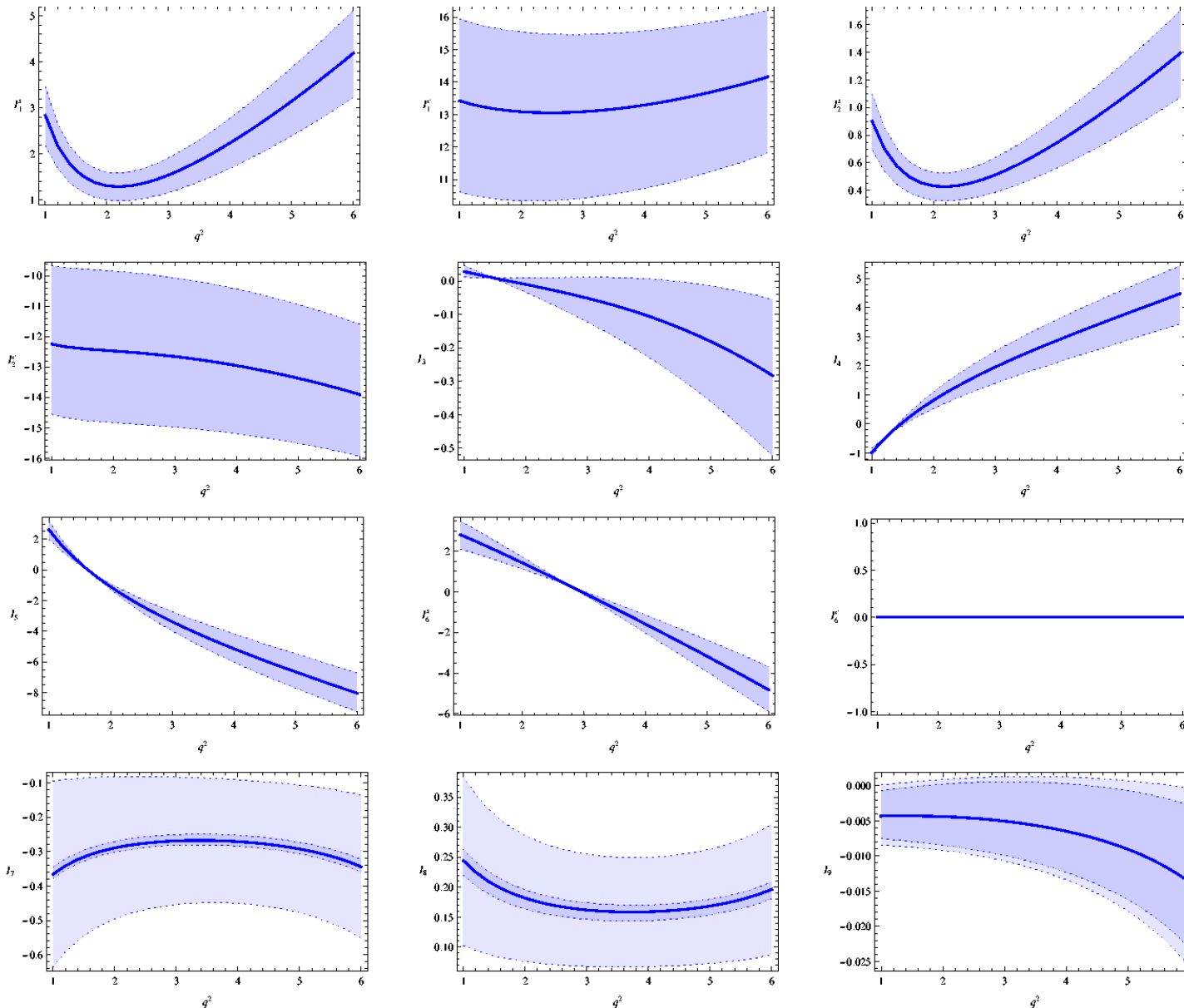
$$A_{\perp L, R} = N\sqrt{2}\lambda^{1/2} \left[\left[(C_9^{\text{eff}} + C_9^{\text{eff}'}) \mp (C_{10} + C'_{10}) \right] \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} (C_7^{\text{eff}} + C_7^{\text{eff}'}) T_1(q^2) \right]$$

$$A_{\parallel L, R} = -N\sqrt{2}(m_B^2 - m_{K^*}^2) \left[\left[(C_9^{\text{eff}} - C_9^{\text{eff}'}) \mp (C_{10} - C'_{10}) \right] \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2} (C_7^{\text{eff}} - C_7^{\text{eff}'}) T_2(q^2) \right]$$

A few comments on I_j



A few comments on I_j



- I_j contain **complete information** to be extracted from the decay
- for massless leptons, $I_1^s = 3I_2^s$ and $I_1^c = -I_2^c$
- $I_6^c = 0$ in SM; a non-zero value is due to **contributions from scalar operators** and $m_l \neq 0$. Potentially good observable for **extended Higgs sector!**

Observables

- order principle: behaviour under **CP trafos**: I_j from b (\bar{B}^0) decay, \bar{I}_j from \bar{d} (B^0) decay
 - CP-even (CP-averaged) $I_j + \bar{I}_j$, CP-odd $I_j - \bar{I}_j$
- normalize by $d(\Gamma + \bar{\Gamma})/dq^2$ (q^2 : invariant dilepton mass)
- **new standard observables:** symmetries S_j and asymmetries A_j :

$$S_j = \frac{I_j + \bar{I}_j}{d(\Gamma + \bar{\Gamma})/dq^2}, \quad A_j = \frac{I_j - \bar{I}_j}{d(\Gamma + \bar{\Gamma})/dq^2}$$

Taking ratios reduces theory errors!

- advantage symmetries: increased statistics
- advantage asymmetries: sensitivity to new CP-violating phases induced by BSM (all A_j very close to 0 in SM)

Relation to Known Observables

Forward-backward asymmetry:

$$A_{\text{FB}} = \frac{3}{8} (2S_6^s + S_6^c)$$

Transverse asymmetries: (Krüger/Matias)

$$A_T^{(2)} = \frac{S_3}{2 S_2^s}$$

$$A_T^{(3)} = \left(\frac{4 S_4^2 + S_7^2}{-2 S_2^c (2 S_2^s + S_3)} \right)^{1/2}$$

$$A_T^{(4)} = \left(\frac{S_5^2 + 4 S_8^2}{4 S_4^2 + S_7^2} \right)^{1/2}$$

Note: while all S_i and A_i are well-behaved, the $A_T^{(i)}$ can, in principle, have (highly-theory dependend) peaks from small denominators.

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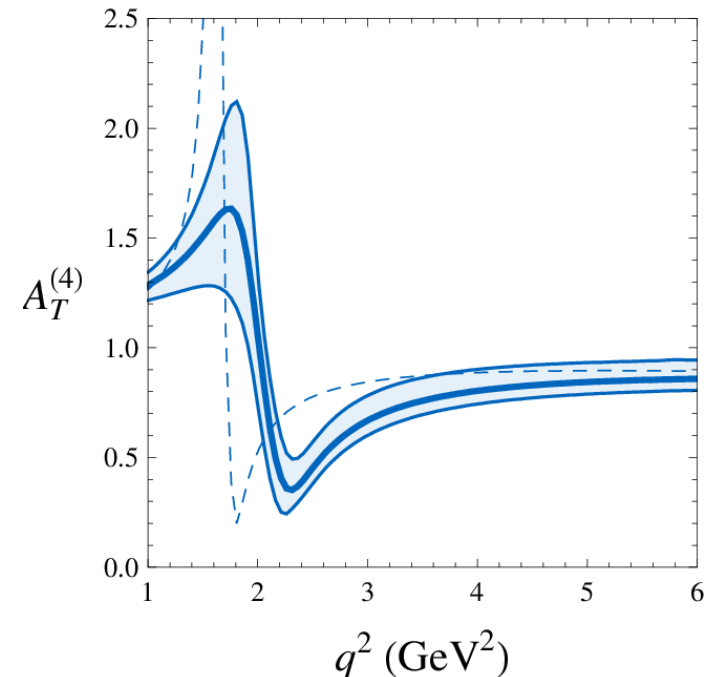
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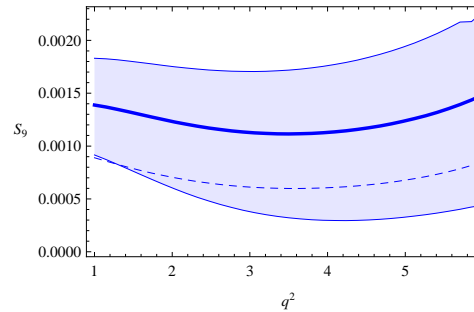
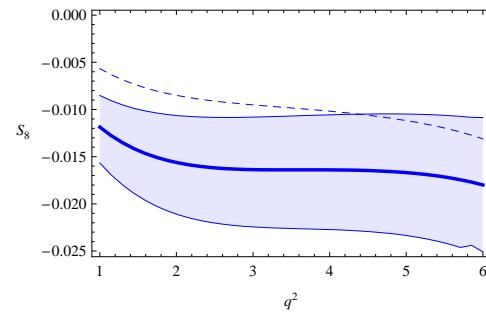
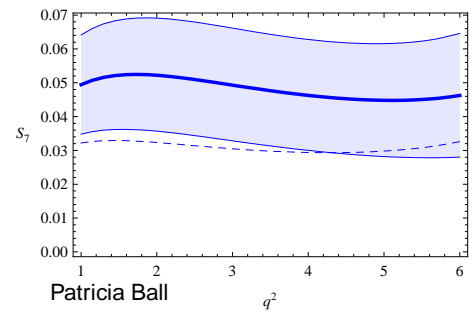
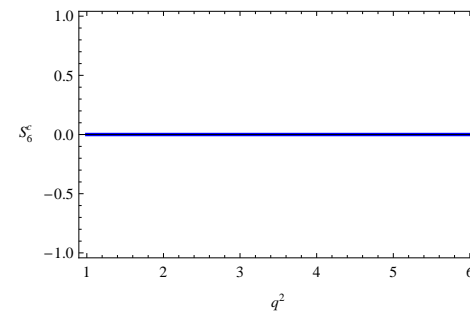
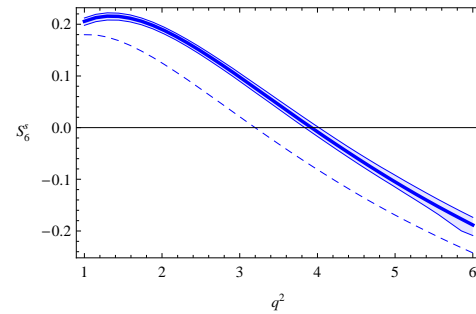
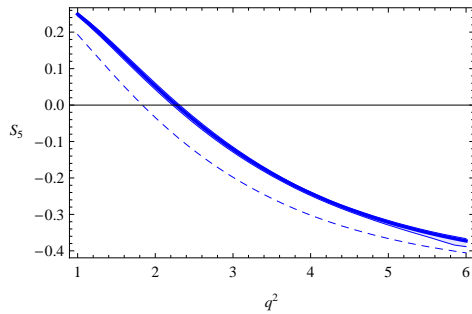
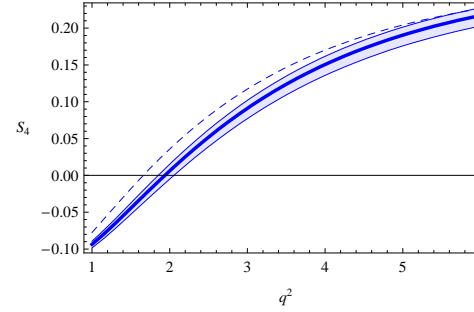
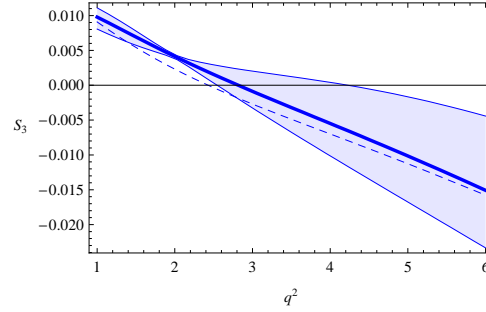
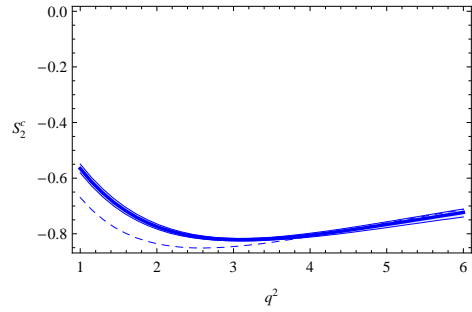
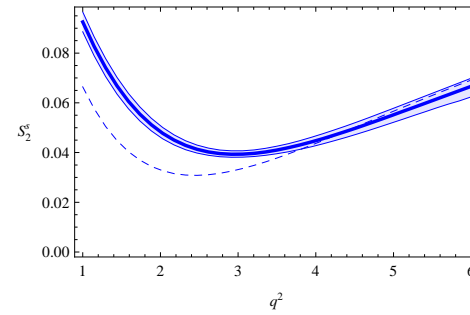
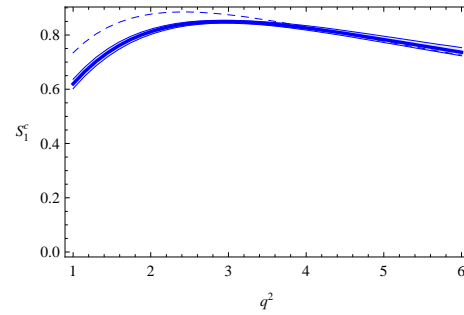
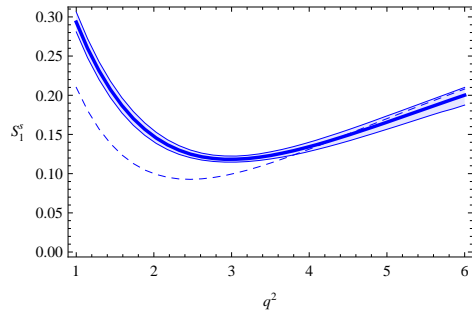
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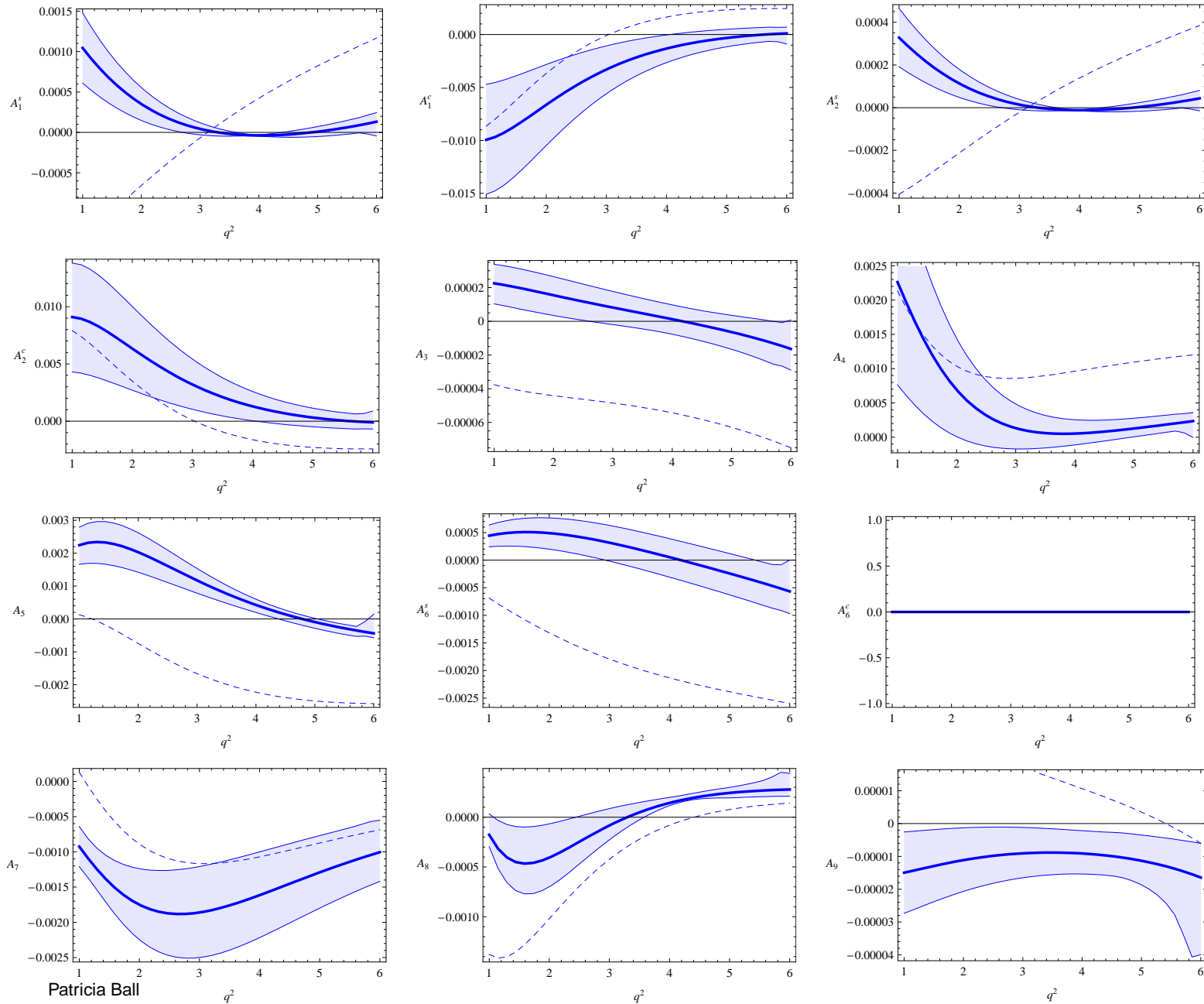
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Observables (SM): Symmetries



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Observables (SM): Asymmetries



How to measure S_i and A_i ?

- full angular spectrum
- or: suitable integrals
 - if observable is smooth in q^2 :

$$S_i(q^2) \rightarrow \langle S_i \rangle \equiv \int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 S_i(q^2)$$

(QCD factorization only valid for small q^2 : choose interval $1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2$ below (theory) charm threshold $4m_c^2$)

- for S_5 , e.g., only need small number of bins in angles:

$$S_5 = -\frac{4}{3} \left[\int_{\pi/2}^{3\pi/2} - \int_0^{\pi/2} - \int_{3\pi/2}^{2\pi} \right] d\phi \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_K \\ \times \frac{d^3(\Gamma - \bar{\Gamma})}{dq^2 d \cos \theta_K d\phi} \bigg/ \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$$

Features to look for in S_i and A_i

- zeros in S_i
 - in S_6 (forward-backward asymmetry), also in $S_{4,5}$
 - correlation between zeros and $\text{BR}(B \rightarrow X_s \gamma)$
- large A_i (larger than $O(1\%)$)
 - due to **new CP-violating phases**
- non-zero S_6^c : evidence for **BSM scalar operators**

How to find New Physics

→ **Aoife Bharucha's talk**

Summary

- $B \rightarrow K^* \mu\mu$ is an ideal channel to find new physics in the flavour sector at the LHC
- high sensitivity to new CP-violating phases
- alternatively, with mass scales of NP fixed by Atlas/CMS, the decay can help to constrain NP couplings
- theoretical description based on
 - **effective Hamiltonian**: known to NNLL accuracy in SM, plus one-loop NP effects (complete MSSM, MSSM with various constraints, little Higgs model) (Buras et al., 200x)
 - **form factors**: QCD sum rules on the light-cone (Ball/Zwicky 04, Ball 08)
 - **QCD factorisation** (Beneke/Feldmann/Seidel 01+04)
- theoretical uncertainty well under control as only ratios (symmetries + asymmetries) considered
- LHC will collect enough data to allow analysis of **full angular spectrum** (may need upgrade?)