# Droplets in the cold and dense chiral phase transition

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## Introduction and motivation

Recently, it was shown that deconfinement can happen during the early post-bounce accretion stage of a core collapse supernova event, which could result not only in a delayed explosion but also in a neutrino burst that could provide a signal of the presence of quark matter in compact stars [1]. However, as was discussed in detail in Ref. [2] (see also Ref. [3]) those possibilities depend on the actual dynamics of phase conversion, more specifically on the time scales that emerge. In a first-order phase transition, as is expected to be the case in QCD at very low temperatures, the process is guided by bubble nucleation (usually slow) or spinodal decomposition ("explosive" due to the vanishing barrier), depending on how fast the system reaches the spinodal instability as compared to the nucleation rate. Nucleation in relatively high-density, cold strongly interacting matter, with chemical potential of the order of the temperature, can also play an important role in the scenario proposed in Ref. [4], where a second (little) inflation at the time of the primordial quark-hadron transition could account for the dilution of an initially high ratio of baryon to photon numbers. A key ingredient in both scenarios is, of course, the surface tension, since it represents the price in energy one has to pay for the mere existence of a given bubble (or droplet), which is not known for cold and dense QCD, as well as the whole nucleation process.

## Effective Theory

Here [5] we consider the linear sigma model coupled with constituent quarks (LSMq) at zero or low temperature and finite quark chemical potential as a model for the thermodynamics of strong interactions in cold and dense matter. Since we are concerned with the chiral phase transition, we neglect the pions [6] and work with the following Lagrangian:

$$\mathcal{L} = \overline{\psi}_f \left[ i\gamma^\mu \partial_\mu - m_q - g\sigma \right] \psi_f + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)$$

$$U(\sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma$$

The scalar field $\sigma$ plays the role of an approximate order parameter for the chiral transition, being an exact order parameter for massless quarks and pions.

We compute the effective potential

$$V_{\rm eff}(\bar\sigma) = U(\bar\sigma) + \Omega_\xi^{\rm ren}(\bar\sigma)$$

for the sigma condensate integrating over the quark fields and keeping quadratic fluctuations of the sigma field around the condensate. Our full thermodynamic potential $\Omega_\xi^{\rm ren}$ incorporates all corrections from the medium and vacuum fluctuations in the MSbar scheme, including logarithmic and scale-dependent contributions from quark and sigma bubble-diagrams.

Up to 1-loop order, the temperature- and chemical-potential-dependent contribution is that of an ideal gas

$$\Omega^{(1)}_{\rm med,Th} = T\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\log\left[1 - e^{-\omega_\sigma/T}\right] - $$
$$-2TN_fN_c\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left\{\log\left[1 + e^{-(E_q-\mu)/T}\right] + \log\left[1 + e^{-(E_q+\mu)/T}\right]\right\}$$

and the quantum vacuum term is given by:

$$\Omega^{(1)}_{\rm vac} = -\frac{M_\sigma^4}{64\pi^2}\left[\frac{3}{2} + \log\left(\frac{\Lambda^2}{M_\sigma^2}\right)\right] + N_fN_c\frac{M_q^4}{64\pi^2}\left[\frac{3}{2} + \log\left(\frac{\Lambda^2}{M_q^2}\right)\right]$$

In the zero-temperature limit, the medium contribution reduces to:

$$\Omega^{(1)}_{\rm med} = -\frac{N_fN_c}{24\pi^2}\left\{2\mu p_f^3 - 3M_q^2\left[\mu p_f - M_q^2\log\left(\frac{\mu + p_f}{M_q}\right)\right]\right\}$$

with the effective masses: $M_q \equiv m_q + g\langle\sigma\rangle$ , $M_\sigma^2 \equiv 3\langle\sigma\rangle^2 - \lambda v^2$.

The conditions for fixing the parameters are imposed on the vacuum effective potential, and therefore will be modified in the presence of vacuum logarithmic corrections.

In order to identify the role played by thermal effects and vacuum logarithmic corrections, we compare three cases:

**(a)** including thermal corrections (with T = 30 MeV): $\Omega_\xi^{\rm ren} = \Omega^{(1)}_{\rm med,Th}$

**(b)** considering quantum vacuum terms: $\Omega_\xi^{\rm ren} = \Omega^{(1)}_{\rm vac} + \Omega^{(1)}_{\rm med}$

**(c)** cold and dense LSMq: $\Omega_\xi^{\rm ren} = \Omega^{(1)}_{\rm med}$

## Surface tension and nucleation

### - Extracting the nucleation parameters:

Following Refs. [6] and aiming for reasonable estimates and functional behavior of the nucleation parameters, we work with approximate analytic relations by fitting the relevant region of the effective potential by a quartic polynomial and working in the thin-wall limit approximation for bubble nucleation. We can express the effective potential over the range between the critical chemical potential, $\mu_c$, and the spinodal, $\mu_{sp}$, in the familiar Landau-Ginzburg form

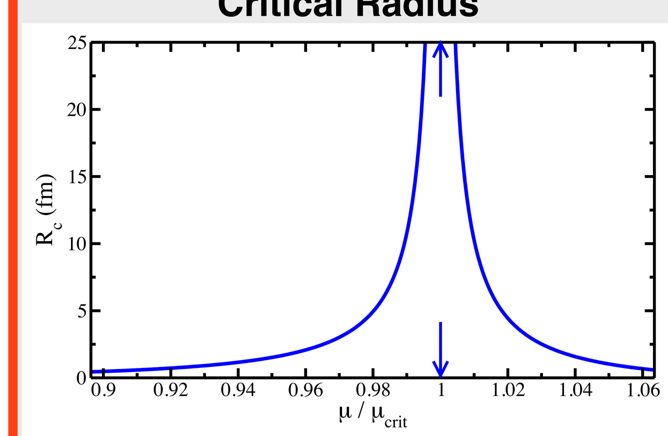
$$V_{\rm eff} \approx \sum_{n=0}^{4} a_n\,\phi^n\ .$$

In the thin-wall approximation, the bubble solution is then written analytically and all the relevant nucleation parameters can be expressed in terms of the coefficients $a_n$.

### - Cold and dense nucleation in the chiral transition:

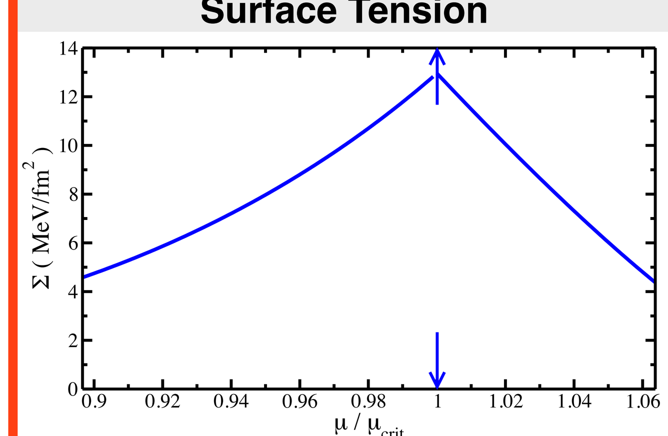
Taking initially the zero-temperature result for the thermodynamic potential neglecting vacuum corrections, we present results for both metastable regions, above and below the critical chemical potential ~305 MeV, which correspond respectively to the nucleation of quark droplets in a hadronic environment and the formation of hadronic bubbles in a partonic medium.

**Critical Radius**



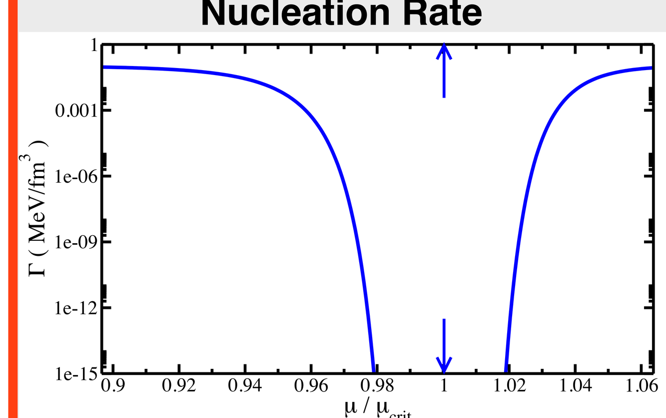
- $R > R_c \Rightarrow$ bubble growth energetically favorable.
- $R_c \to 0$ at spinodals (instability) $\Rightarrow$ explosive spinodal decomposition

**Surface Tension**



- amount of energy per unit of area that is spent in the construction of a surface between the phases
- well below previous estimates [7,8]

**Nucleation Rate**



- estimated as $\Gamma \sim T_f^4 e^{-F_b/T_f}$ [9,10], where $F_b$ is the free energy of the critical bubble configuration and $T_f$ = 30 MeV is an *ad hoc* temperature
- negligibly small in the vicinity of the criticality.

### - Thermal effects versus vacuum quantum corrections:

We find that the nucleation parameters up to T=10 MeV present variations within ~10% of the zero-temperature values. Here, we use T=30 MeV.
The metastable region in the presence of quantum vacuum terms is ~40% bigger than in its absence, indicating possible large differences in the dynamics of the system, even though the critical chemical potential itself shifts only ~2%.

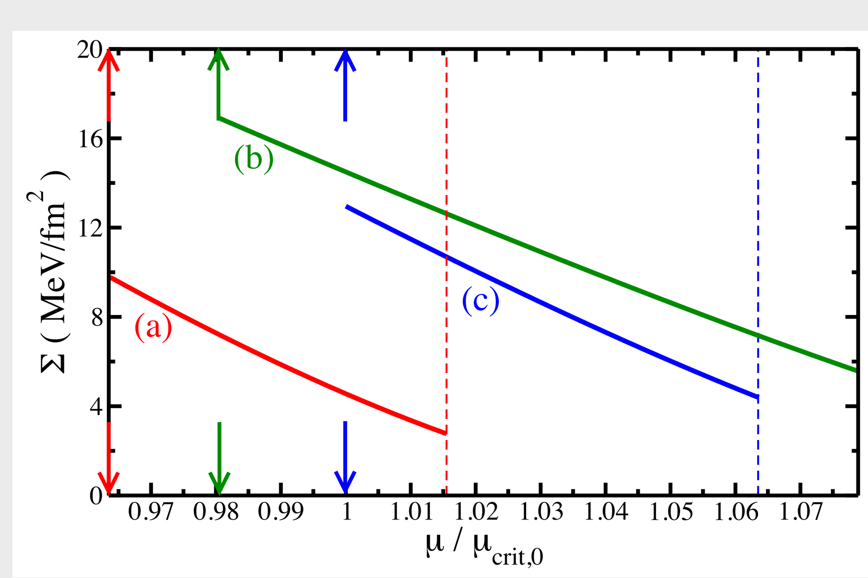


**Fig. 4:** Surface tension for quark-matter nucleation in the cold and dense LSMq **(c)**, including vacuum terms **(b)** and thermal corrections **(a)**.

**Density X quark chemical potential**

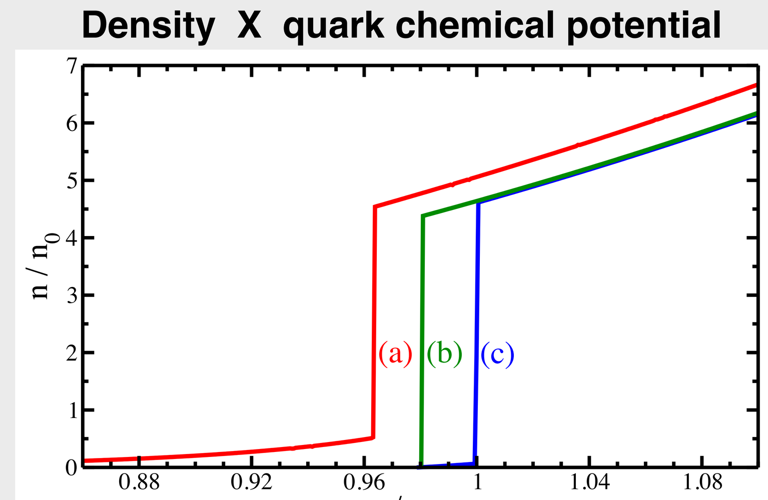
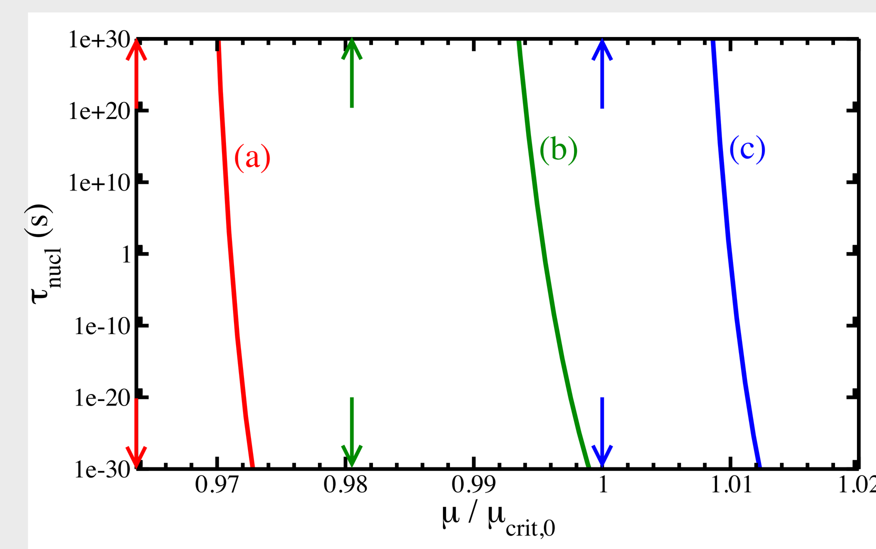


**Fig. 5:** Density in units of the nuclear saturation density.

## Consequences for quark-matter-induced supernova explosions

To contribute to the investigation of the possibility of nucleating quark matter droplets during the early post-bounce stage of core collapse supernovae, we follow Ref. [2] and define the nucleation time as being the time it takes for the nucleation of a single critical bubble inside a volume of 1 km³, which is typical of the core of a protoneutron star, i.e.

$$\tau_{nucl} \equiv \left(\frac{1}{1{\rm km}^3}\right)\frac{1}{\Gamma}$$



The relevant time scale to compare is the time interval the system takes from the critical chemical potential to the spinodal during the supernova event, if it ever reaches such high densities in practice. The typical time scale for the early post-bounce phase is of the order of a fraction of a second, so that the time within the metastable region is smaller, in the ballpark of milliseconds.

Consistently, we find that, within LSMq, vacuum corrections tend to increase the density depth required for efficient nucleation whereas thermal corrections push in the opposite direction. Our results for the nucleation time, together with those for the surface tension, tend to favor the best scenario for nucleation of quark matter in the supernova explosion scenario considered in Ref. [2], especially when thermal corrections with physical temperatures are included. To a great measure, this happens because the nucleation time, and the whole process of phase conversion, depends very strongly on the surface tension, since it enters cubed in the Boltzmann exponential of the nucleation rate.

## Conclusions

We have quantified approximately the homogeneous nucleation in the linear sigma model with quarks in the MSbar scheme at zero and low temperature and finite quark chemical potential, including vacuum and medium fluctuations [5]. All the relevant quantities were computed as functions of the chemical potential, the key function being the surface tension.

The inclusion of temperature and vacuum logarithmic corrections revealed a clear competition between these features in characterizing the dynamics of the chiral phase conversion, so that if the temperature is low enough the consistent inclusion of vacuum corrections could help preventing the nucleation of quark matter during the collapse process. In particular, we show that the model predicts a surface tension of ~5 -15 MeV/fm², rendering nucleation of quark matter possible during the early post-bounce stage of core collapse supernovae [2] and also allowing for mixed quark-hadron phases in compact-star structure [11].

The LSMq, however, does not contain essential ingredients to describe nuclear matter, e.g. it does not reproduce features such as the saturation density and the binding energy. Therefore, the results obtained here should be considered with caution when applied to compact stars or the early universe. It is an effective theory for a first-order chiral phase transition in cold and dense strongly interacting matter, and allows for a clean calculation of the physical quantities that are relevant for homogeneous nucleation in the process of phase conversion. In the spirit of an effective model description, our results should be viewed as estimates that indicate that the surface tension is reasonably low and falls with baryon density, as one increases the supercompression. First-principle calculations in QCD in this domain are probably out of reach in the near future. Therefore, estimates within other effective models would be very welcome.

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