



Determination of relaxation times at weak and strong coupling

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Motivation/Introduction

- Relativistic **fluid dynamics** has played a key role in our current understanding of the novel “**near perfect**” fluid behavior displayed by the Quark-Gluon Plasma (QGP)
- However, inclusion of dissipation is problematic because of the nontrivial consequences of **acausality** in a **covariant** setup.
 **Failure of Navier-Stokes theory**
- Do we fully understand **relativistic** dissipative fluids ?

Solution: Inclusion of relaxation times

Transient theories of fluid dynamics Can be causal (stable)!

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta \nabla_\alpha u^\alpha + \dots$$

$$\tau_n \dot{n}^{<\alpha>} + n^\alpha = \kappa \nabla^\alpha (\mu/T) + \dots$$

$$\tau_\pi \dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} = 2\eta \nabla^{<\mu} u^{\nu>} + \dots$$

Israel&Stewart(1978)

- Because of the inclusion of a relaxation time, this theory can be **causal** and **stable** !
 - Dissipative currents are independent dynamical variables

→ How to compute the additional transport coefficient?
How does this change the fluid-dynamical picture?

Old picture: gradient expansion

Burnett equation: only shear stress tensor

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + C_1 \frac{d}{d\tau} \sigma^{\mu\nu} + C_2 \omega_\lambda^{\langle\mu} \omega^{\nu\rangle\lambda} + C_3 \sigma_\lambda^{\langle\mu} \sigma^{\nu\rangle\lambda} + \dots$$

- For dilute gases, can be derived from Chapman-Enskog expansion
- Extension of NS theory; no additional dynamical variables
- Acausal and unstable **even** in nonrelativistic regime

Can **transient** fluid dynamics be
derived from the gradient expansion ?

Can **transient** fluid dynamics be derived from the gradient expansion ?

No

Simple example, add the terms

$$\tau'_\pi \frac{d}{d\tau} \pi^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + C_1 \frac{d}{d\tau} \sigma^{\mu\nu} + 2\eta\tau'_\pi \frac{d}{d\tau} \sigma^{\mu\nu} + \dots$$

- For any value of relaxation time, the asymptotic solution corresponds to the **same** gradient expansion
- Relaxation time becomes ambiguous ...

→ Transient dynamics **can never** be obtained from asymptotic solution!

→ How can one obtain relaxation equations?

To understand the problem:

- ✓ We consider an **arbitrary** linear theory

$$J(X) = \int d^4x' G_R(X - X') F(X')$$

In Fourier space,

$$\tilde{J}(Q) = \tilde{G}_R(Q) \tilde{F}(Q)$$

Notation,

$$\tilde{A}(Q) = \int d^4X \exp(iQ \cdot X) A(X) ,$$

$$A(X) = \int \frac{d^4Q}{(2\pi)^4} \exp(-iQ \cdot X) \tilde{A}(Q) .$$

Gradient expansion as a Taylor series

The traditional approach is based on the **Taylor expansion** of the Green's function around the origin

→ Linear theory

$$\tilde{J}(Q) = \tilde{G}_R(Q) \tilde{F}(Q)$$

→ Taylor expansion



$$\tilde{G}_R(Q) \sim \tilde{G}_R(0, 0) + \partial_\omega \tilde{G}_R(0, 0) \omega + \dots,$$

→ Gradient expansion !



$$J(X) = C F(X) + C_1 \partial_t F(X) + \dots,$$



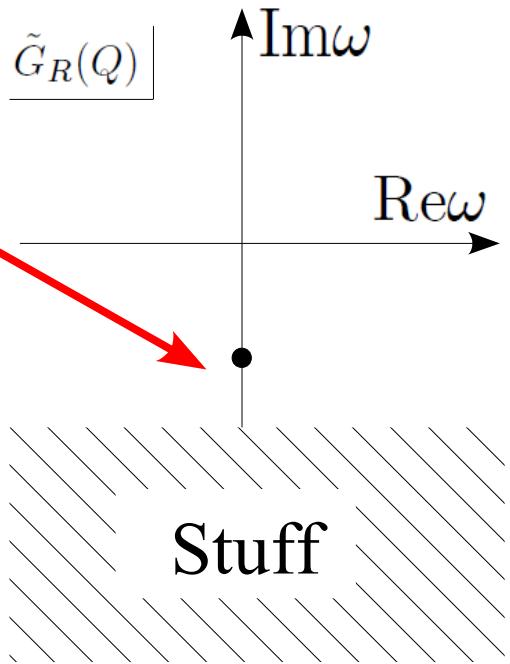
$$C = \tilde{G}_R(0, 0), \quad C_1 = \left. \partial_\omega \tilde{G}_R(Q) \right|_{\omega, \mathbf{k}=0}$$

Limitations of the Gradient expansion

→
$$\tilde{G}_R(Q) = \frac{f(Q)}{\omega - \omega_0(\mathbf{k})} + \dots$$

→ If the Green's function has poles the **radius of convergence** of the gradient expansion is **limited**

→ At long times (small frequencies), only the poles **nearest** to origin are important.



Limitations of the Gradient expansion

G.S.D., J. Noronha, H. Niemi, D.H. Rischke,
Phys.Rev. D83 (2011) 074019,
arXiv:1102.4780 [hep-th]

- Linear theory

$$\tilde{J}(Q) = \tilde{G}_R(Q) \tilde{F}(Q)$$

- Laurent expansion



$$\tilde{G}_R(Q) = \frac{f(\omega_0, \mathbf{k})}{\omega - \omega_0(\mathbf{k})} + \left[\tilde{G}_R(0, \mathbf{k}) + \frac{f(\omega_0, \mathbf{k})}{\omega_0(\mathbf{k})} \right] + \dots$$

- Relaxation equation !



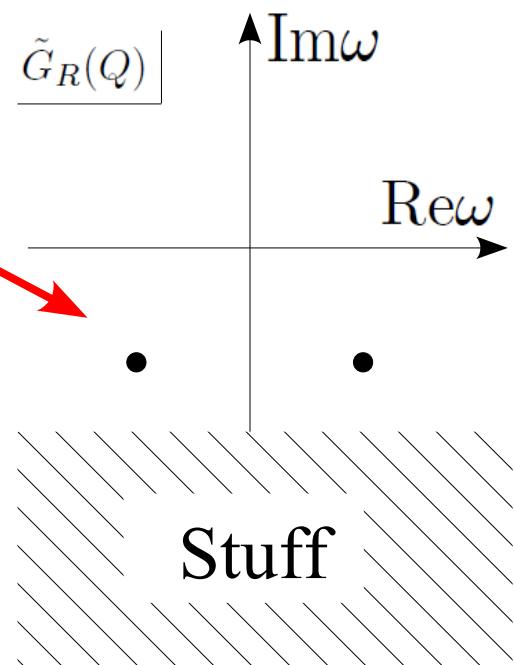
$$\tau_R \partial_t J + J = DF + \dots$$

$$\text{L} \rightarrow \tau_R = \frac{1}{i\omega_0(\mathbf{0})}, \quad D = \tilde{G}_R(0, \mathbf{0})$$

Limitations of the Gradient expansion

Two Poles:

$$\tilde{G}_R(Q) = \frac{f_1(Q)}{\omega - \omega_1(\mathbf{k})} + \frac{f_2(Q)}{\omega - \omega_2(\mathbf{k})} + \dots$$



→ At long times (small frequencies),
both poles contribute **equally** to the dynamics.

Neither pole can be neglected

Limitations of the Gradient expansion

→ Linear theory

$$\tilde{J}(Q) = \tilde{G}_R(Q) \tilde{F}(Q)$$

→ Laurent expansion

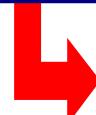


$$\tilde{G}_R(Q) = \frac{f_1(\omega_1, \mathbf{k})}{\omega - \omega_1(\mathbf{k})} + \frac{f_2(\omega_2, \mathbf{k})}{\omega - \omega_2(\mathbf{k})} + \left[\tilde{G}_R(0, \mathbf{k}) + \frac{f_1(\omega_1, \mathbf{k})}{\omega_1(\mathbf{k})} + \frac{f_2(\omega_2, \mathbf{k})}{\omega_2(\mathbf{k})} \right] + \dots$$

→ **Not** a relaxation equation !



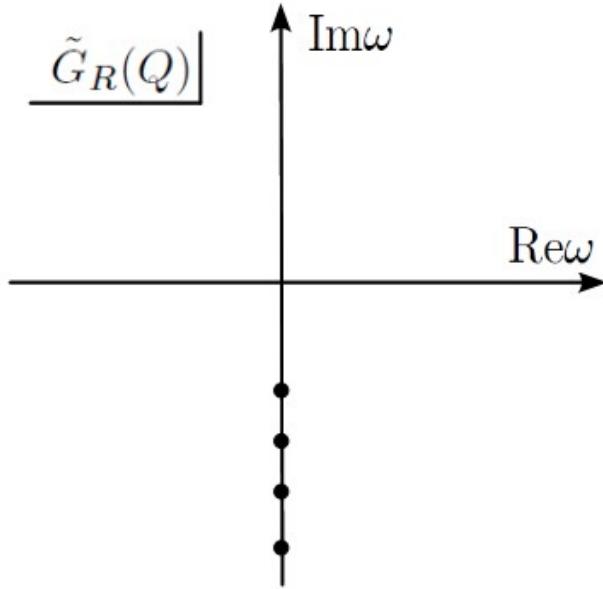
$$\chi_2 \partial_t^2 J + \chi_1 \partial_t J + J = DF + \dots$$



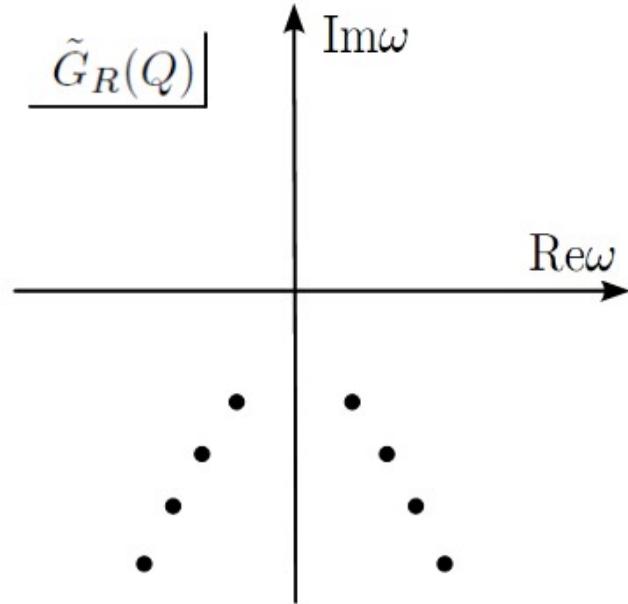
$$\chi_2 = \frac{-1}{\omega_1(\mathbf{0}) \omega_2(\mathbf{0})}, \quad \chi_1 = \frac{\omega_1(\mathbf{0}) + \omega_2(\mathbf{0})}{i\omega_1(\mathbf{0}) \omega_2(\mathbf{0})}, \quad D = \tilde{G}_R(0, \mathbf{0})$$

Be careful ...

Dilute Gas



AdS/CFT



If the pole closest to the origin is **purely imaginary**, the long-time dynamics is given by **relaxation type** equations

If nearest poles have **real parts**, long time dynamics is given by a **second order** differential equation (oscillations)

Application: Boltzmann equation

$$\rightarrow \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \dots$$

Simplest scenario: → Israel-Stewart 14-moment approx.

→ Massless/classical limits

→ constant cross section

In this case, $\eta = \frac{4}{3}T\sigma$

$$\tau_\pi = \frac{5}{3n\sigma}$$

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Exactly the same as in the moments method!

G.S.D., T. Koide, D. Rischke (2010),

Application: AdS/CFT

$$\rightarrow \chi_2 \ddot{\pi}^{\langle\mu\nu\rangle} + \chi_1 \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \dots$$

The transient dynamics of AdS/CFT theories is **not** described by relaxation equations, **even at long times**.

Poles were calculated by Starinets (2002). The coefficients are,

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$$\chi_1 = \frac{1.27}{4\pi T}$$

$$\chi_2 = \frac{0.93}{(4\pi T)^2}$$

χ_1 is **not** a relaxation time!

Summary

- Gradient expansion is an **asymptotic** solution of the transient theory. Asymptotic solution can be obtained from the transient theory, **but not vice versa**.
- Relaxation time in the Israel-Stewart theory is given by the location of the **pole** of the Green's function **nearest to the origin** (slowest quasi-normal mode).
- If the nearest pole is not purely imaginary, long-time dynamics is **not** given by IS type theory.
- AdS/CFT **cannot** be described by Israel-Stewart-like theories