Shear Viscosity in a partonic cascade using pQCD-based cross sections

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Motivation

Recent results from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven indicate the formation of a new state of matter, the quark gluon plasma (QGP) in ultra relativistic heavy ion collisions.

Large values of the elliptic flow v_2 lead to the indication that the QGP behaves like a nearly perfect fluid which makes dissipative hydrodynamic a promising candidate to describe the phase of collective flow. One of its crucial transport parameter is the shear viscosity which is investigated in this work.

Two different methods are used to extract the shear viscosity coefficient η from the partonic cascade BAMPS, a relativistic static gradient method and a Green-Kubo relation. Both methods are tested with an analytic relation and are applied to calculations with pQCD-based cross sections.

Th BAMPS cascade

We use the parton cascade BAMPS [1] (Boltzmann Approach for Multi Parton Scattering) which simulates the full space-time evolution of the QGP.

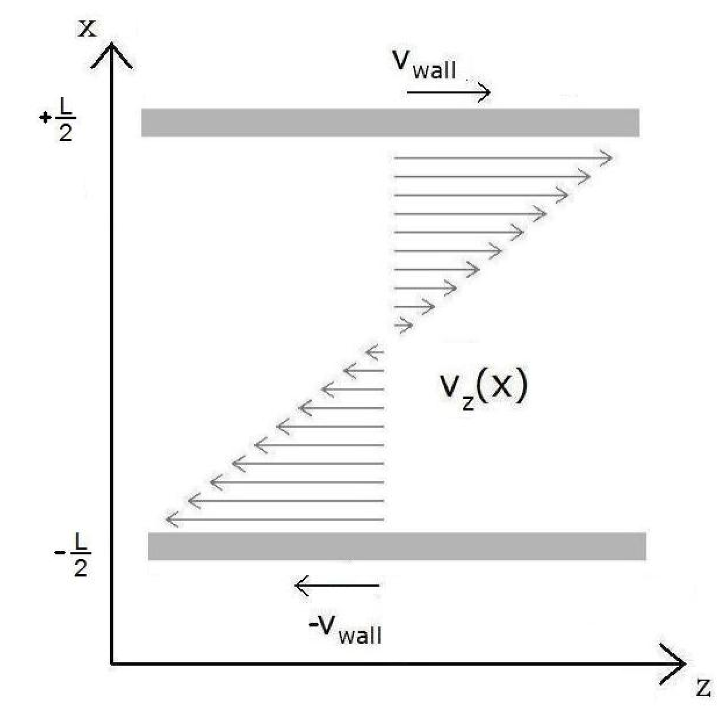
The kinetic on-shell Boltzmann equation for partons is solved with Monte-Carlo techniques, including elastic and inelastic pQCD processes:

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}_i}{\mathbf{E}_i}\frac{\partial}{\partial \mathbf{r}}\right) f_i(\mathbf{r}, \mathbf{p}_i, t) = \mathcal{C}_i^{2\to 2} + \mathcal{C}_i^{2\to 3} + \mathcal{C}_i^{3\to 2} \quad (11)$$

For the collisions terms two different types of cross sections are used, constant and isotropic two- and three-particle cross sections as well as pQCD-based two- and three-particle cross sections.

Shear stress and shear flow

Starting from a classical ansatz we want to describe shear flow in a stationary as-simple-as-possible geometry. We chose the velocity direction along the z-axis, its absolute value can only vary in x-direction, as shown in the figure below.
We consider a particle system embeded between two plates which move with a velocity v_{wall} in opposite directions.



A stationary profile of the flow velocity $v_z(x)$ is established due to the interactions among particles. A naive solution of $v_z(x)$ is a linear function. However this is an approximate solution, which is only valid for nonrelativistic fluids. For a relativistic fluid we find that

$$v_z(x) = \tanh(\theta x) \quad (1)$$

with a constant θ.

$$y_z(x) = \theta x \quad (2)$$

is defined as rapidity, which is linear in the relativistic case. Since it is only shifted by a constant under a Lorentz transformation the profile of $v_z(x)$ looks the same in the comoving frame of each layer being parallel to the plates.
In the Navier-Stokes-limit the shear stress $\pi^{\mu\nu}$ is proportional to the gradient of the flow four velocity u^ν.

$$\pi^{\mu\nu} = -2\eta\partial^{\langle\mu}u^{\nu\rangle} \quad (3)$$

where η is the shear viscosity.
In the gemonetry we use this equation simplifies to

$$\pi^{xz} = -\eta\gamma\theta \quad (4)$$

Green-Kubo Relation

Green [6] and Kubo [7] showed that for a linear consecutive relation like the Navier-Stokes definition of the shear viscosity any linear transport coefficient can be related to the equilibrium correlation function of its underlying flux. For the shear viscosity η the related flux is the off-diagonal energy momentum tensor $\pi^{\mu\nu}$.

$$\pi^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{\text{eq}} \quad (5)$$

The Green-Kubo relation relates the shear viscosity with the correlation function of the shear tensor:

$$\eta = \frac{V}{T}\int_0^\infty \mathrm{d}t \int_V \mathrm{d}^3r\, \mathrm{Cor}^{\mu\nu}(t) \quad (6)$$

with the correlation function:

$$\mathrm{Cor}^{\mu\nu}(t) = \lim_{T\to\infty}\frac{1}{2T}\int_{-\infty}^{\infty}\pi^{\mu\nu}(t')\cdot\pi^{\mu\nu}(t'+t)\mathrm{d}t \quad (7)$$

For a thermalized medium the first momentum of the shear tensor vanishes $\langle\pi^{\mu\nu}\rangle = 0$ while the second moment has a non-zero value $\langle\pi^{\mu\nu}(t)^2\rangle = \mathrm{Cor}^{\mu\nu}(0) \neq 0$. The shear viscosity coefficient η can be extracted from a numerical box simulation with periodic wall conditions. The correlation function is calculated for two chosen spatial dimensions, for example (xy). The correlation function for solutions of the Boltzmann-equation have the form of an an exponential decay:

$$\mathrm{Cor}^{\mu\nu}(t) \sim \exp(-t/\tau) \quad (8)$$

For an ultrarelativistic gluon gas simulated with the Boltzmann-equation the zero-time correlation function can be calculated analytically:

$$C^{(\mu=x,\nu=y)}(t=0) = \left(\frac{4}{5}\right)\frac{nT^2}{V} \quad (9)$$

Using the equilibrium gluon density $n = 16/\pi^2\, T^3$, the equilibrium entropy density $s = 4n$ and the exponential ansatz the shear viscosity to entropy ratio can be brought into a simple relation:

$$\left(\frac{\eta}{s}\right) = \left(\frac{T}{5}\right)\cdot\tau \quad (10)$$

In general τ has a complex dependence on different parameters like the temperature and inter-particle cross sections.

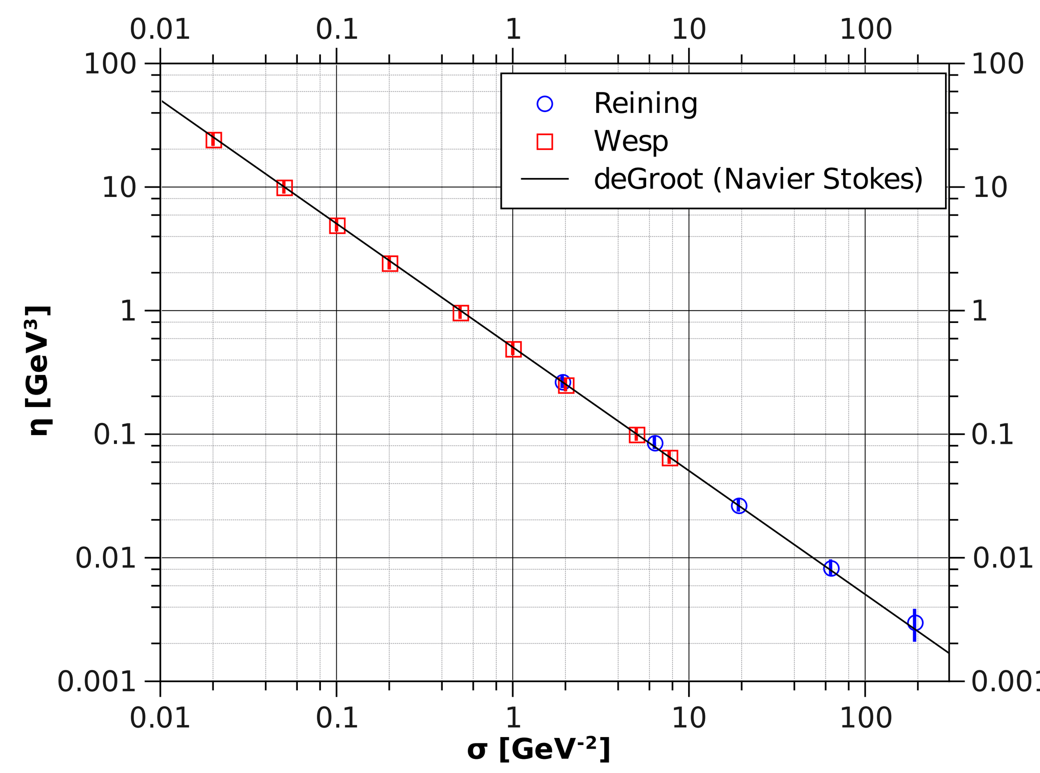
Consistency Check

For constant elastic isotropic two particle cross sections the relation for the shear viscosity is known from an analytical calculation. DeGroot derived the following expression for two particle cross sections from the Navier-Stokes framework of hydrodynamics [5]:

$$\eta \approx 0.8436\frac{T}{\sigma_{tr}} = 1.2654\frac{T}{\sigma_{22}} \quad (12)$$

For both methods the shear viscosity is numerically extracted from BAMPS and compared to the analytic relation. Over five orders of magnitude an excellent agreement is found. For both methods the shear viscosity can be extracted reliably, even though they use very different approaches. The errorbars include both statistical and systematic errors.

Comparison to analytic relation



Results

Within this work the shear viscosity for different kind of cross sections has been extracted. The shear viscosity for two- and three-particle pQCD-based cross section is extracted from BAMPS and compared to other published work from our workgroup. In addition, the shear viscosity for an constant and isotropic three-particle cross section is compared to the analytical result for two particle cross section. From an analytical calculation using the Grad's ansatz the shear viscosity ratio for two particle to three particle interactions is found to be:

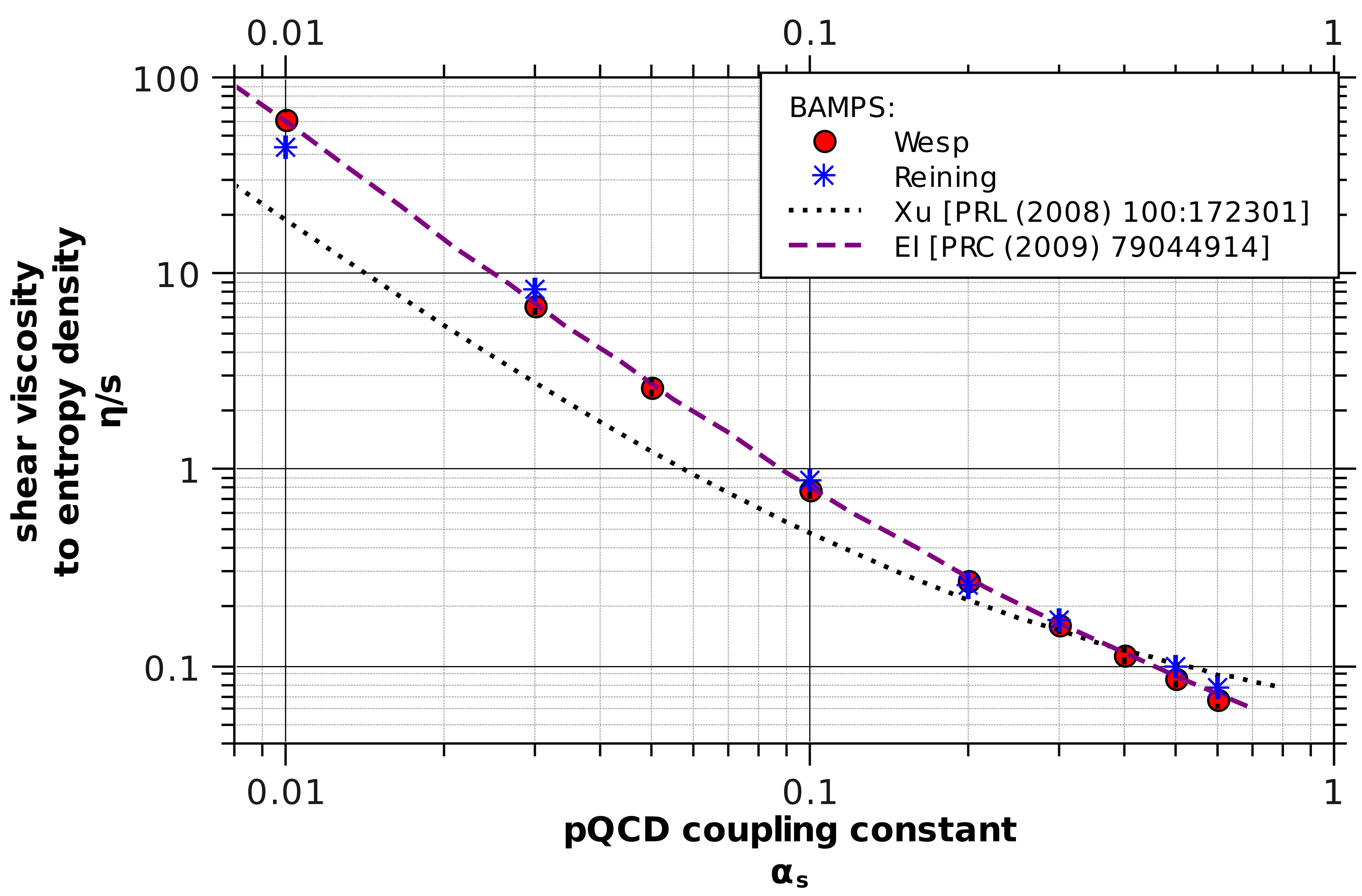
$$\frac{\eta_{2\leftrightarrow 2}}{\eta_{2\leftrightarrow 3}} = \frac{3}{2} \quad (13)$$

for the same constant cross section. Numerical calulations within this work confirm this result.

For a gluonic medium modeled with elastic $gg \leftrightarrow gg$ and inelastic $gg \leftrightarrow ggg$ pQCD-based processes the shear visocsity in dependence of the pQCD coupling constant α_s is extracted from BAMPS.
We find an excellent agreement for El [3], Reining and Wesp. For small $\alpha_s < 0.3$ a different scaling behavior is found for Xu [2].
For our numerical simulations the $\eta(\alpha_s)/s$ dependence can be fitted with the following relation, known from analytic pQCD calculations:

$$\frac{\eta}{s} \propto \frac{1}{\alpha_s^2 \ln(\alpha_s)} \quad (14)$$

pQCD-Results



Shear viscosity to entropy ratio η/s for a gluonic medium simulated with the BAMPS cascade. Elastic and inelastic pQCD-based processeses are implemented.

References

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