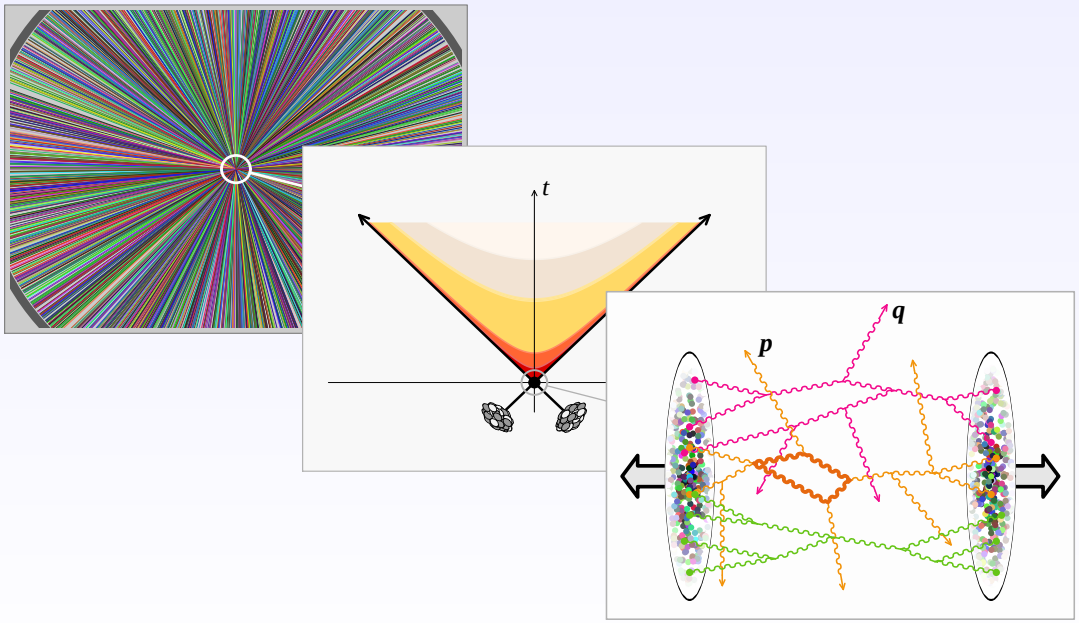
Heavy-ion collisions and QCD: the big picture

Quark Matter 2011, Annecy



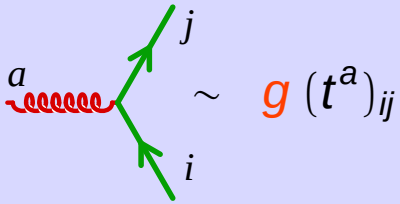
François Gelis
IPhT, Saclay

Outline

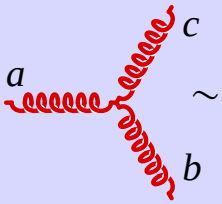
Quarks and gluons

Strong interactions : Quantum Chromo-Dynamics

- Matter : quarks ; Interaction carriers : gluons



$\sim \; g\,(t^a)_{ij}$



$\sim \; g\,(T^a)_{bc}$

- i, j : quark colors ; a, b, c : gluon colors
- $(t^a)_{ij}$: 3×3 SU(3) matrix ; $(T^a)_{bc}$: 8×8 SU(3) matrix

Lagrangian

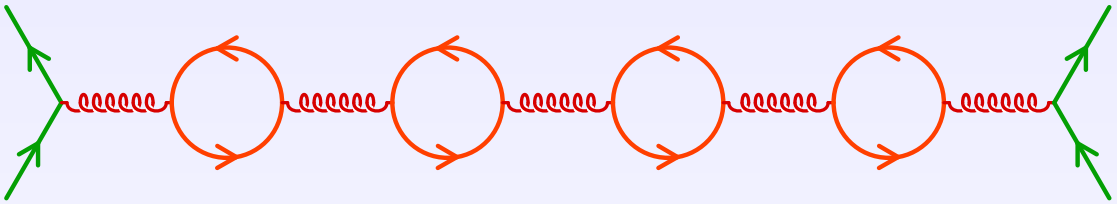
$$\mathcal{L} = -\frac{1}{4}F^2 + \sum_f \overline{\psi}_f (i\not{D} - m_f)\psi_f$$

- Free parameters : quark masses m_f, scale $\Lambda_{\rm QCD}$

Asymptotic freedom

- Running coupling : $\alpha_s = g^2/4\pi$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$



- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)

Asymptotic freedom

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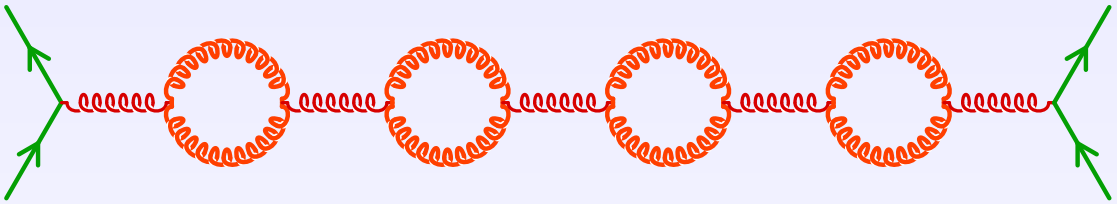
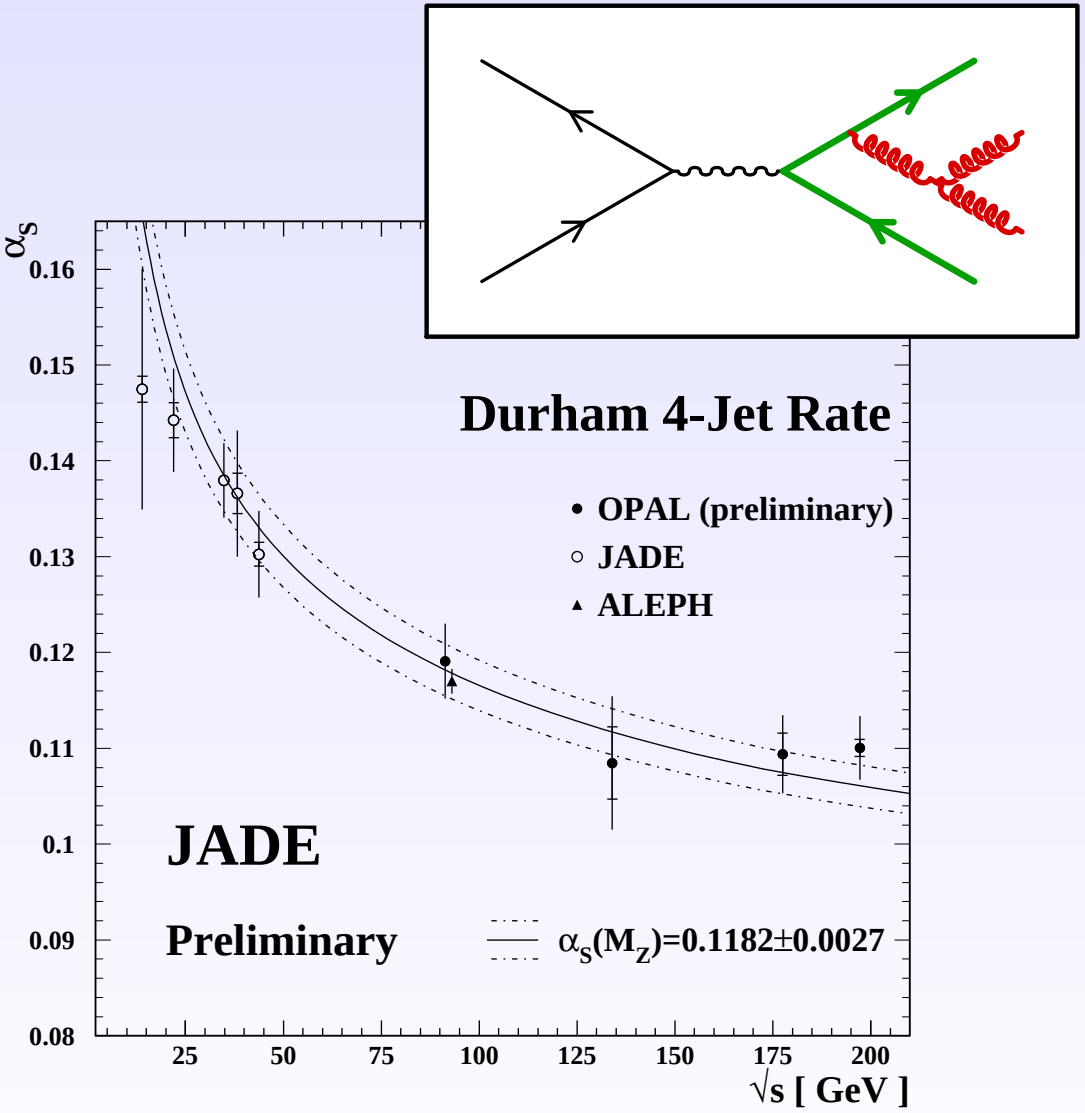
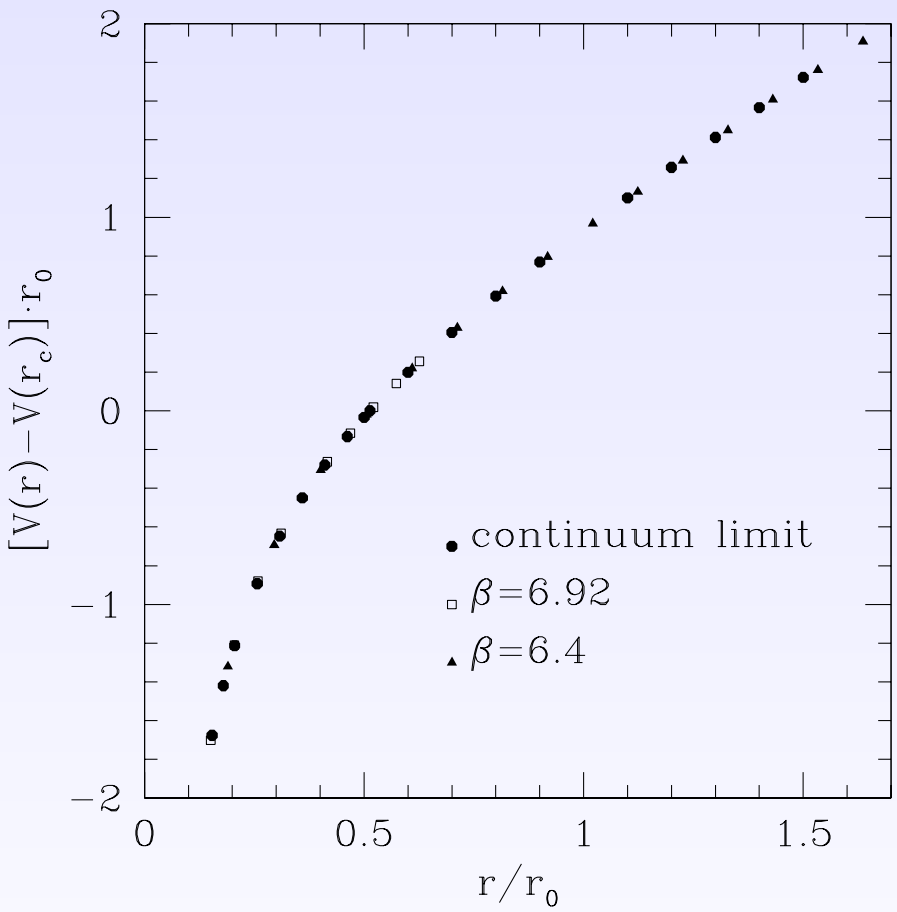


- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)
- But gluonic vacuum fluctuations produce an anti-screening (because of the non-abelian nature of their interactions)
- As long as $N_f < 11N_c/2 = 16.5$, the gluons win...

Asymptotic freedom



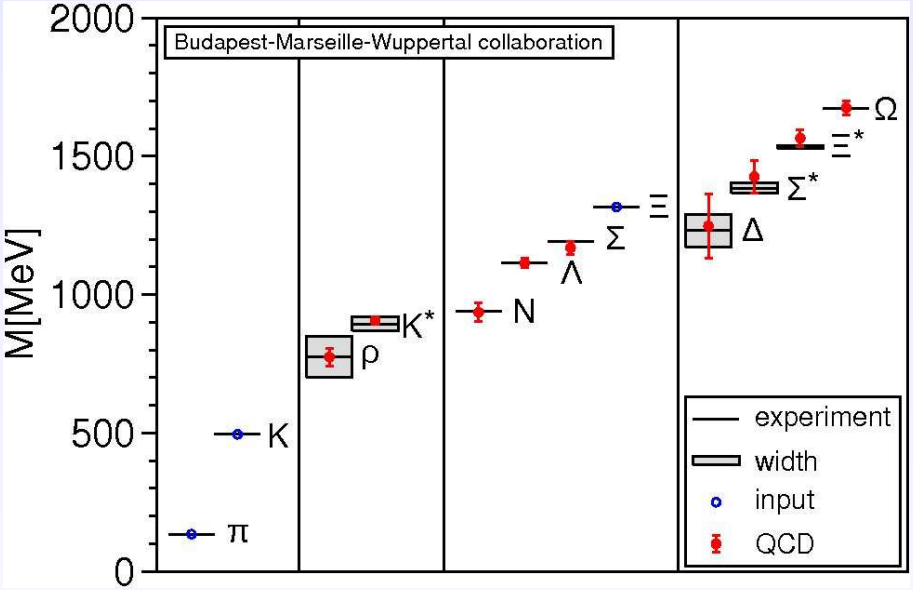
Color confinement



- The quark potential increases linearly with distance

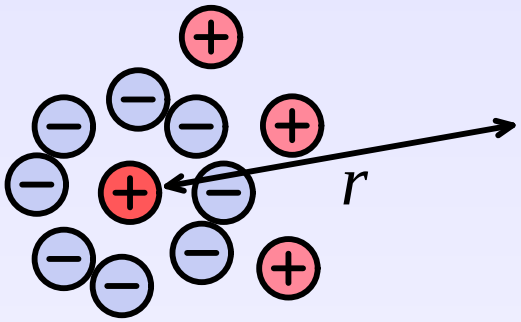
Color confinement

- In nature, we do not see free quarks and gluons (the closest we have to actual quarks and gluons are jets)
- Instead, we see hadrons (quark-gluon bound states):



- The hadron spectrum is uniquely given by $\Lambda_{\mathrm{QCD}}, m_f$
- But this dependence is non-pertubative (it can now be obtained fairly accurately by lattice simulations)

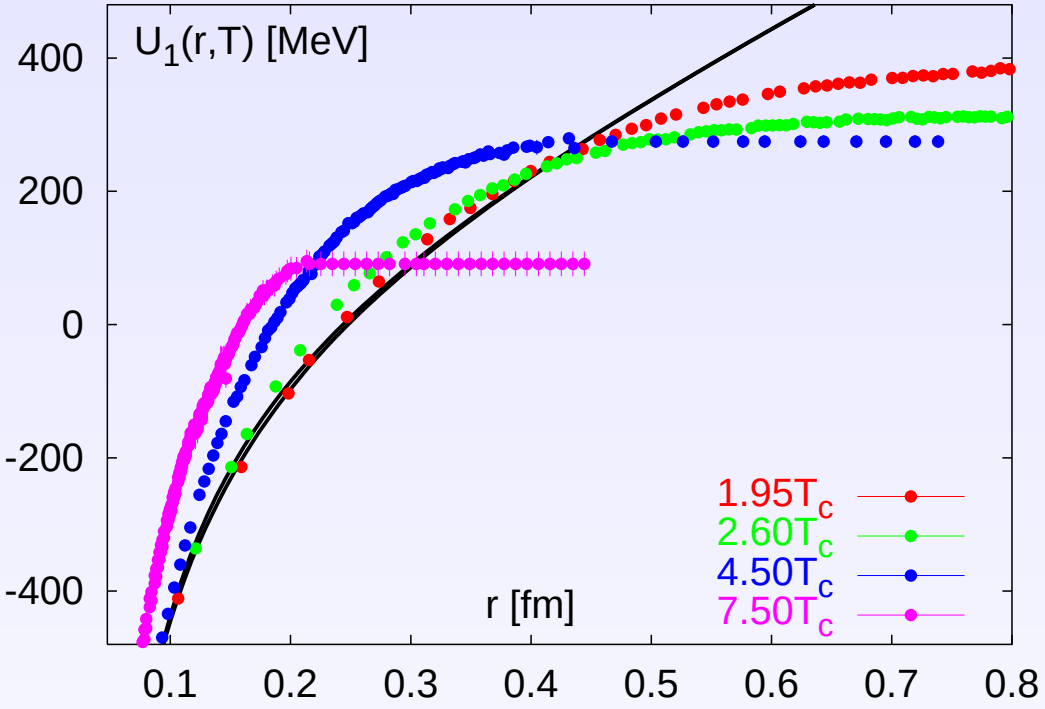
Debye screening



$$V(r) = \frac{\exp(-r / r_{debye})}{r}$$

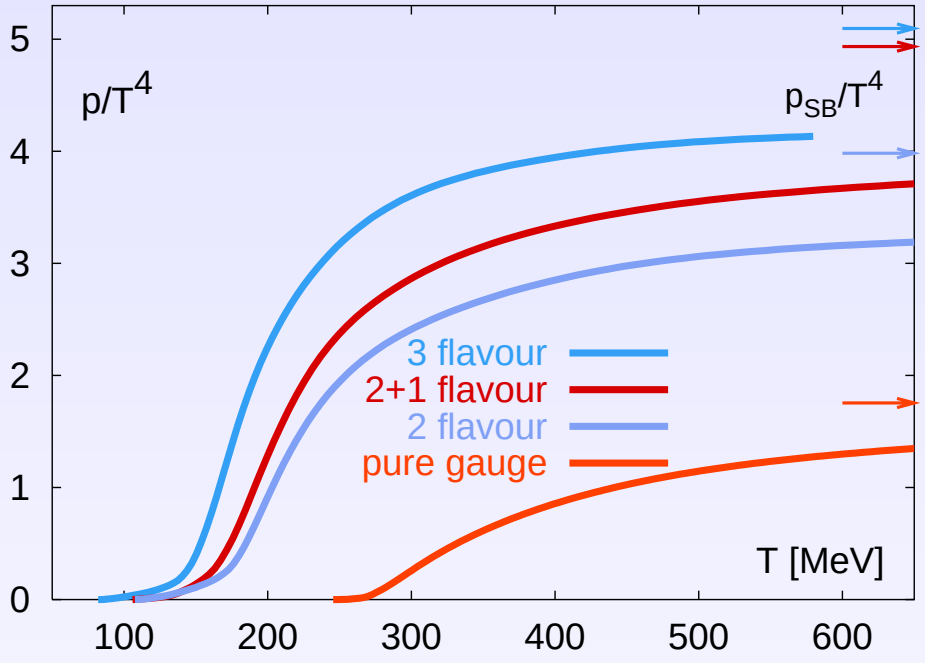
- In a dense medium, color charges are screened by their neighbours
- The interaction potential decreases exponentially beyond the Debye radius r_{debye}
- Hadrons whose radius is larger than r_{debye} cannot bind

Debye screening



- In lattice calculations, one sees the $q\bar{q}$ potential flatten at long distance as T increases

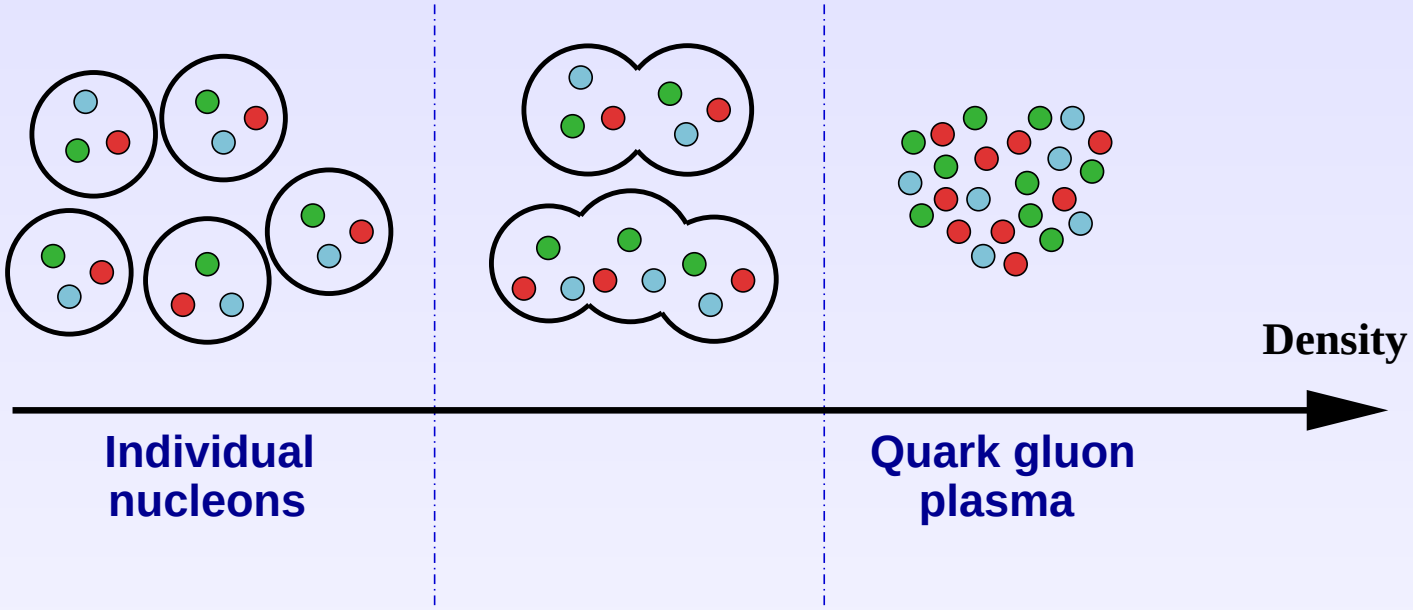
Deconfinement transition



- Rapid increase of the pressure :
  - at $T \sim 270$ MeV, with gluons only
  - at $T \sim 150$ to 180 MeV, with light quarks

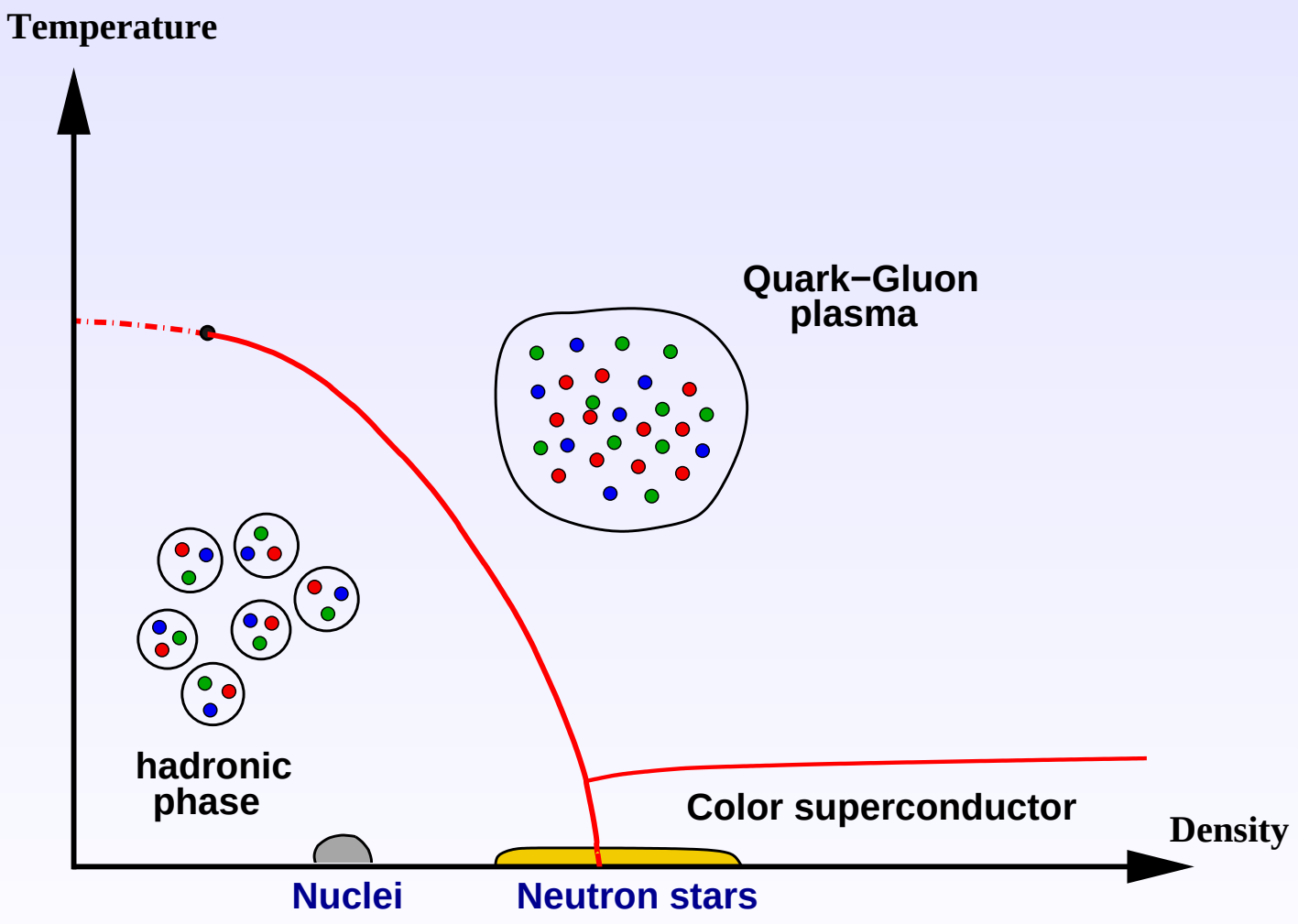
$\triangleright$ interpreted as the increase in the number of degrees of freedom due to the liberation of quarks and gluons

Deconfinement transition

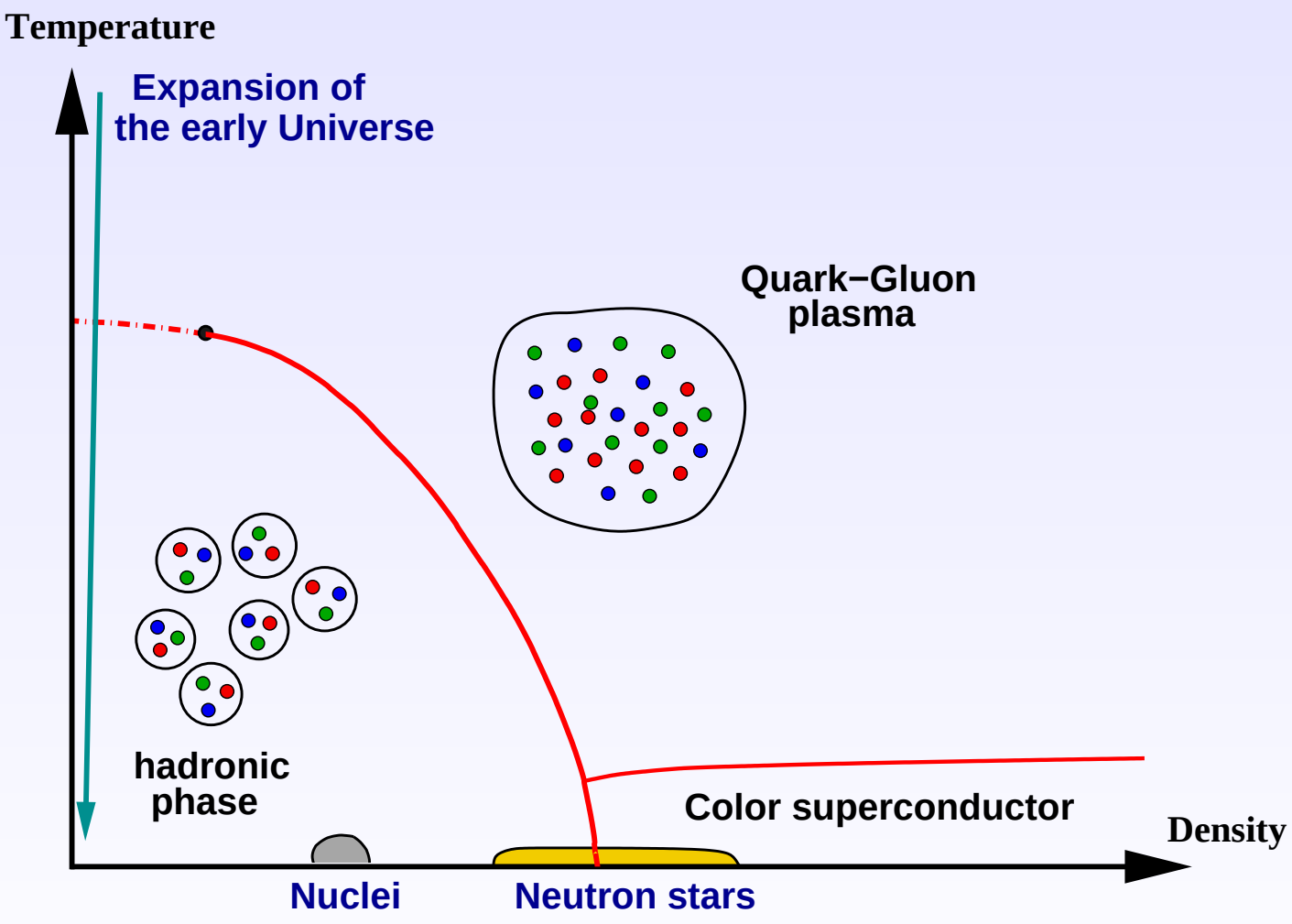


- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- This phenomenon extends to the whole volume when the phase transition ends
- Note: if the transition is first order, it goes through a mixed phase containing a mixture of nucleons and plasma

QCD phase diagram



QGP in the early universe



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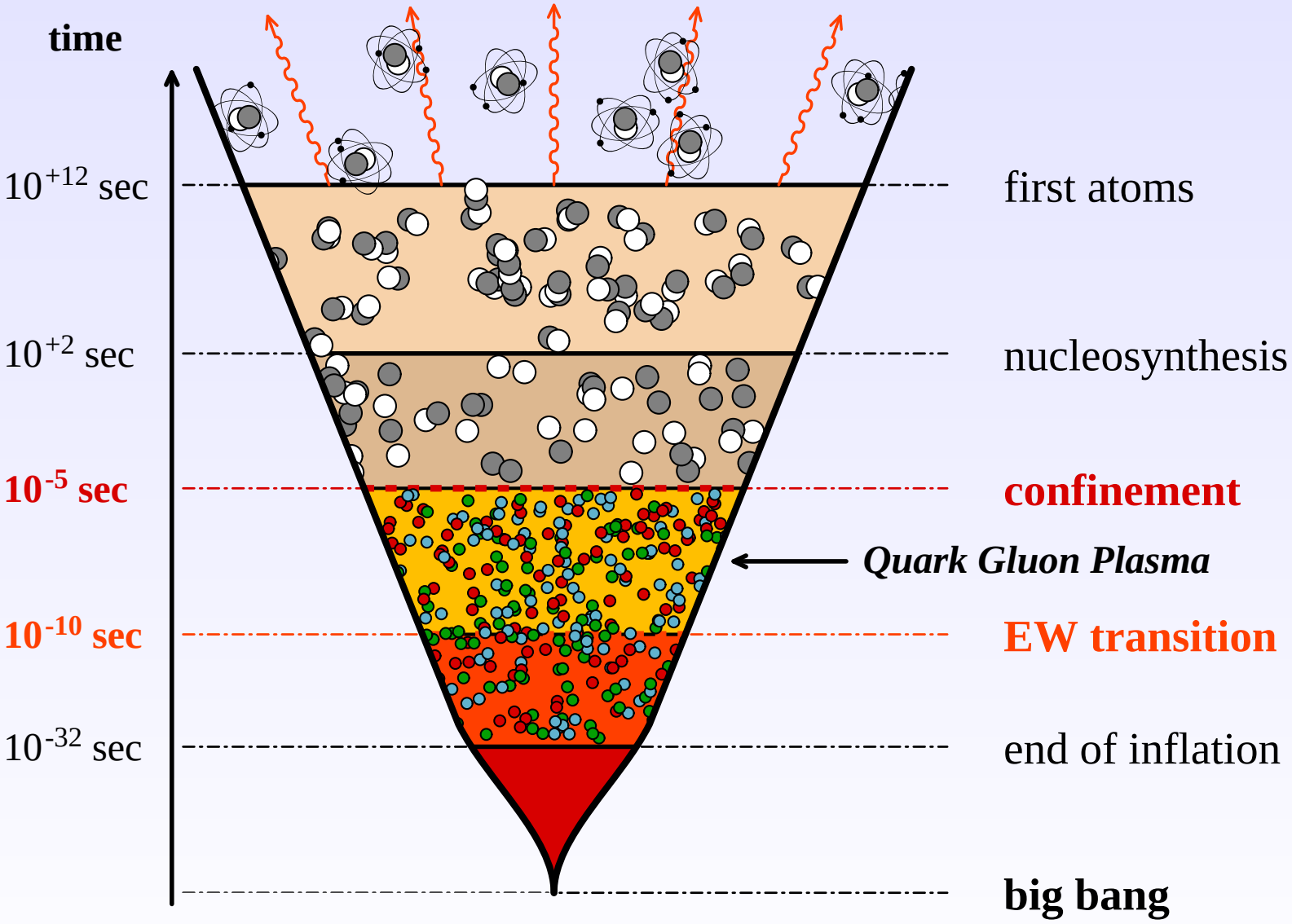
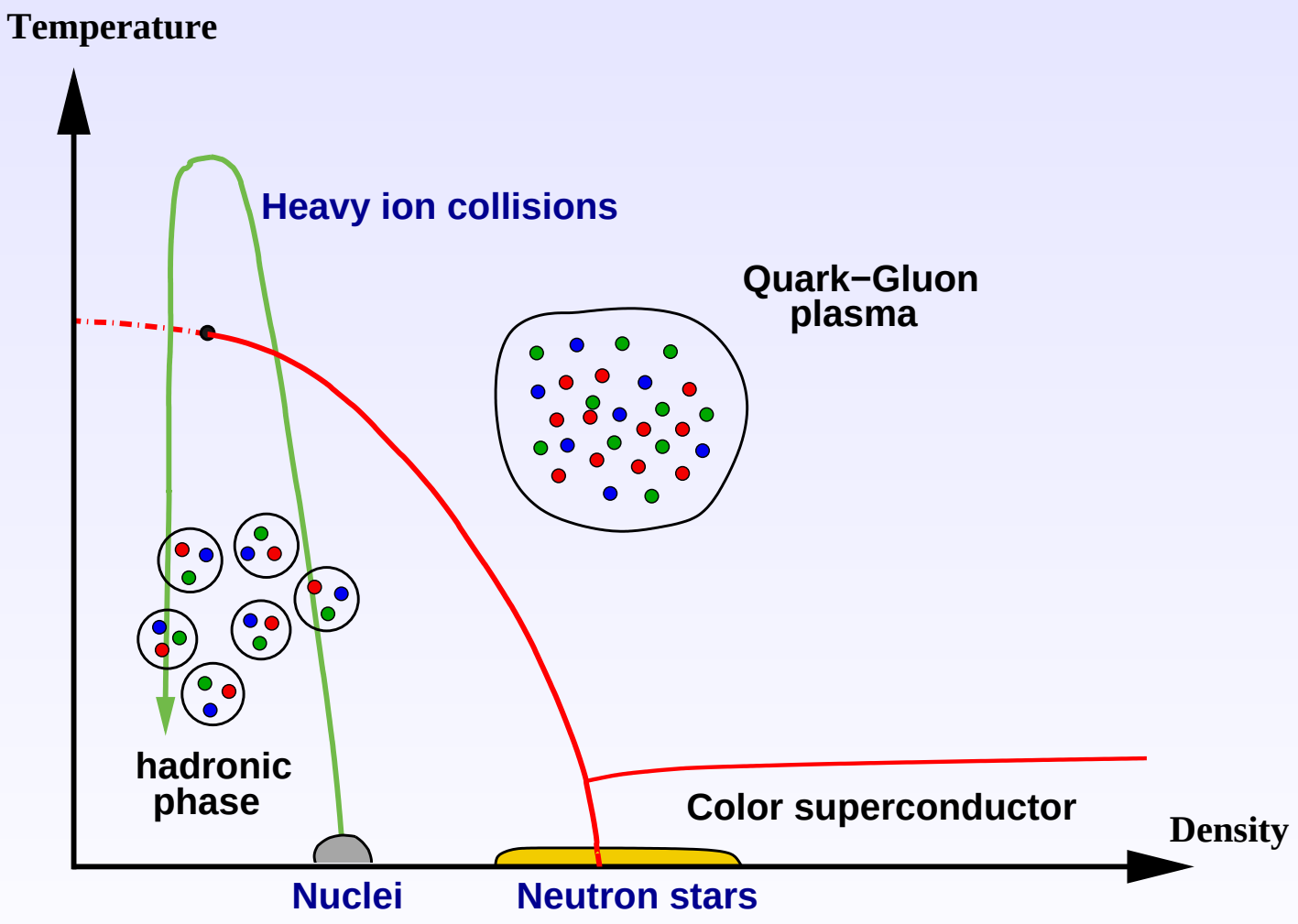


Heavy ion collisions



What would we like to learn?

i. Establish the existence of a phase transition

ii. Parameters of the transition: T_c, ϵ_c

iii. Equation of state of nuclear matter

iv. Transport properties of nuclear matter

v. Do some hadrons survive in the QGP?

vi. Dynamics of the collision, evolution at early time, formation of the QGP and thermalization

What we must get out of the way first...

- Unfortunately, heavy ion collisions also depend on a number of other trivial facts:

i. Lead nuclei are approximately spherical

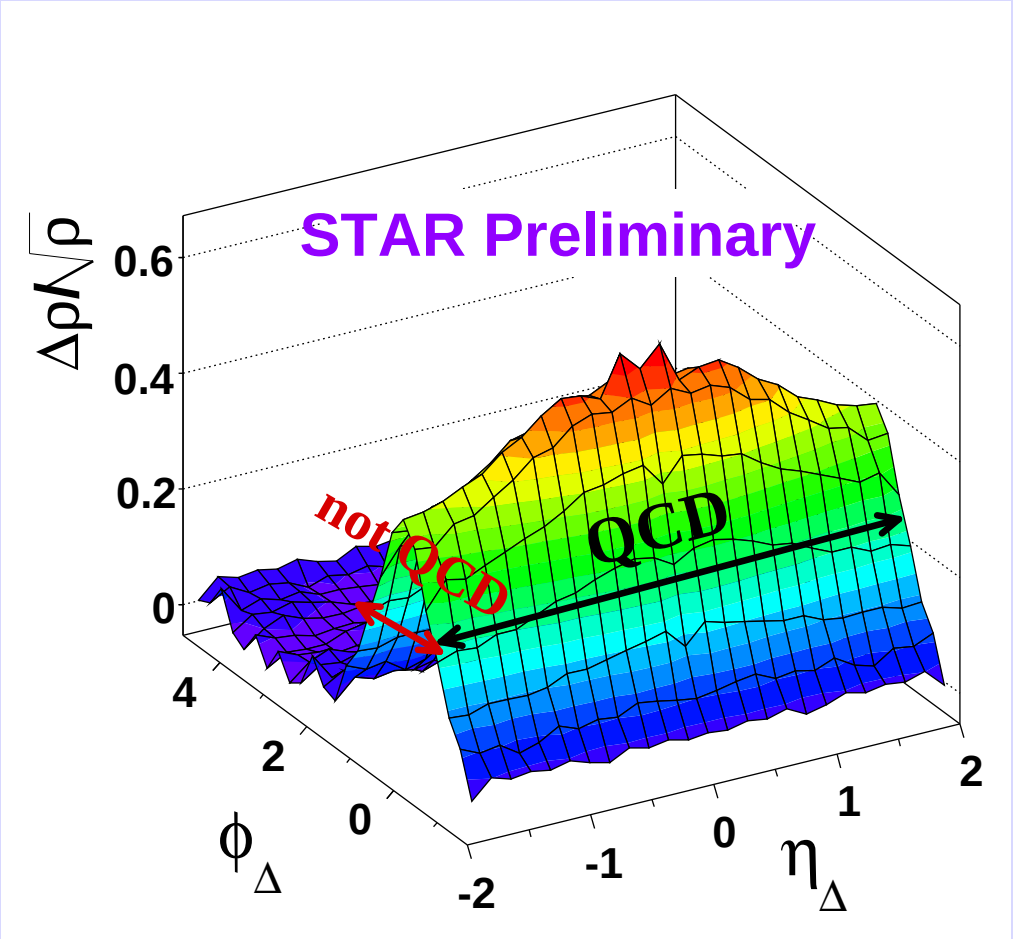
ii. Their diameter is about 12 fermis

iii. They contain $A \approx 200$ nucleons

iv. The positions of these nucleons fluctuate

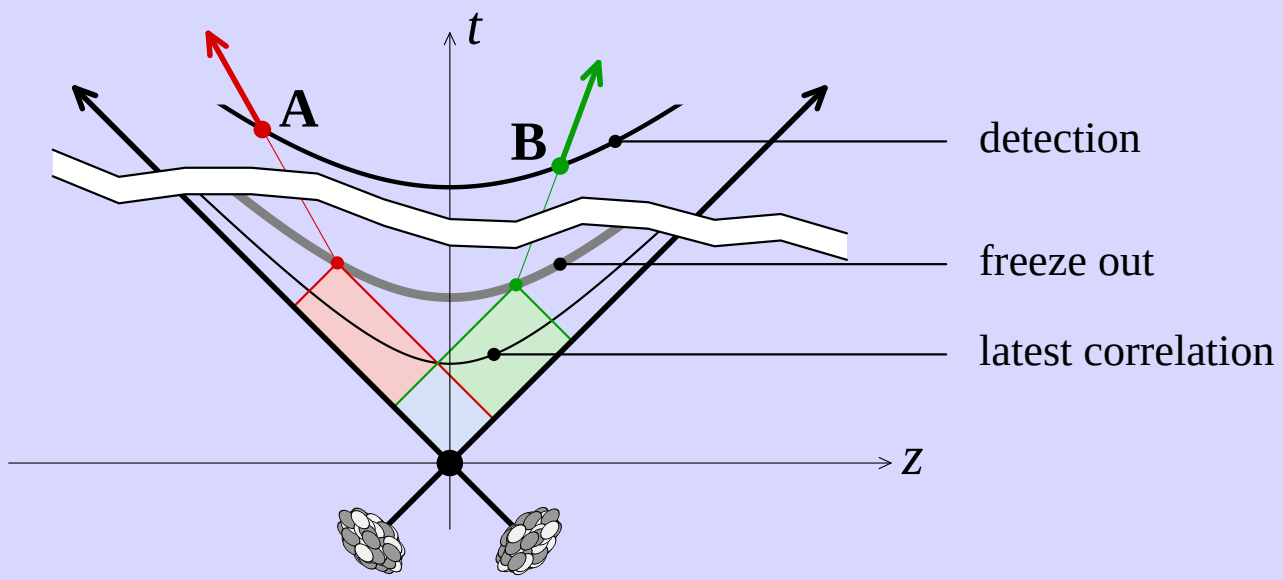
- These properties have all an incidence on observables
- None of them is interesting from the point of view of QCD
- We need ways to make observables independent of these trivial aspects of nuclear physics

Example: 2-hadron correlations (aka “the ridge”)



- Long range correlation in $\Delta\eta$ (rapidity)
- Narrow correlation in $\Delta\varphi$ (azimuthal angle)

Long range rapidity correlations



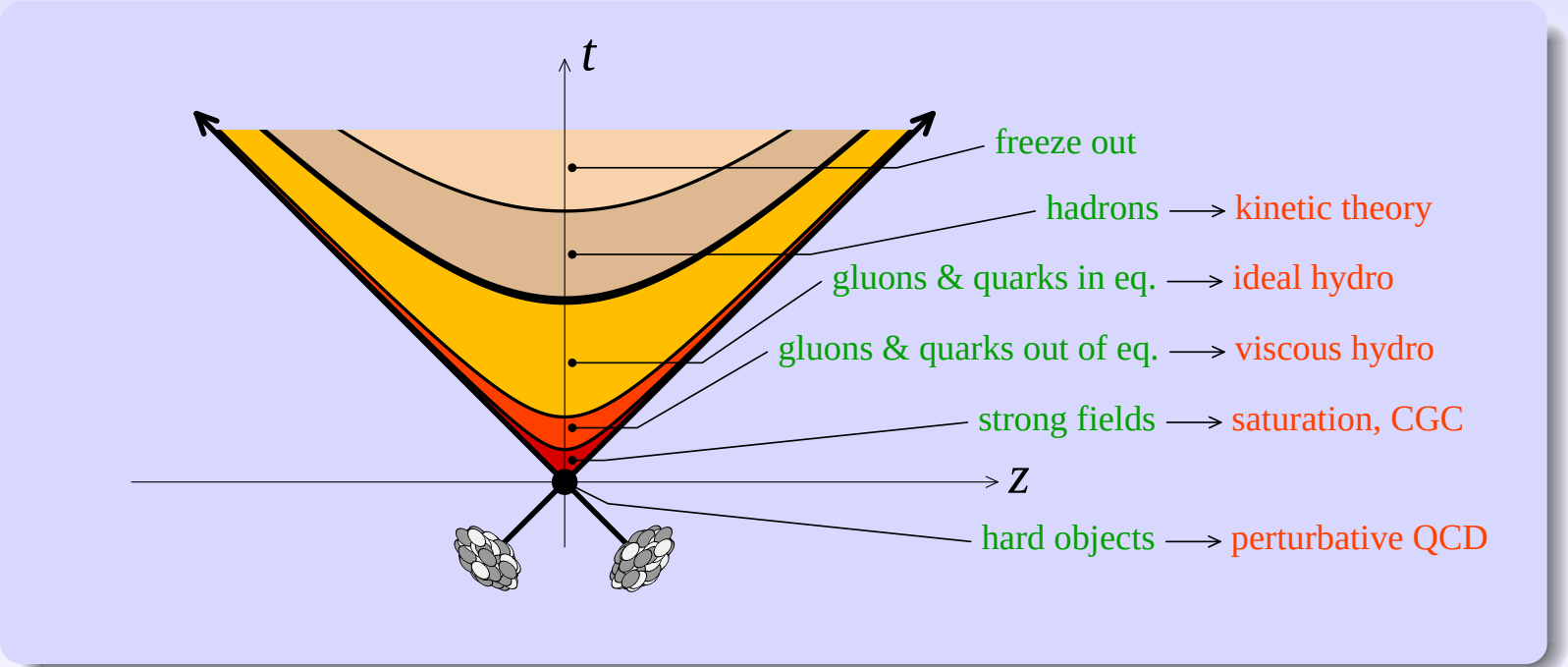
Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \leq t_{\text{freeze out}} \; e^{-\frac{1}{2}|y_A - y_B|}$$

With $t_{\text{freeze out}} = 10$ fm/c, $|y_A - y_B| = 6$: $t_{\text{correlation}} \leq 0.5$ fm/c

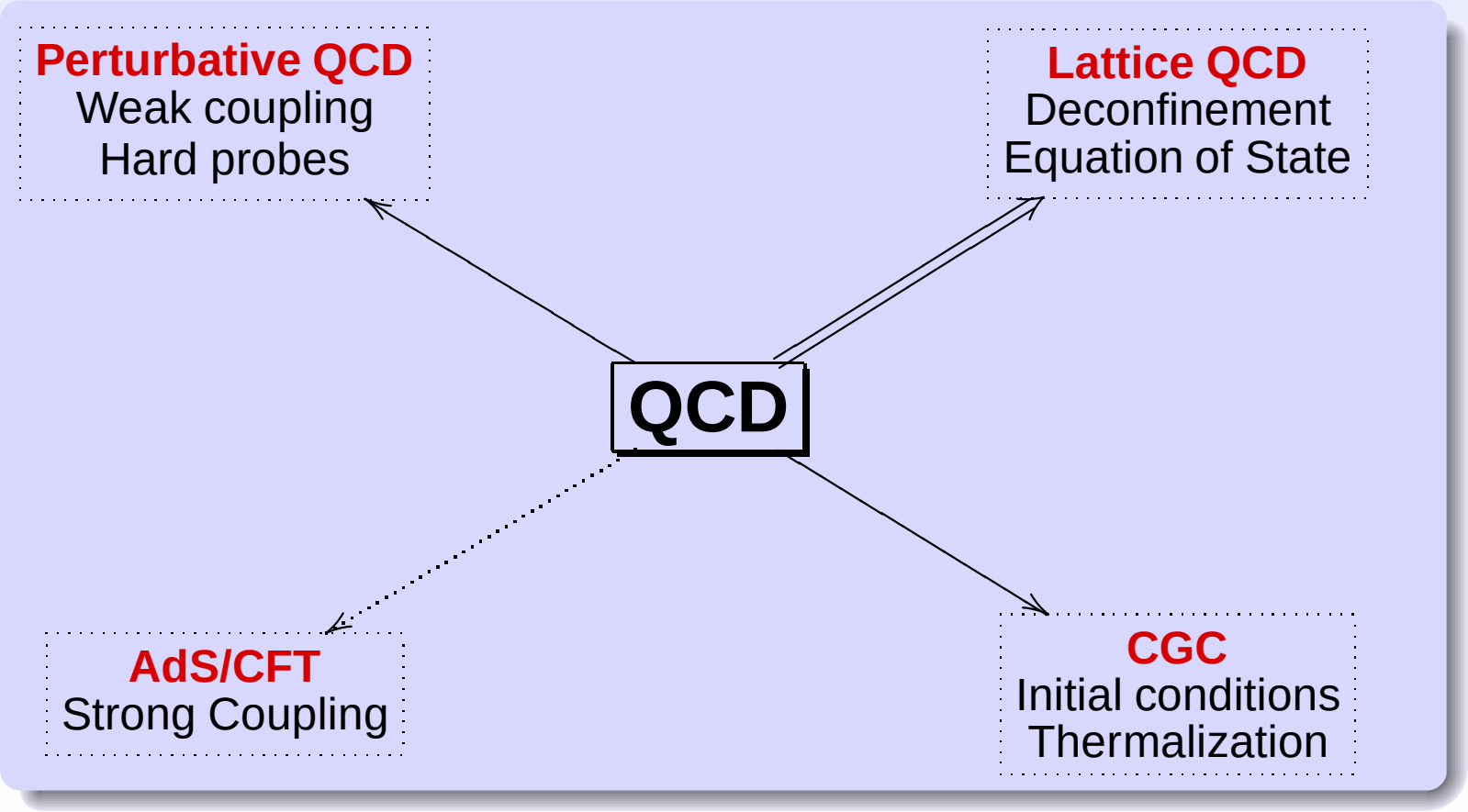
The multiple facets of QCD in HIC



- Except for the production of hard objects (jets, heavy quarks, direct photons) at the impact of the two nuclei, we have to deal with strong interactions in a non-perturbative regime
  NOTE: non-perturbative $\neq$ strongly coupled!!!
- One treats these situations with a range of effective descriptions (CGC, hydrodynamics, kinetic theory) that are more or less closely related to QCD, but always require some QCD input

The multiple facets of QCD in HIC

- The simple formulation of QCD is deceptive: Ab initio calculations are very difficult, and feasible only for a handful of questions
- In many instances, it is more efficient to use an effective theory in which inessential degrees of freedom have been integrated out



The multiple facets of QCD in HIC

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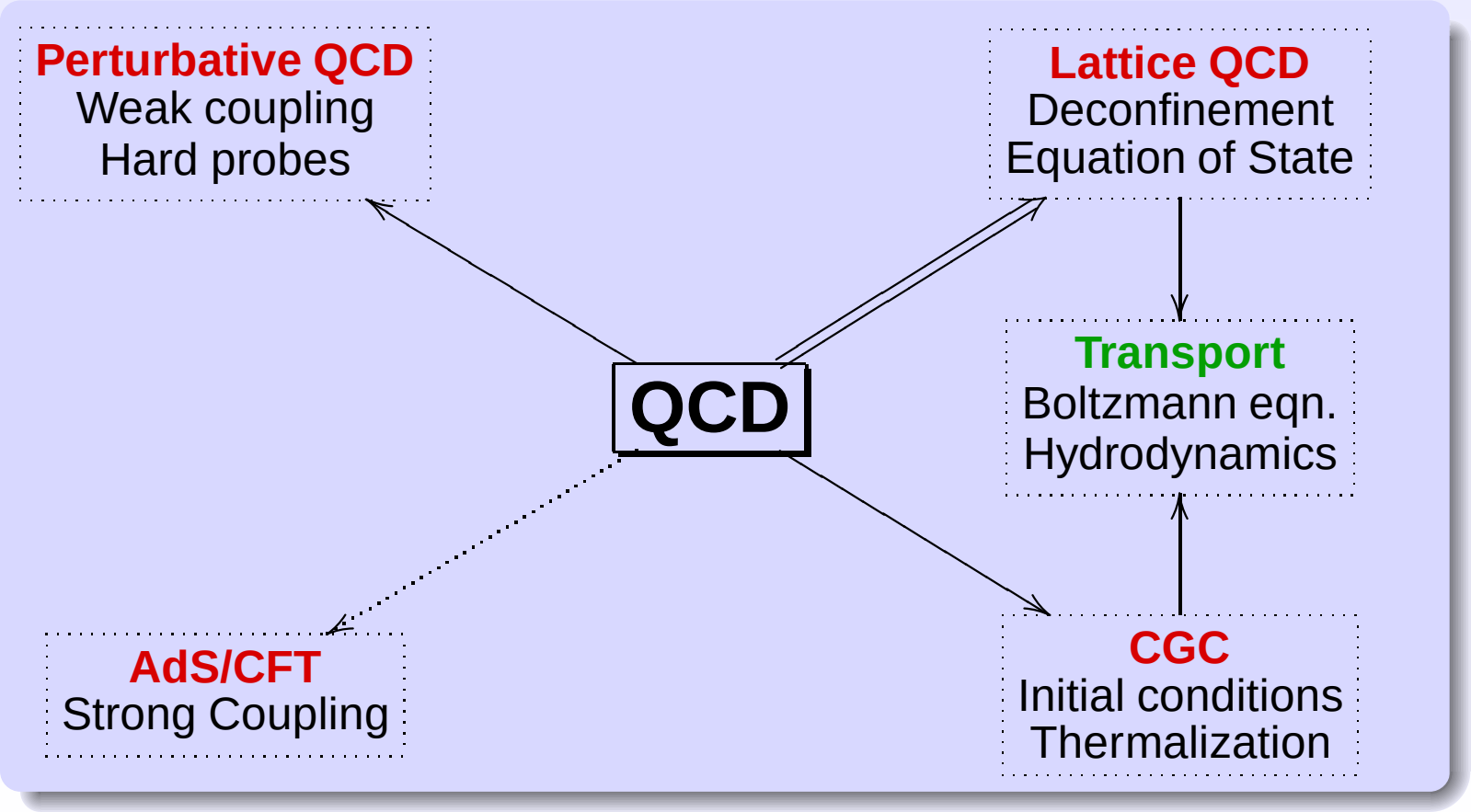


Transport models

- In many cases, the description of the system can be done at a scale large enough for the microscopic details to become irrelevant:
  - Kinetic theory
  - Hydrodynamics
- To a large extent, the evolution of the system is driven by conservations laws (energy, momentum, baryon number...)
- The microscopic dynamics is relegated into a handful of quantities that enter in these mesoscopic descriptions

Kinetic theory

- The system is described by a particle distribution

$$f(t,\vec{\boldsymbol{x}},\vec{\boldsymbol{p}}) = \frac{dN}{d^3\vec{\boldsymbol{x}}d^3\vec{\boldsymbol{p}}}$$

  (in most cases, this distribution is spin and color averaged)

- The evolution of f is driven by the interactions between these particles

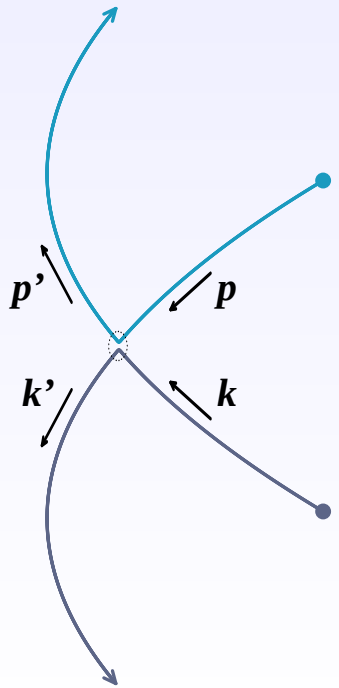
- The only QCD input is a set of cross-sections

Boltzmann equation

- The Boltzmann equation describes the evolution of a distribution of particles that undergo short range collisions

$$\left[\partial_t + \vec{v}_p \cdot \vec{\nabla}_x\right] f(t,\vec{x},\vec{p}) = \underbrace{\mathcal{C}_p[f]}_{\text{collisions}} \qquad \text{with } \vec{v}_p \equiv \frac{\vec{p}}{E_p}$$

- Elementary 2-body collision :



Boltzmann equation

- For $2 \to 2$ collisions, the collision term reads :

$$\mathcal{C}_{\boldsymbol{p}}[f] = \frac{1}{2E_{\boldsymbol{p}}} \int \frac{d^3\vec{\boldsymbol{p}}'}{(2\pi)^3 2E_{\boldsymbol{p}'}} \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}} \int \frac{d^3\vec{\boldsymbol{k}}'}{(2\pi)^3 2E_{\boldsymbol{k}'}} \underbrace{(2\pi)^4 \delta(p+k-p'-k')}_{E,\vec{\boldsymbol{p}} \text{ conservation}}$$

$$\times \underbrace{\Big[f(\vec{\boldsymbol{p}}')f(\vec{\boldsymbol{k}}')(1+f(\vec{\boldsymbol{p}}))(1+f(\vec{\boldsymbol{k}})) - f(\vec{\boldsymbol{p}})f(\vec{\boldsymbol{k}})(1+f(\vec{\boldsymbol{k}}'))(1+f(\vec{\boldsymbol{p}}')) \Big]}_{\text{micro-reversibility, detailed balance}} \underbrace{\left|\mathcal{M}\right|^2}_{\text{QCD}}$$

▷ Most of the equation relies on conservation laws and general principles of statistical physics. Only the cross-section depends on QCD

Inputs

i. Cross-sections

ii. Initial condition $f(t_0, \vec{\boldsymbol{x}}, \vec{\boldsymbol{p}})$

Hydrodynamics : limit of kinetic theory when $\ell_{\rm mfp} \to 0$

Equations of hydrodynamics (conservation laws)

$$\partial_\mu T^{\mu\nu} = 0 \qquad , \qquad \partial_\mu J_B^\mu = 0$$

Assumptions and inputs

i. Near equilibrium form of $T^{\mu\nu}$:

$$T^{\mu\nu} = \underbrace{(p+\epsilon)\, v^\mu v^\nu - p\, g^{\mu\nu}}_{\text{ideal hydro}} \oplus \underbrace{(\eta, \zeta)\partial v}_{\text{viscous terms}} \oplus \cdots$$

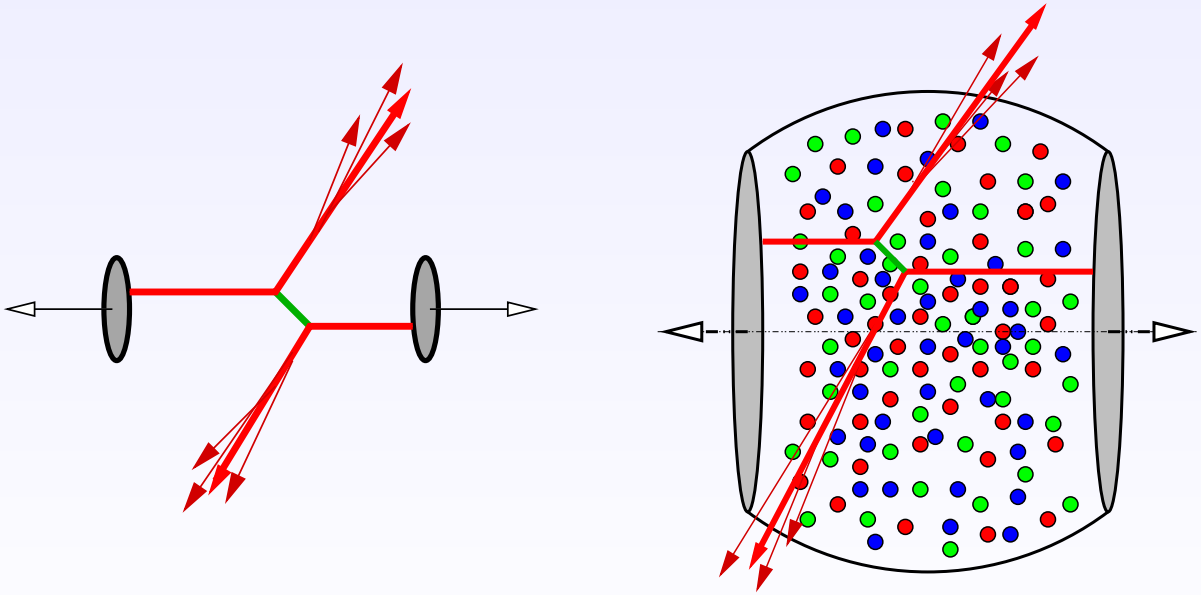
ii. Equation of State: $p = f(\epsilon)$

iii. Transport coefficients: $\eta, \zeta, \cdots$

iv. Initial condition for ϵ and $\vec{v}$ at some t_0

Jet quenching

- The basis of perturbative QCD is asymptotic freedom
- pQCD is the tool of choice for computing the production of hard objects (high $p_\perp$ jets, direct photons, heavy quarks)
- In heavy ion collisions, a new challenge for QCD is the study of the propagation of a hard object in a dense quark-gluon medium



Partition function

- Partition function :

$$Z \equiv \mathrm{Tr}\,(e^{-\beta H}) = \int [\mathcal{D}A^{\mu}\mathcal{D}\overline{\psi}\mathcal{D}\psi]\; e^{-S_E\,[A^{\mu},\overline{\psi},\psi]}$$

- S_E is the Euclidean action, with imaginary time in $[0, \beta = 1/T]$. The Matsubara formalism provides a way to do perturbative calculations at finite T

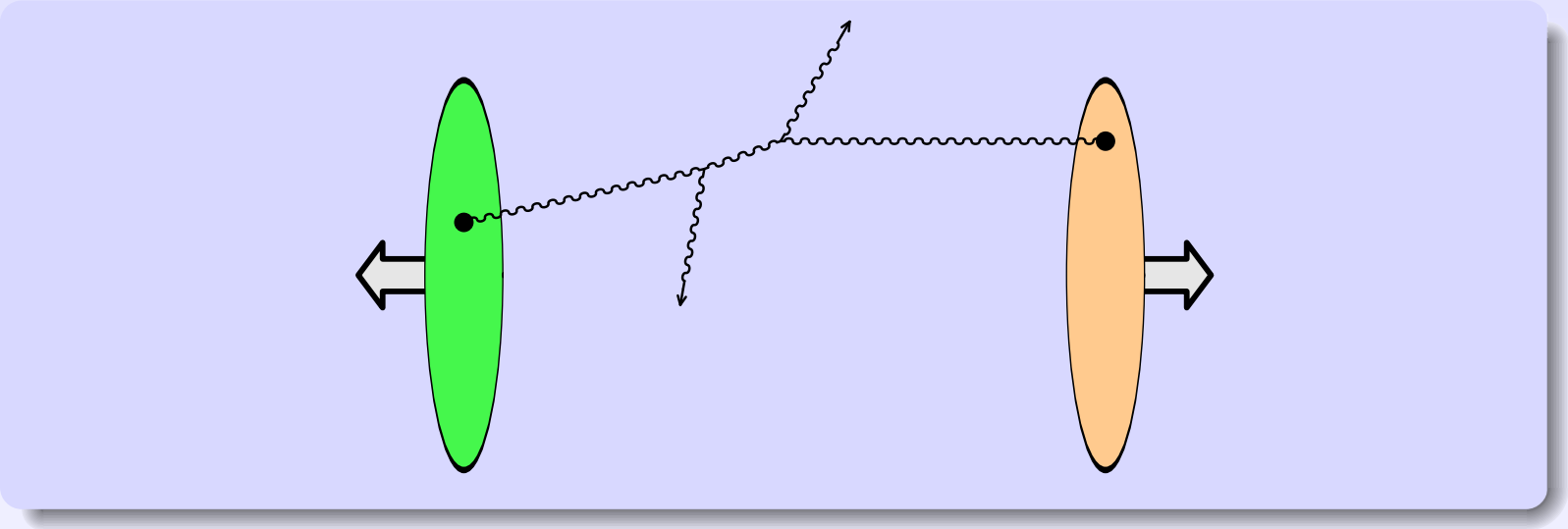
- Z knows everything about the QGP thermodynamics :

$$E = -\frac{\partial Z}{\partial \beta}$$
$$S = \beta E + \ln(Z)$$
$$F = E - TS = -\frac{1}{\beta}\ln(Z)$$

Lattice QCD

- Lattice QCD : discretize space-time, and approximate the functional integration by a Monte-Carlo sampling
- "Sign problem" :
  - does not work for "real time" correlation functions
    ▷ limited to static properties of the QGP (thermodynamics)
  - does not work with a baryon chemical potential
- Light quarks with realistic masses are computationally expensive

Longitudinal momentum fraction in AA collisions



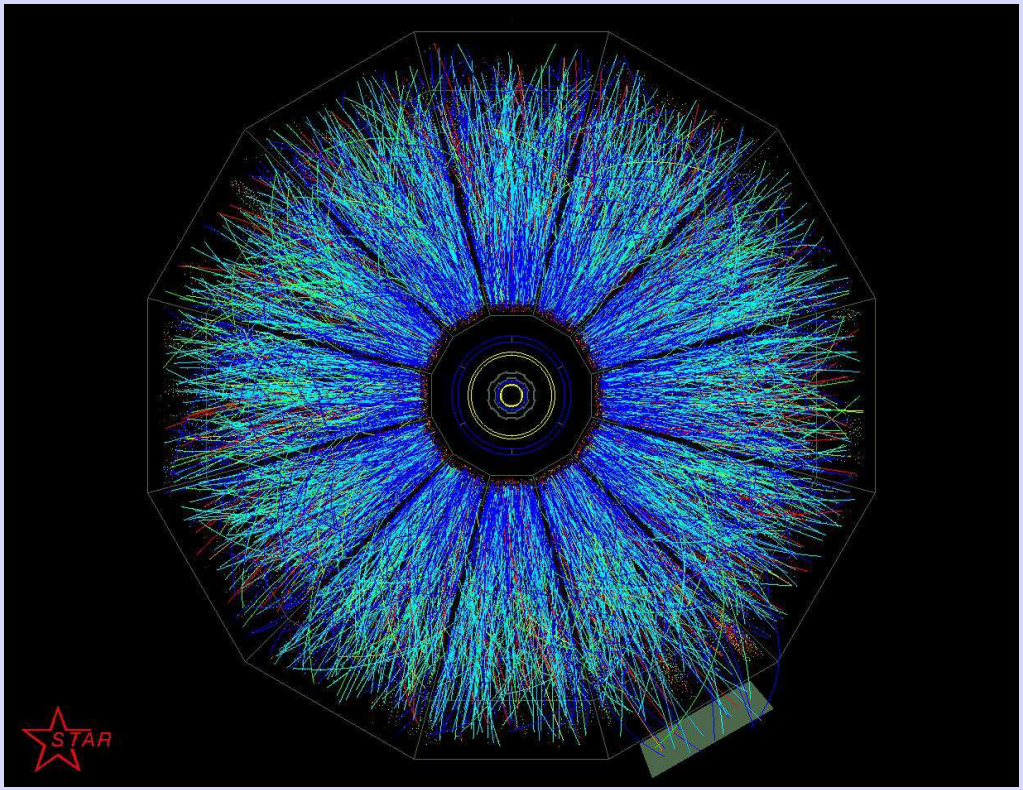
- The partons that are relevant for the process under consideration carry the longitudinal momentum fractions:

$$x_{1,2} = \frac{P_\perp}{\sqrt{s}} e^{\pm Y}$$

  - $P_\perp$: transverse momentum
  - Y : rapidity
  - $\sqrt{s}$: collision energy

Longitudinal momentum fraction in AA collisions

Nucleus-Nucleus collision

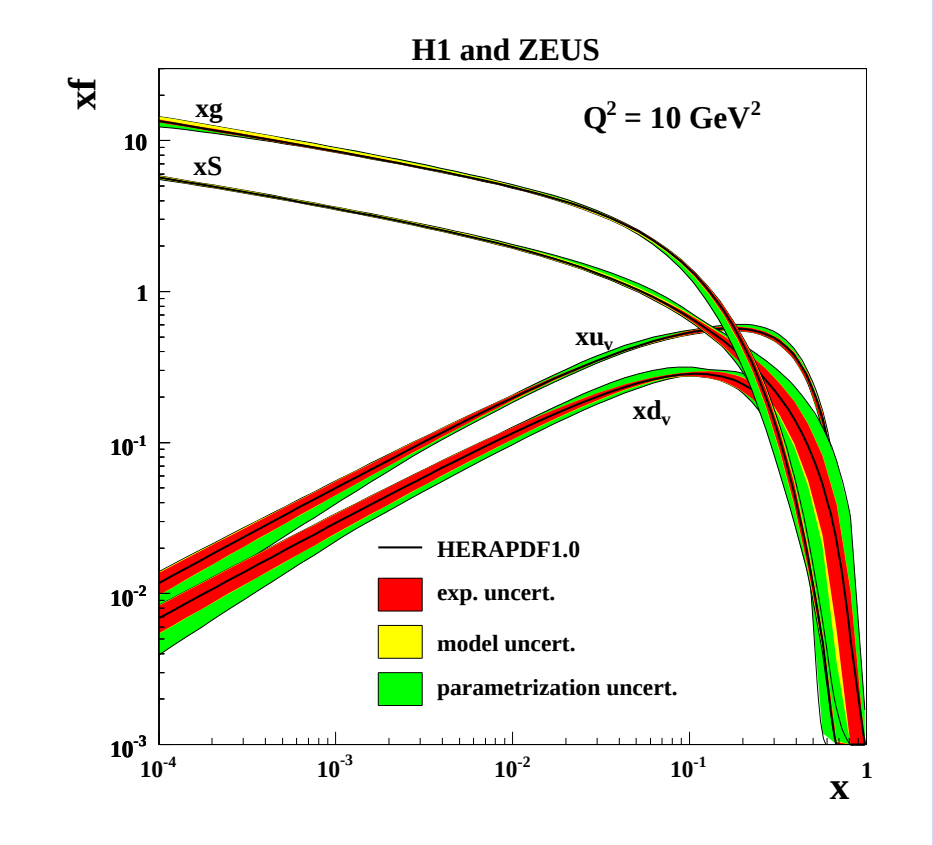


- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

  $\triangleright$ partons at small x are the most important

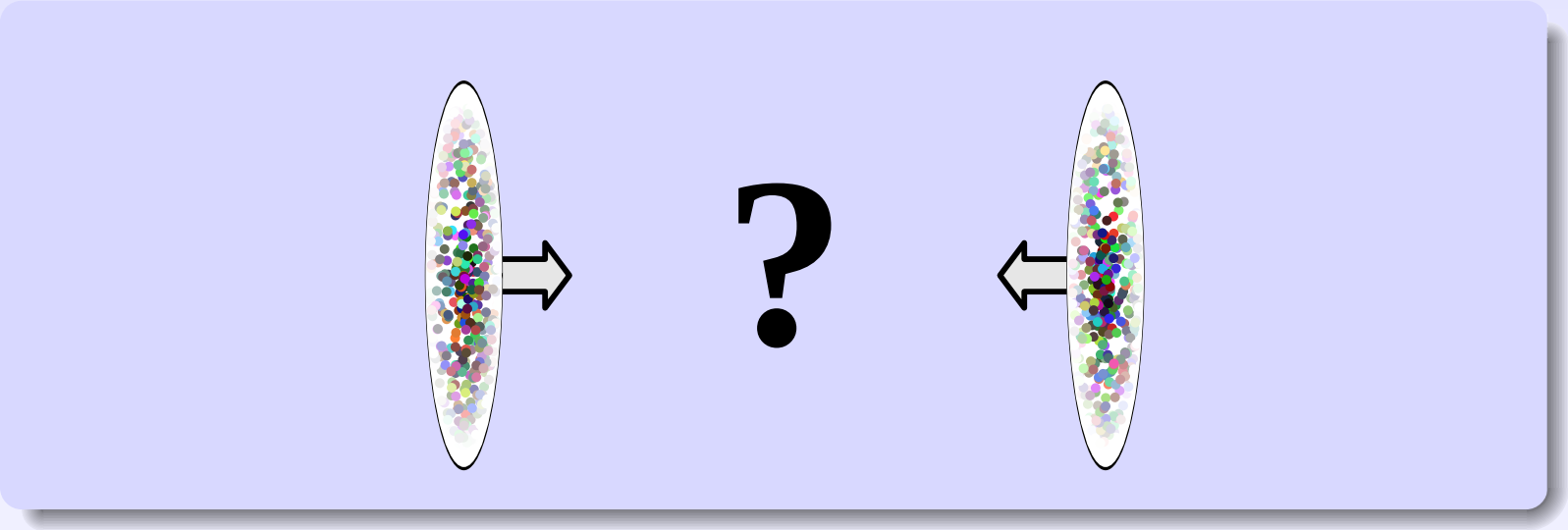
Growth of the gluon distribution at small x

Parton distributions at small x



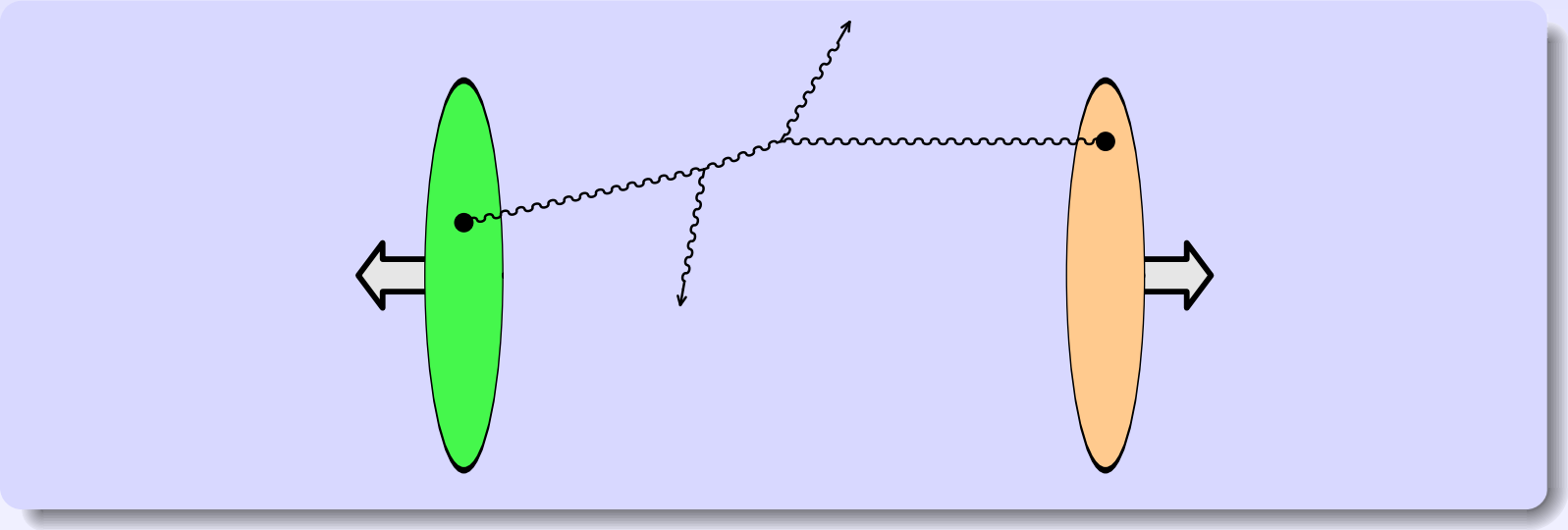
- Gluons dominate at any $x \leq 10^{-1}$

Multiple scatterings and gluon recombination



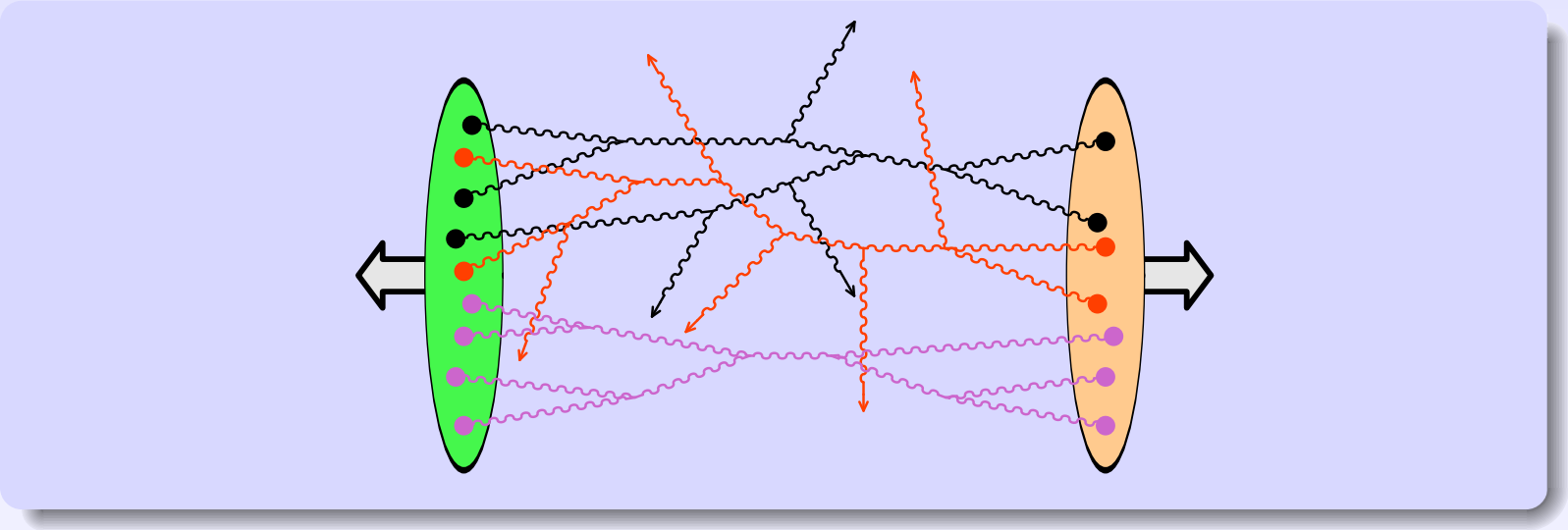
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ large Q^2, no small-x effects
  - ▷ usual PDFs + DGLAP evolution

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial
  - ▷ gluon recombinations are important (saturation)
  - ▷ multi-parton distributions + JIMWLK evolution
  - ▷ new techniques are required (Color Glass Condensate):

$$\mathcal{L} = -\frac{1}{4}F^2 + J \cdot A$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

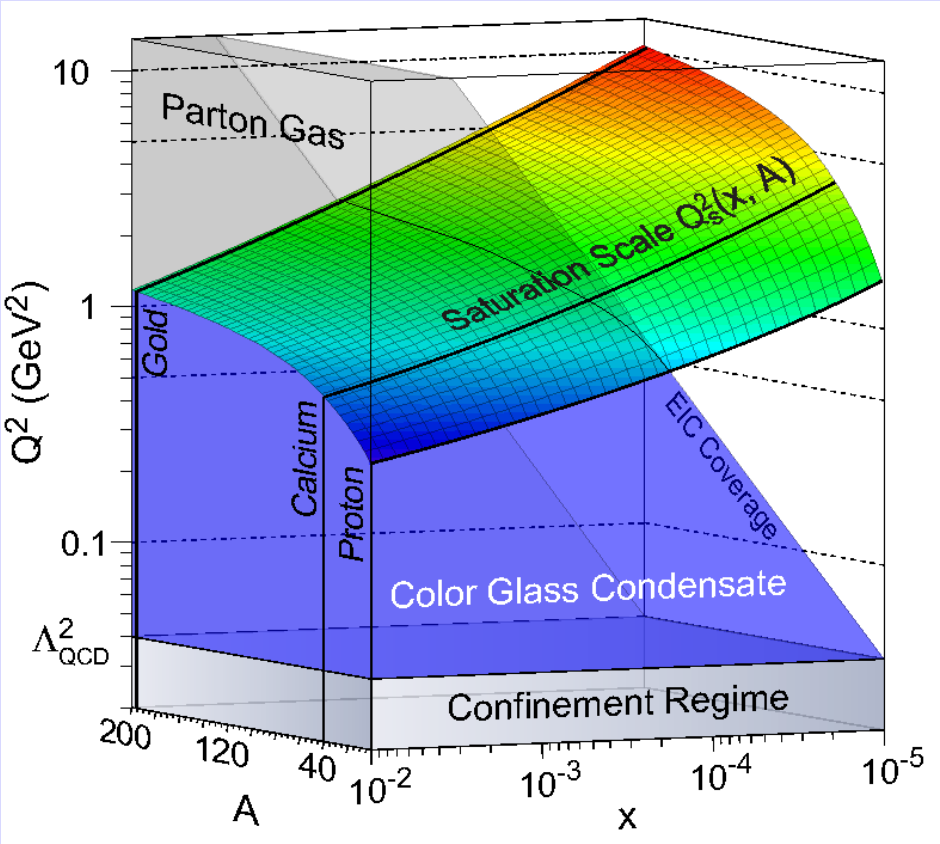
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Saturation domain

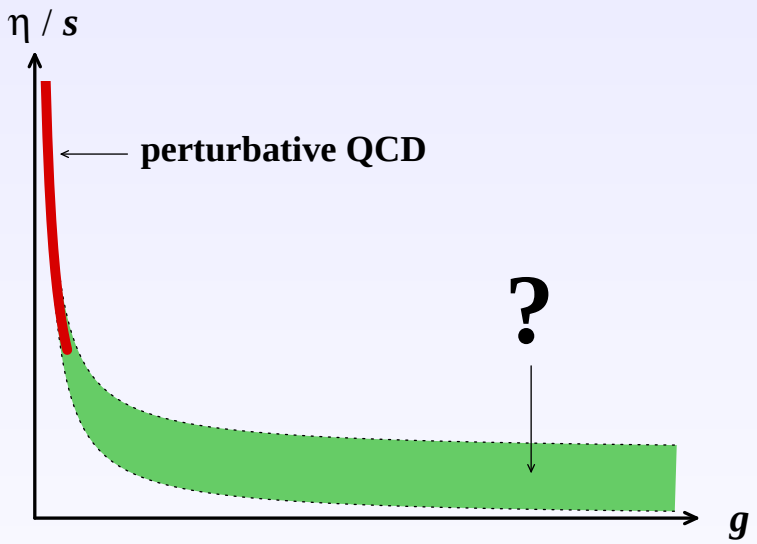
Saturation scale as a function of x and A



Viscosity at weak coupling

- The shear viscosity has been calculated in QCD at weak coupling ($g \to 0$), and it is quite large :

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



- However, η/s decreases quickly when the coupling increases $\triangleright$ Can we calculate it?

AdS/CFT duality at T=0

- Maximally super-symmetric $SU(N)$ Yang-Mills theories in the limit $g^2N \to +\infty$ are dual to classical super-gravity on an $\mathrm{AdS}_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2}(\underbrace{-dt^2 + d\vec{x}^2}_{\text{we live here... (at z=0)}} + dz^2) + R^2 d\Omega_5^2$$

- If an operator $\mathfrak{O}$ of our world is coupled on the boundary to a field φ_0 that extends in the bulk, the duality states that :

$$e^{-S_{\mathrm{cl}}[\phi]} = \left\langle e^{\int_{\mathrm{boundary}} \mathfrak{O}\,\varphi_0} \right\rangle$$

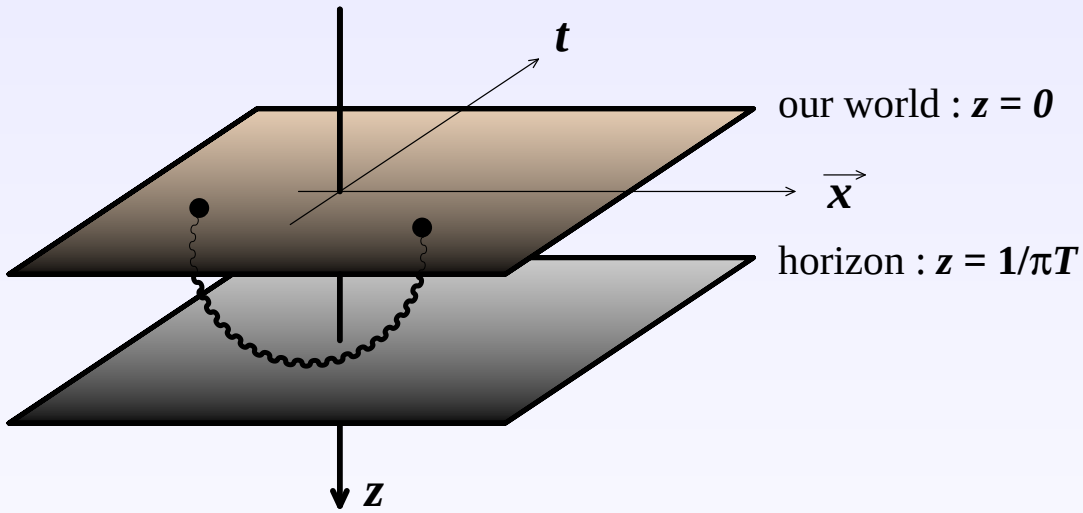
  - The right hand side is a generating functional for the correlators of operators $\mathfrak{O}$ in the 4-dim gauge theory
  - The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition φ_0)

AdS/CFT duality at high T

- At finite temperature T :

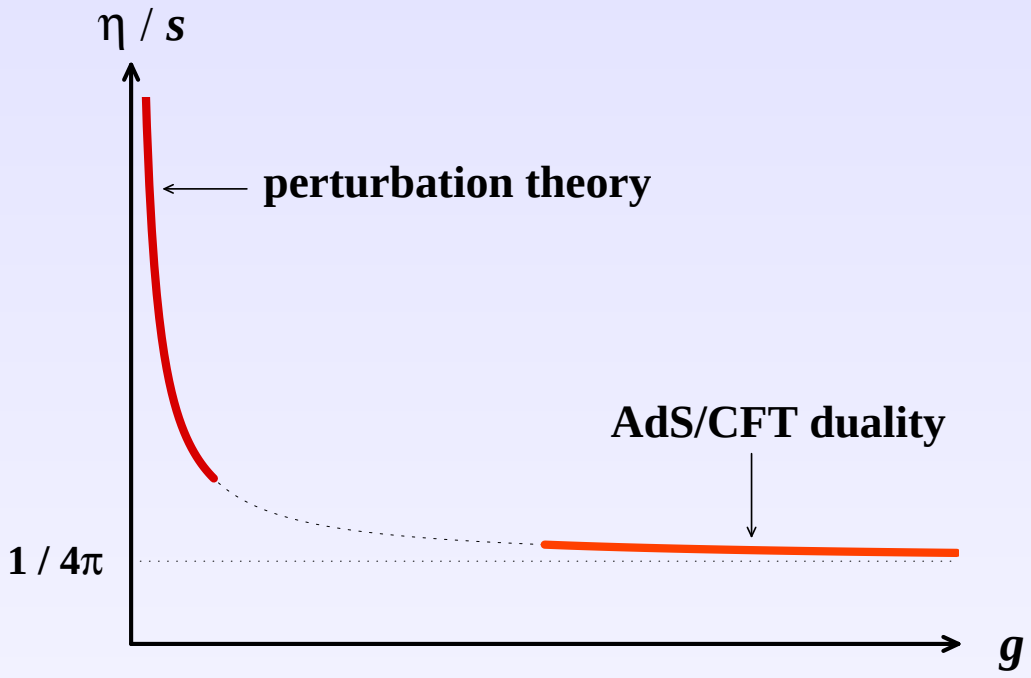
$$-dt^2 + dz^2 \to -f(z)dt^2 + dz^2/f(z) \quad \text{with } f(z) = 1 - (\pi z T)^4$$

- $f(z) = 0$ at $z = 1/\pi T \quad \Rightarrow$ black hole horizon



- Ordinary particles in 4-dimensions are the end points of strings living in the bulk. Temperature effects occur when a string gets close to the BH horizon

Viscosity in SUSY Yang-Mills



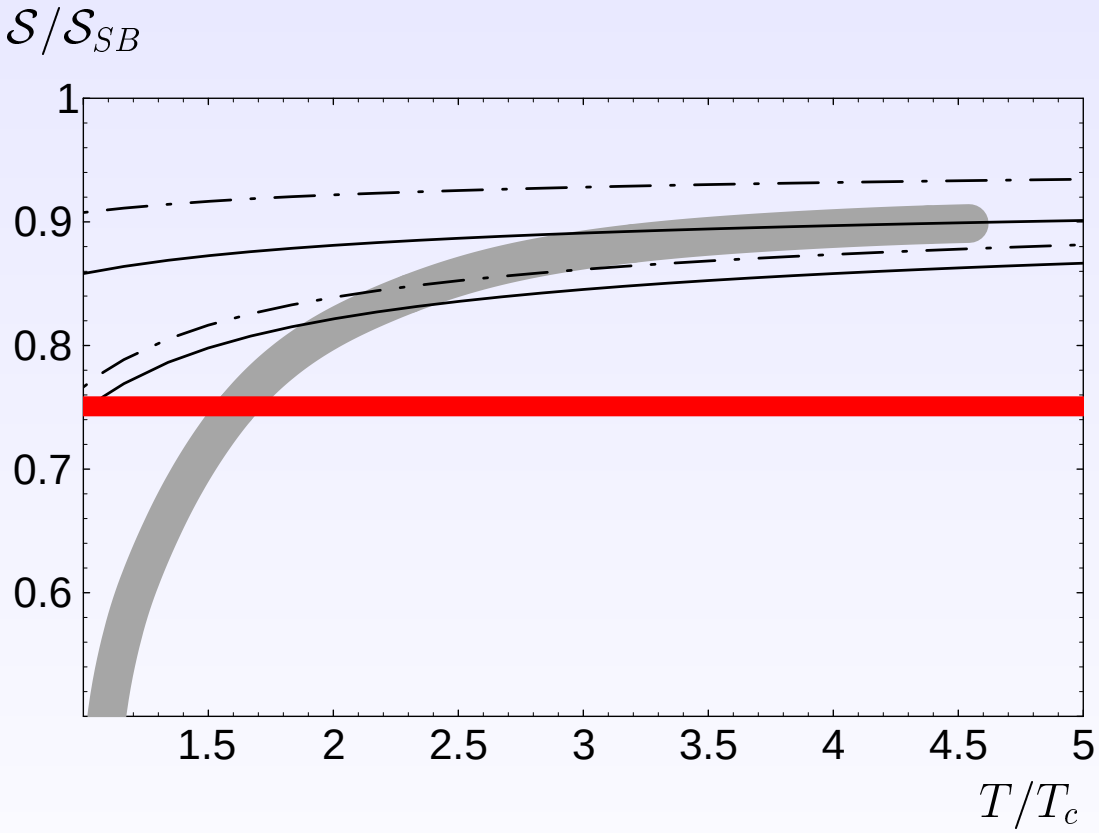
- In SYM at $g^2N \to \infty$, one gets $\eta/s = 1/4\pi$
- Conjecture : $1/4\pi$ is the lowest possible value for η/s
- Note: all the known substances have a viscosity to entropy ratio (much) larger than that

SUSY YM $\neq$ QCD

- AdS/CFT only applies to maximally super-symmetric Yang-Mills theories. Such theories are scale invariant, have no running coupling, no chiral symmetry breaking, and no confinement
- Whether what we learn about these theories is accurate for QCD (that has broken scale invariance, running coupling, chiral symmetry breaking, confinement, and quite different matter fields...) is at best a wishful thinking
- Nevertheless an interesting playground in order to realize how wrong one's weak coupling prejudices may be...

Importance of scale violations

- Is the QGP at $T/T_c \sim 2-3$ really strongly coupled? For quantities such as the entropy, perturbative techniques (+resummations) lead to sensible results in this region



- At $T < T_c$, the coupling may indeed be strong, but scale violations make AdS/CFT unreliable

Summary

- QCD in heavy ion collisions displays a very rich spectrum of phenomena
- Ab initio methods (lattice) are often impractical in QCD
- The consequence of this is the diversity of tools and techniques that have been developed to study various aspects of strong interactions in heavy ion collisions
- QCD also plays a role in providing inputs into a number of effective descriptions such as kinetic theory and hydrodynamics

Thank You!