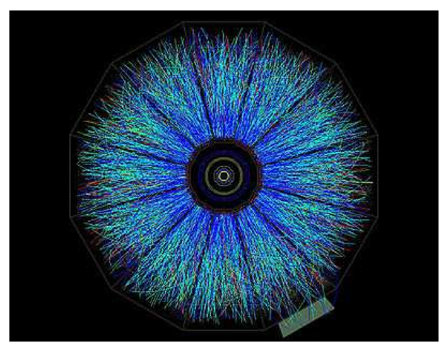
The phase diagram in T, μ, N_c-space

G.Torrieri, with I.Mishustin, S.Lottini, P.Nicolini

FIAS, J.W. Goethe–Universität, D-60438 Frankfurt am Main, Germany



Introduction: Large N_c

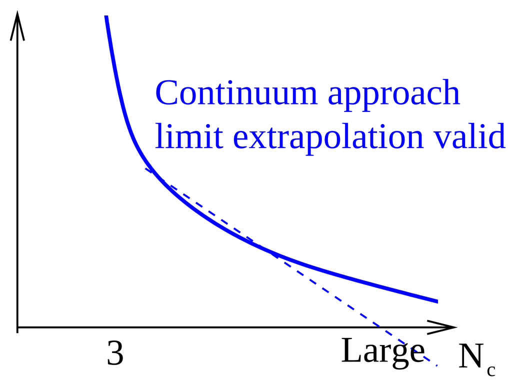
For a Non-abelian Gauge theory $SU(N_c)$ ($N_c = 3$ for QCD), significant simplifications occur in the "large number of colors" limit $N_c \to \infty, g_{YM}^2 N_c \to \lambda|_{constant}$

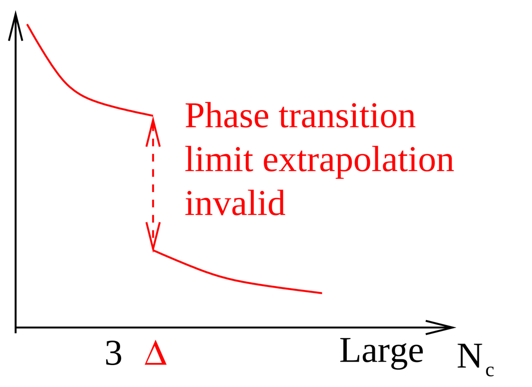
- Planar diagrams dominate Strong force $\to$ strings
- Mesons $\to$ weakly interacting quasiparticles
- Baryons $\to$ strongly interacting semi-classical states

A lot of recent field theory advances (eg AdS/CFT) derive from this limit. But...

Real N_c not large, corrections $\sim 1/3$, $N_f/N_c \to \mathcal{O}(1)$ precision at most (qualitative!)

Phase transitions in N_c and N_f/N_c? Yang-Mills weird and non-trivial [1–3]



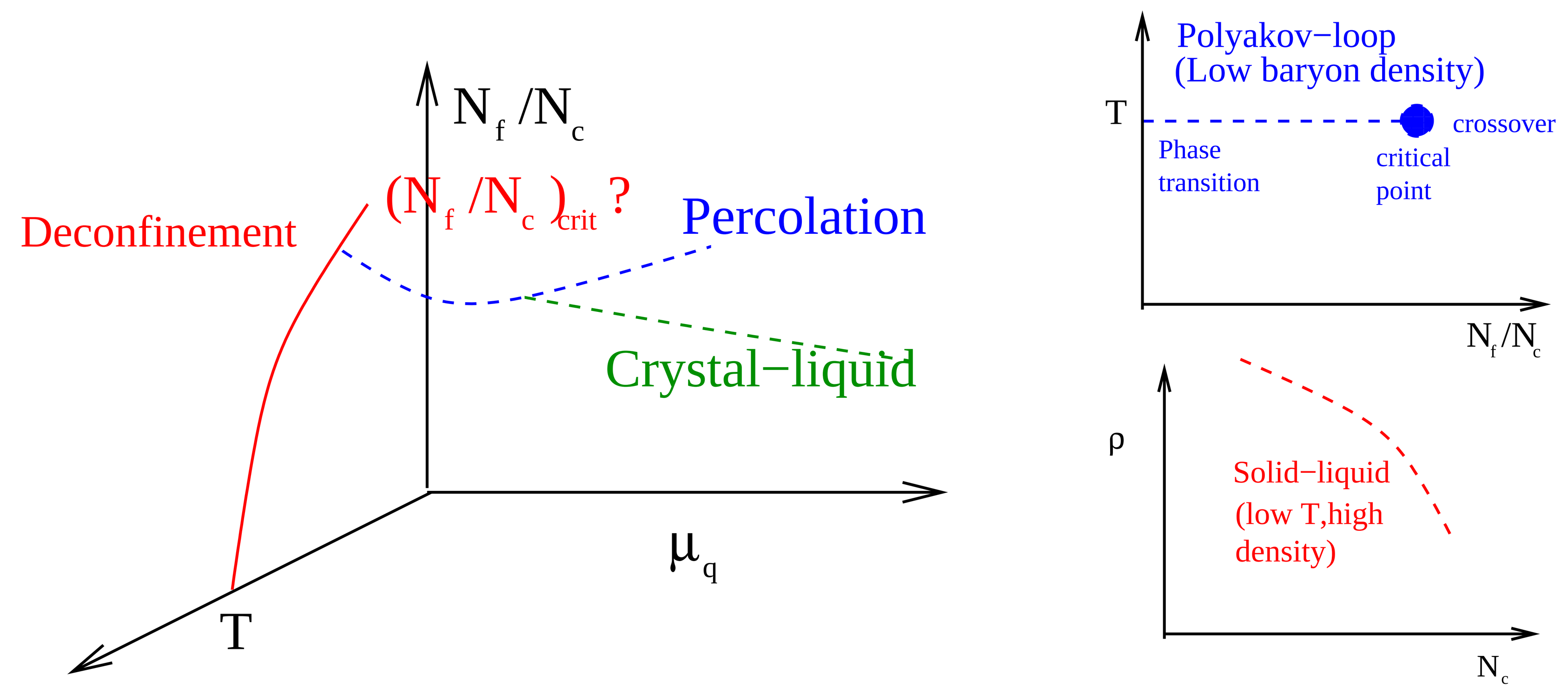


[1] Giorgio Torrieri and Igor Mishustin. . *Phys. Rev.*, C82:055202, 2010.

[2] Stefano Lottini and Giorgio Torrieri. . *arXiv:*, 1103.4824, 2011.

[3] Piero Nicolini and Giorgio Torrieri. . *arXiv:*, 1105.0188, 2011.

A phase transition in N_c: The evidence



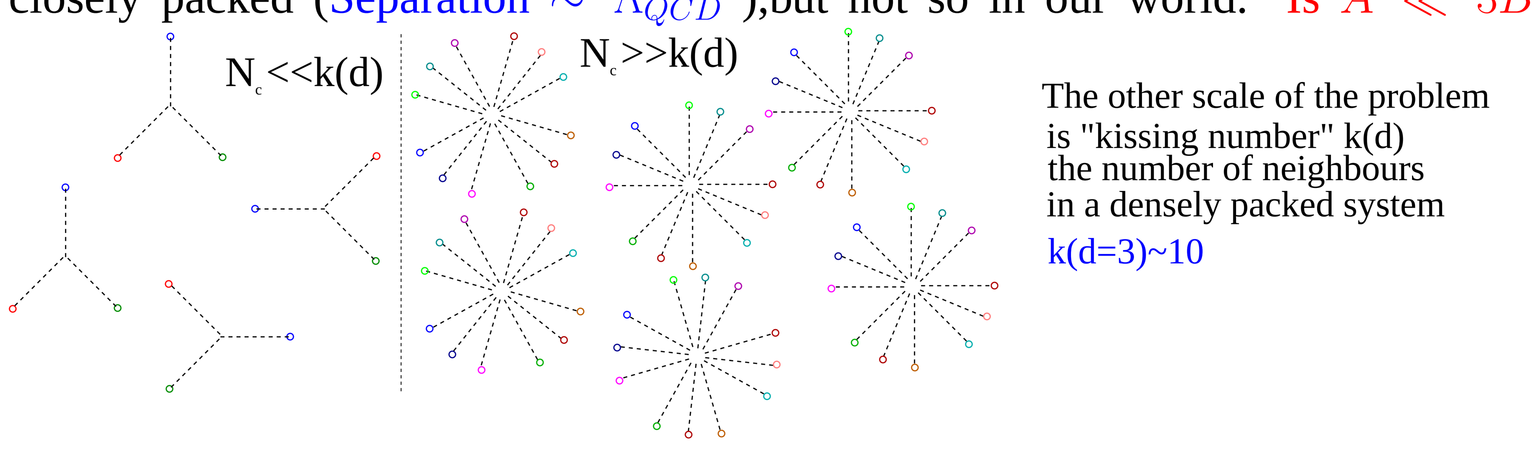
Polyakov loops: Confined $SU(N_c)_{N_f=0}$ invariant under symmetry Z_N, spontaneously broken by deconfinement at high T. Symmetry principles dictate that deconfinement is a phase transition, at $N_f = 0$. At $N_f/N_c \sim 1$, according to the lattice, deconfinement is a cross-over. So most likely there is a critical point in N_c for confinement. And this is controlled by factors subleading in N_f/N_c!

Crystal-liquid transition: At $N_c \to \infty, \mu_B/N_c \sim \Lambda_{QCD}$, the ground state of nuclear matter is widely understood to be a skyrme crystal From: I.Klebanov, Nucl.Phys.B262:133,1985 *...This treatment ignores the kinetic energy of skyrmions. It can be roughly estimated to be* $1/Mca^2 \sim 100$ *MeV* ($\sim N_c^{1/2}$, binding energy $\sim N_c$) *Energy of this order is enough to unbind the crystal a t* $N_c = 3$ This must be a phase transition, as symmetries change!

Phenomenology: Large N_c qualitatively correct for vacuum, not for nuclear matter (Binding energy $\sim m_{baryon}$,baryon level density $\ll \Lambda_{QCD}$). Related to a phase transition?

Hints from Van Der Waals (Rough but universal) [1]

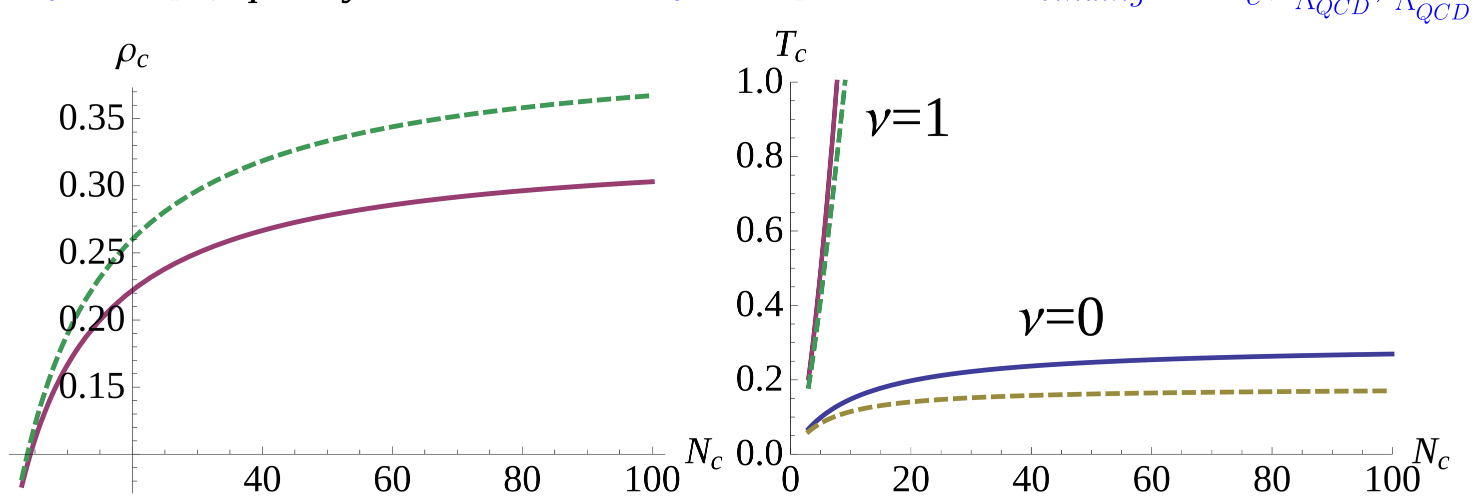
Lets investigate how each parameter changes with N_c . Only possible dependence of excluded volume is $V_{excluded} \sim A + \frac{B}{N_c^\alpha}$ At large N_c natural to assume gas is closely packed (Separation $\sim \Lambda_{QCD}$),but not so in our world. Is $A \ll 3B$? Why?



$N_f N_c \gg k(d)$ Exclusion principle satisfiable just with Ψ_{color}

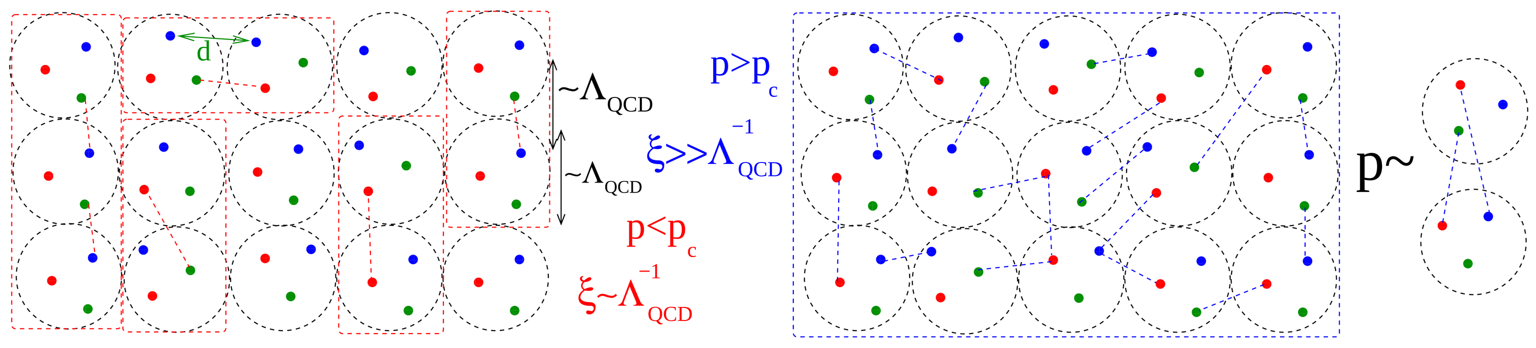
$N_f N_c \ll k(d)$ Exclusion principle needs Ψ_{space} $\to$ cost in energy $\sim \Lambda_{QCD}$

If one assumes that $A \sim 1$ and $B \sim N_N \sim 10$, phase diagram close to nuclear liquid at $N_c \ll N_N$, quarkyonic matter at $N_c \gg N_N$ [1]. For $E_{binding} \sim N_c^\nu, \frac{T_c}{\Lambda_{QCD}}, \frac{\rho}{N_{QCD}^3}$ are...



It makes sense, but its not a proof! A dependence on N_N suggests percolation dynamics

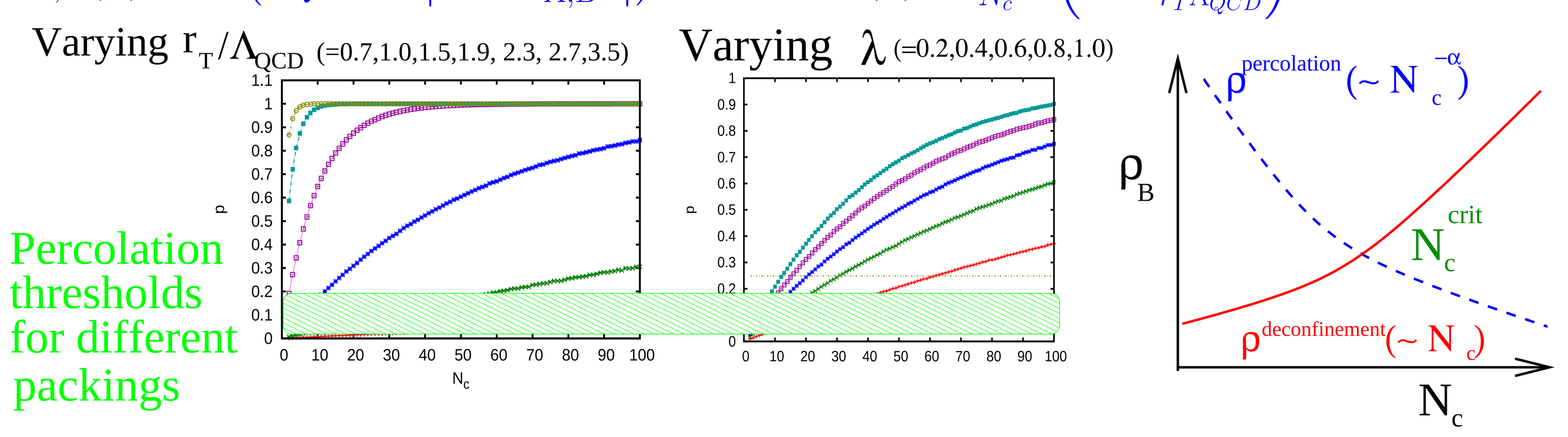
Percolation at finite N_c [2]



$$p = 1 - \left(q_{(1),ij}\right)^{(N_c)^\alpha} \qquad q_{(1),ij} = \int f_A(x_i)\mathrm{d}x_i \int f_B(x_j)\mathrm{d}x_j \left(1 - F(|x_i - x_j|)\right)$$

We assume [2] a "hard-sphere" distribution for $f_{A,B}(x)$ and a Θ -function propagator for the exchange $i \leftrightarrow j$ (modeling confinement within a range $r_T\Lambda_{QCD}$). Mathematically very similar to Glauber model $\to$ Insensitive to most details of propagator!

$$f_{A,B}(x) = \Theta\left(\Lambda_{QCD} - \left|x - x_{A,B}^{\text{center}}\right|\right) \qquad F(d) = \frac{\lambda}{N_c}\Theta\left(1 - \frac{d}{r_T\Lambda_{QCD}}\right)$$



Rapid growth with N_c at $p = p_c$ for all curves. Transition seems universal at $N_c \sim \mathcal{O}(10)$ We can interpret these links as "tunneling" probabilities of quasi-free quarks. Quark wavefunctions above percolation correlated across the whole volume, quark wavefunctions below percolation correlated within the hadron. Percolation regime looks very "quarkyonic". Not deconfinement ($\mu_q^{deconf} \sim N_c\Lambda_{QCD}$) . but percolation, deconfinement cross@N_c^{crit}

$N_c < N_c^{crit}$ $\rho_{deconfinement} \le \rho_{percolation}$, ie percolation transition does not exist

$N_c > N_c^{crit}$ Percolation and deconfinement are separate (Quarkyonic phase?)

What is N_c^{crit} ? Relationship to $\left(N_c^{crit}\right)_{confinement}$? Perhaps! AdS/CFT helps!

Critical N_c with quantum gravity ansatz [3]

In Gauge-gravity picture, the phase transition(s) we are talking about are inherently quantum-gravitational, since varying $N_c @ Fixed \lambda \equiv$ varying g_s .

In normal space, black hole evaporates $\to$ Thermodynamically unstable state!

Put it in a reflecting box (eg A negative cosmological constant $\Lambda = 1/L_{ADS} < 0$!)

$L_{ADS} < x_+^{crit}$ BH+photons heat capacity < 0 , BH decays

$L_{ADS} > x_+^{crit}$ BH+photons heat capacity > 0 , BH stable (BH and photons in equilibrium)

The two regimes connected by Hawking-page (HP) phase transition (1st order).

It is thought confinement$_d$ $\Leftrightarrow$ HP$_{d+1}$

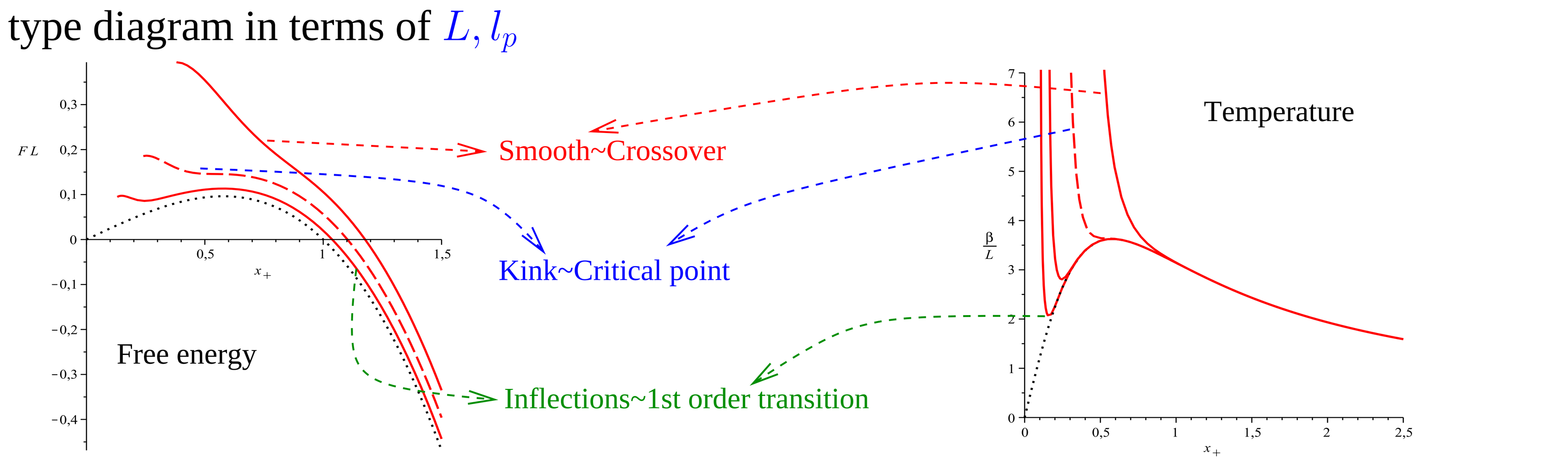
In Gauge world , confinement critical point is understood in terms of symmetries (Z_N).

In Gravity world , HP is most likely a transition because of naked singularity conjecture. You either have a black hole, with a singularity, or you dont!

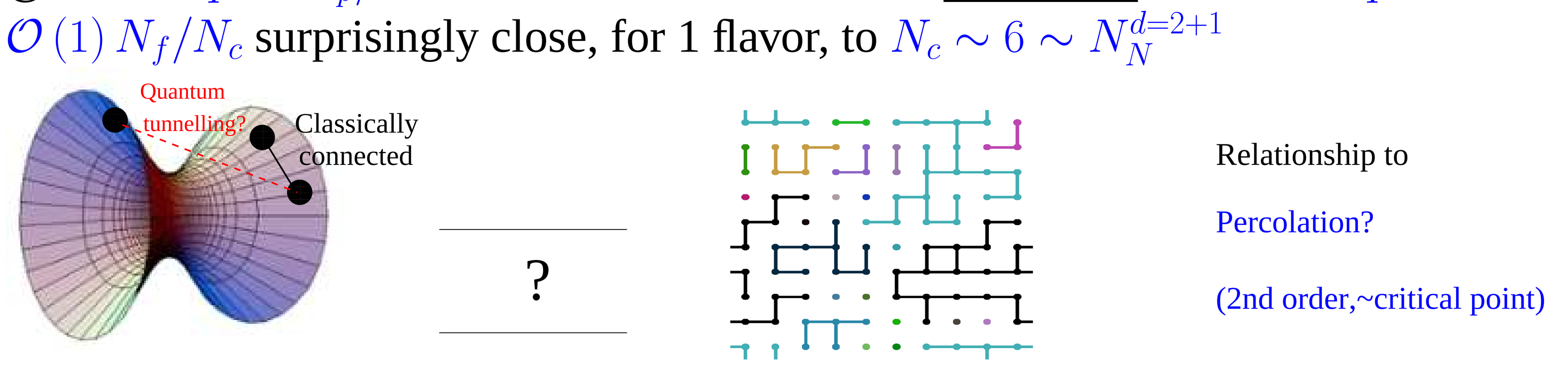
Hence, making confinement into a cross-over is equivalent to smoothening black hole singularity. This can be done via an ansatz inspired by non-commutative geometry [3]: Black hole problem reduces to solving Einsteins equations for

$T_0^0 = \frac{1}{(2\pi l_p)^{3/2}} \exp\left[-\frac{x^2}{2l_p^2}\right] \Rightarrow \delta(x)_{l_p \to 0} \quad T^{\mu\nu}_{;\nu} = 0$ and calculating horizon entropy.

In Flat space Black hole heat capacity becomes positive after critical radius $x_+^{planck} \sim l_p$ (Ansatz used to study remnants). In AdS space , ($x_+^{planck} \sim L_{AdS}$) we get a Van der Waals-type diagram in terms of L, l_p



@critical $q = l_p/L$ HP transition becomes a cross-over. Critical $q^* \simeq 0.18$ If $\Leftrightarrow$ $\mathcal{O}(1)$ N_f/N_c surprisingly close, for 1 flavor, to $N_c \sim 6 \sim N_N^{d=2+1}$



HP coincides with critical density for a gas of black holes in AdS space to collapse. Driven by interplay of inter- black hole distance and the horizon. Non-commutativity fuzzes this over , so black holes can interact over super-horizon distances via tunneling probability, $\sim$ percolation! Connection between deconfinement cross-over and percolation not trivial in SU(N), but understandable in gravity. More rigorous work needed!

Conclusions, discussion and outlook

Several phase transitions possible In N_c space They might be linked when the full $T - \mu - N_c$ plane is considered $N_c^{crit} \sim N_N \sim 10_{3d}$. Large N_c should be done with caution , but plenty to discover with the VARIABLE N_c collider (The lattice)