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Quark Matter 2011
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Outline: Flow fluctuations

Take-away messages

- Flow encompasses more phenomena than previously realized.
- New flow observables will tightly constrain models.

1. Long-range two-particle correlations

2. Flow fluctuations

3. New flow observables
Elliptic Flow c. QM09

Azimuthal distribution of emitted particles:

\[ \frac{dN}{d\phi} \propto 1 + 2v_2^2 \cos 2\phi + 2v_4^2 \cos 4\phi + \ldots \]

Elliptic flow:

\[ v_2^2 \equiv \langle \cos 2\phi \rangle \]

\[ \propto \epsilon^2 \equiv \left\{ y^2 - x^2 \right\} \left\{ y^2 + x^2 \right\} \]

Measured from 2-particle correlation:

\[ \langle e^{i2(\phi_1 - \phi_2)} \rangle \equiv \langle e^{2\phi_1} \rangle \langle e^{-2\phi_2} \rangle \]

\[ \propto v_2 \]

\[ v_2 \{ EP \}^2 \]
Elliptic Flow \( v_2 \equiv \langle \cos 2\phi \rangle \propto \varepsilon^2 \equiv \left\{ y^2 - x^2 \right\} \left\{ y^2 + x^2 \right\} \)

Measured from 2-particle correlation:
\[
\left\langle e^{i2(\phi_1 - \phi_2)} \right\rangle = \left\langle e^{i2\phi_1} \right\rangle \left\langle e^{-i2\phi_2} \right\rangle = \left\langle v_2 \right\rangle^2 \equiv v_2^{EP} \]

\[
\psi_R \vec{p}_t \phi_x y \]

(MATT LUZUM (IPhT))

FLOW FLUCTUATIONS

QUARK MATTER 2011
Elliptic Flow c. QM09

Azimuthal distribution of emitted particles:
\[ dN/d\phi \propto 1 + 2v^2 \cos 2\phi + 2v^4 \cos 4\phi + \ldots \]

Elliptic flow:
\[ v_2 \equiv \langle \cos 2\phi \rangle \]

\[ \propto \varepsilon^2 \equiv \left\{ y^2 - x^2 \right\} \left\{ y^2 + x^2 \right\} \]

Measured from 2-particle correlation:
\[ \langle e^{i2(\phi_1 - \phi_2)} \rangle \approx v_2^2 \equiv v_2^2 \{2\}^2 \]

\[ \varepsilon_{\text{RP}} \left\{ \mathbf{p}_t x y \right\} \]

\[ \text{LOW FLUCTUATIONS} \]

\[ \text{QUA}

Quark Matter 2011
**Elliptic Flow**

Elliptic flow:

\[ v_2 \equiv \langle \cos 2\phi \rangle \]

\[ \propto \varepsilon^2 \equiv \left\{ y^2 - x^2 \right\} \left\{ y^2 + x^2 \right\} \]

Measured from 2-particle correlation:

\[ \langle e^{i2(\phi_1 - \phi_2)} \rangle \]

\[ = \langle e^{i2\phi_1} \rangle \langle e^{-i2\phi_2} \rangle \]

\[ \equiv v_2^2 \]

\[ \approx v_2^2 \left\{ \frac{\psi_{RP}}{\frac{\psi}{\psi_{IP}}} \right\}^2 \]
Azimuthal distribution of emitted particles:

\[ dN/dφ \propto 1 + 2v^2 \cos 2φ + 2v^4 \cos 4φ + \ldots \]

Elliptic flow:

\[ v_2 \equiv \langle \cos 2φ \rangle \propto \epsilon^2 \equiv \left\{ y^2 - x^2 \right\} \left\{ y^2 + x^2 \right\} \]

Measured from 2-particle correlation:

\[ \langle e^{i2(φ_1 - φ_2)} \rangle = \cos(2φ_1) \cos(2φ_2) \]

\[ \sim v_2^2 \left\{ \psi \right\} \]

Azimuthal distribution of emitted particles:

\[
\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots
\]
Azimuthal distribution of emitted particles:

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Azimuthal distribution of emitted particles:

\[ \frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots \]

Elliptic flow:

\[ v_2 \equiv \langle \cos 2\phi \rangle \propto \varepsilon_2 \equiv \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \]
Azimuthal distribution of emitted particles:

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\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots
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Measured from 2-particle correlation:

\[ \langle \langle e^{i2(\phi_1 - \phi_2)} \rangle \rangle \]
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Elliptic flow: \( v_2 \equiv \langle \cos 2\phi \rangle \propto \varepsilon_2 \equiv \frac{y^2 - x^2}{y^2 + x^2} \)

Measured from 2-particle correlation:

\[
\langle \langle e^{i2(\phi_1 - \phi_2)} \rangle \rangle \overset{(\text{flow})}{=} \langle \langle e^{i2\phi_1} \rangle \langle e^{-i2\phi_2} \rangle \rangle = \langle v_2^2 \rangle
\]
Azimuthal distribution of emitted particles:

\[ \frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots \]

Elliptic flow: \( v_2 \equiv \langle \cos 2\phi \rangle \propto \varepsilon_2 \equiv \frac{\{y^2-x^2\}}{\{y^2+x^2\}} \)

Measured from 2-particle correlation:

\[ \left\langle \left\langle e^{i2(\phi_1-\phi_2)} \right\rangle \right\rangle \stackrel{\text{flow}}{=} \left\langle \left\langle e^{i2\phi_1} \right\rangle \left\langle e^{-i2\phi_2} \right\rangle \right\rangle = \left\langle v_2^2 \right\rangle \equiv v_2 \{2\}^2 \]
Azimuthal distribution of emitted particles:

\[ \frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots \]

Elliptic flow: \( v_2 \equiv \langle \cos 2\phi \rangle \propto \varepsilon_2 \equiv \frac{y^2-x^2}{y^2+x^2} \)

Measured from 2-particle correlation:

\[
\langle \langle e^{i2(\phi_1-\phi_2)} \rangle \rangle ^{(\text{flow})} = \langle \langle e^{i2\phi_1} \rangle \langle e^{-i2\phi_2} \rangle \rangle = \langle v_2^2 \rangle \equiv v_2 \{2\}^2 \approx v_2 \{EP\}^2
\]
Azimuthal distribution of emitted particles:

\[
\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots
\]

Elliptic flow: \(v_2 \equiv \langle \cos 2\phi \rangle \propto \varepsilon_2 \equiv \frac{y^2 - x^2}{y^2 + x^2}\)

Measured from 2-particle correlation:

\[
\left\langle \frac{dN_{\text{pairs}}}{d\Delta \phi} \right\rangle^{(\text{flow})} \propto 1 + \left\langle v_2^2 \right\rangle \cos 2(\Delta \phi) + \left\langle v_4^2 \right\rangle \cos 4(\Delta \phi) + \ldots
\]
Unique long-range correlations in heavy-ion collisions.

\[
\begin{align*}
R(\Delta \eta, \Delta \phi) & \quad \text{for p+p} \\
R(\Delta \eta, \Delta \phi) & \quad \text{for Au+Au}
\end{align*}
\]

\[\Delta \phi = \phi - \phi_{\text{trig}}\]

\[\Delta \eta = \eta - \eta_{\text{trig}}\]

**Two-particle correlations**

Unique long-range correlations in heavy-ion collisions...

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Unique long-range correlations in heavy-ion collisions...

\[ \frac{dN}{d\Delta \phi} \left( \frac{1}{N_{\text{trig}}} \right) \]

\begin{align*}
\Delta \phi &\quad 0 &\pi/2 &\pi &3\pi/2 \\
\Delta \eta &\quad -6 &-4 &-2 &0 &2 &4 &6 \\
\end{align*}

\begin{align*}
0 &0.01 &0.02 &0.03 &0.04 &0 &\pi/2 &\pi &3\pi/2 \\
\end{align*}

\text{Au+Au}

\text{(STAR, arXiv:1010.0690)}

Two-particle correlations

Unique long-range correlations in heavy-ion collisions...

\[
\frac{dN}{d\Delta \phi} (\pi \text{trig})
\]

\[
\text{Au+Au}
\]

(Subtracted) correlation

Hydrodynamic calculation


Event centrality

0-10%
40-60%

\text{LOW FLUCTUATIONS}

\text{Quark Matter 2011}
Unique long-range correlations in heavy-ion collisions...
TWO-PARTICLE CORRELATIONS

Unique long-range correlations in heavy-ion collisions...

Ridge

‘Subtracted’ correlation

Au+Au

(START, arXiv:1010.0690)

**TWO-particle correlations**

Unique long-range correlations in heavy-ion collisions...

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**Ridge**

'Subtracted' correlation

**Shoulder**

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(Star, arXiv:1010.0690)

Unique long-range correlations in heavy-ion collisions...
Two-particle correlations

Unique long-range correlations in heavy-ion collisions...

Ridge  'Subtracted' correlation  Shoulder

Hydrodynamic calculation

**Two-particle correlations**

Unique *long-range* correlations in heavy-ion collisions...

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**Ridge**

- 'Subtracted' correlation

**Shoulder**

![Graph showing correlations](image)

**Hydrodynamic calculation**

- (STAR, arXiv:1010.0690)

...can be generated by purely collective flow.
LOW FLUCTUATIONS

\[ \frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots \]

\[
\langle dN_{\text{pairs}} d\Delta \phi \rangle \propto 1 + \infty \sum_{n=1}^{\infty} 2v_n \cos n(\Delta \phi)
\]

\[ v_2 \epsilon_2 e^{2i\Phi_2} \equiv -\{ r_2 e^{i\phi} \} \{ r_2 \} \]


\[ \vec{p}_t \]

\[ \phi \]

\[ \psi_{RP} \]
LOW FLUCTUATIONS

\[
\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots
\]

(Flow fluctuations)
Flow fluctuations

\[
\frac{dN}{d\phi} \propto 1 + 2v_1 \cos \phi + 2v_2 \cos 2\phi + 2v_3 \cos 3\phi + \ldots
\]

Flow fluctuations

\[
\frac{dN}{d\phi} \propto 1 + 2v_1 \cos \phi + 2v_1^s \sin \phi + 2v_2 \cos 2\phi + 2v_2^s \sin 2\phi + 2v_3 \cos 3\phi + \ldots
\]
Flow fluctuations

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n)
\]
LOW FLUCTUATIONS

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n)
\]

\[
\Rightarrow \left\langle \frac{dN_{\text{pairs}}}{d\Delta\phi} \right\rangle \propto 1 + \sum_{n=1}^{\infty} 2 \left\langle v_n^2 \right\rangle \cos n(\Delta\phi)
\]
LOW FLUCTUATIONS

\[ \frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n) \]

\[ \Rightarrow \left\langle \frac{dN_{\text{pairs}}}{d\Delta\phi} \right\rangle_{(\text{flow})} \propto 1 + \sum_{n=1}^{\infty} 2 \left\langle v_n^2 \right\rangle \cos n(\Delta\phi) \]
LOW FLUCTUATIONS

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Flow fluctuations

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n)
\]

\[\implies \left\langle \frac{dN_{\text{pairs}}}{d\Delta \phi} \right\rangle \propto 1 + \sum_{n=1}^{\infty} 2 \left\langle v_n^2 \right\rangle \cos n(\Delta \phi)
\]

\[
v_2 e^{2i\psi_2} \propto \varepsilon_2 e^{2i\Phi_2} \equiv -\frac{\{r^2 e^{2i\phi}\}}{\{r^2\}}
\]

LOW FLUCTUATIONS

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n)
\]

\[
\rightarrow \left\langle \frac{dN_{\text{pairs}}}{d\Delta \phi} \right\rangle_{(\text{flow})} \propto 1 + \sum_{n=1}^{\infty} 2 \left\langle v_n^2 \right\rangle \cos n(\Delta \phi)
\]

\[
v_3 e^{3i\psi_3} \propto \varepsilon_3 e^{3i\Phi_3} \equiv -\frac{\{r^3 e^{3i\Phi}\}}{\{r^3\}}
\]

(Qin, Petersen, Bass, Muller, Phys. Rev. C 82, 064903 (2010); Qiu, Heinz, arXiv:1104.0650)
LOW FLUCTUATIONS

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n)
\]

\[
\implies \left\langle \frac{dN_{\text{pairs}}}{d\Delta\phi} \right\rangle \propto 1 + \sum_{n=1}^{\infty} 2 \left\langle v_n^2 \right\rangle \cos n(\Delta\phi)
\]

\[
v_1 e^{i\psi_1} \propto \varepsilon_1 e^{i\Phi_1} \equiv -\left\{ \frac{r^3 e^{i\phi}}{\{r^3\}} \right\}
\]

(Teaney & Yan, arXiv:1010.1876; Gardim, Grassi, Hama, Luzum, Ollitrault, arXiv:1103.4605)
Flow explains long-range correlations

Quantitative evidence of flow hypothesis:

1. Centrality dependence, size, of $v_3$ and $v_2$

2. $p_t$-dependence and orientation with respect to event plane

3. Centrality dependence of “ridge amplitude”
   (Sorensen, Bolliet, Mocsy, Pandit, Pruthi, arXiv:1102.1403)

4. Factorization, mass dependence, . . .
   (See plenary/parallel talks from ALICE, ATLAS, CMS, . . .)
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Current long-range 2-particle data can be explained by flow alone.*

$\langle \cos \Delta \phi \rangle = v(t)_{1} v(t)_{2} - p(t)_{1} p(t)_{2} \langle \sum p^2 \rangle$
Flow explains long-range correlations

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4. Factorization, mass dependence, …
   (See plenary/parallel talks from ALICE, ATLAS, CMS, …)

Current long-range 2-particle data can be explained by flow alone*.

\[ * \langle \cos \Delta \phi \rangle = v_1^{(t)} v_1^{(a)} - \frac{p_t^{(t)} p_t^{(a)}}{\langle \sum p_t^2 \rangle} \]
Flow explains long-range correlations

Quantitative evidence of flow hypothesis:

1. Centrality dependence, size, of $v_3$ and $v_2$
   
   

2. $p_t$-dependence and orientation with respect to event plane
   
   
   \(\text{(Luzum & Ollitrault, Phys.Rev.Lett. 106 (2011) 102301)}\)

3. Centrality dependence of “ridge amplitude”
   
   \(\text{(Sorensen, Bolliet, Mocsy, Pandit, Pruthi, arXiv:1102.1403)}\)

4. Factorization, mass dependence, …
   
   \(\text{(See plenary/parallel talks from ALICE, ATLAS, CMS, …)}\)

Current long-range 2-particle data can be explained by flow alone*.

\[\text{can accurately measure many new flow observables with little non-flow contamination}\]
NEW FLOW OBSERVABLES: $v_3$, $v_4$, $v_5$, …

CMS preliminary, Velkovska plenary

LESSONS:

$v_3$ is a more sensitive probe of $\eta/s$
NEW FLOW OBSERVABLES: $v_3, v_4, v_5, \ldots$

**PHENIX, arXiv:1105.3928**

**Lessons:**
Combining $v_2$ and $v_3$ can rule out IC models
**NEW FLOW OBSERVABLES: \( v_3, v_4, v_5, \ldots \)**

**PHENIX, arXiv:1105.3928**

**Lessons:**
Combining \( v_2 \) and \( v_3 \) can rule out IC models (CGC is *not* ruled out)
NEW FLOW OBSERVABLES: $v_3$, $v_4$, $v_5$, …

ALICE, arXiv:1105.3865

LESSONS:
Glauber may not work either
**New Flow Observables: \( v_3, v_4, v_5, \ldots \)**

*ATLAS, Jia plenary*

**Lessons:**

Higher coefficients are measurable and add more constraints
NEW FLOW OBSERVABLES: $v_3$, $v_4$, $v_5$, ... 

STAR preliminary, Sorensen plenary

**Lessons:**
Stay tuned (see Paul Sorensen’s talk later this session)
Many other independent flow observables can be measured:

\[ \nu \{ n_1, n_2, \ldots, n_k \} \equiv \langle \cos (n_1 \phi_1 + \ldots + n_k \phi_k) \rangle \]
Many other independent flow observables can be measured:

\[ \nu\{n_1, n_2, \ldots, n_k\} \equiv \langle \cos (n_1 \phi_1 + \ldots + n_k \phi_k) \rangle \]

\[ \nu_n\{2\}^2 \equiv \nu\{n, -n\} = \langle \nu_n^2 \rangle \]

\[ 2\nu_n\{2\}^4 - \nu_n\{4\}^4 \equiv \nu\{n, n, -n, -n\} = \langle \nu_n^4 \rangle \]

\[ \nu_{24} \equiv \nu\{2, 2, -4\} = \langle \nu_2^2 \nu_4 \cos 4(\psi_2 - \psi_4) \rangle \]

\[ \nu_{23} \equiv \nu\{2, 2, 2, -3, -3\} = \langle \nu_2^3 \nu_3^2 \cos 6(\psi_2 - \psi_3) \rangle \]

\[ \nu_{12} \equiv \nu\{1, 1, -2\} = \langle \nu_1^2 \nu_2 \cos 2(\psi_1 - \psi_2) \rangle \]

\[ \nu_{13} \equiv \nu\{1, 1, 1, -3\} = \langle \nu_1^3 \nu_3 \cos 3(\psi_1 - \psi_3) \rangle \]

\[ \nu_{123} \equiv \nu\{1, 2, -3\} = \langle \nu_1 \nu_2 \nu_3 \cos (\psi_1 + 2\psi_2 - 3\psi_3) \rangle \]
OThER FLOW OBSERVABLES

Many other independent flow observables can be measured:

\[ \nu \{ n_1, n_2, \ldots, n_k \} \equiv \langle \cos (n_1 \phi_1 + \ldots + n_k \phi_k) \rangle \]

\[ \nu_n \{ 2 \}^2 \equiv \nu \{ n, -n \} = \langle \nu_n^2 \rangle \]

\[ 2 \nu_n \{ 2 \}^4 - \nu_n \{ 4 \}^4 \equiv \nu \{ n, n, -n, -n \} = \langle \nu_n^4 \rangle \]

\[ \nu_{24} \equiv \nu \{ 2, 2, -4 \} = \langle \nu_2^2 \nu_4 \cos 4(\psi_2 - \psi_4) \rangle \]

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**Other flow observables**

Many other independent flow observables can be measured:

\[ \nu\{n_1, n_2, \ldots, n_k\} \equiv \langle \cos (n_1\phi_1 + \ldots + n_k\phi_k) \rangle \]

\[ \nu_n\{2\}^2 \equiv \nu\{n, -n\} = \langle \nu_n^2 \rangle \]

\[ 2\nu_n\{2\}^4 - \nu_n\{4\}^4 \equiv \nu\{n, n, -n, -n\} = \langle \nu_n^4 \rangle \]

\[ \nu_{24} \equiv \nu\{2, 2, -4\} = \langle \nu_2^2\nu_4 \cos 4(\psi_2 - \psi_4) \rangle \]

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\[ \nu_{123} \equiv \nu\{1, 2, -3\} = \langle \nu_1\nu_2\nu_3 \cos(\psi_1 + 2\psi_2 - 3\psi_3) \rangle \]
**Other flow observables**

Many other independent flow observables can be measured:

\[ v\{n_1, n_2, \ldots, n_k\} \equiv \langle \cos (n_1 \phi_1 + \ldots + n_k \phi_k) \rangle \]

\[ v_n \{2\}^2 \equiv v\{n, -n\} = \langle v_n^2 \rangle \]

\[ 2v_n \{2\}^4 - v_n \{4\}^4 \equiv v\{n, n, -n, -n\} = \langle v_n^4 \rangle \]

\[ v_{24} \equiv v\{2, 2, -4\} = \langle v_2^2 v_4 \cos 4(\psi_2 - \psi_4) \rangle \]

\[ v_{23} \equiv v\{2, 2, 2, -3, -3\} = \langle v_2^3 v_3^2 \cos 6(\psi_2 - \psi_3) \rangle \]

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Many other independent flow observables can be measured:

\[ \nu \{ n_1, n_2, \ldots, n_k \} \equiv \langle \cos (n_1 \phi_1 + \ldots + n_k \phi_k) \rangle \]

\[ \nu_n \{ 2 \}^2 \equiv \nu \{ n, -n \} = \langle \nu_n^2 \rangle \]

\[ 2 \nu_n \{ 2 \}^4 - \nu_n \{ 4 \}^4 \equiv \nu \{ n, n, -n, -n \} = \langle \nu_n^4 \rangle \]

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Flow fluctuations have important, non-negligible effects on measured correlations

⇒ flow has further-reaching effects than was previously realized

With this understanding comes many new possible independent flow measurements

In the near future, look for more precise extraction of medium properties (e.g., $\eta/s$) in addition to strong constraints on geometry and fluctuations of the early-time evolution.
Flow fluctuations have important, non-negligible effects on measured correlations. This implies that flow has further-reaching effects than was previously realized. With this understanding comes many new possible independent flow measurements. In the near future, look for more precise extraction of medium properties (e.g., $\eta/s$) in addition to strong constraints on geometry and fluctuations of the early-time evolution.
Summary

- Flow fluctuations have important, non-negligible effects on measured correlations
- Flow has further-reaching effects than was previously realized
- With this understanding comes many new possible independent flow measurements
- In the near future, look for more precise extraction of medium properties (e.g., \( \eta/s \)) in addition to strong constraints on geometry and fluctuations of the early-time evolution.
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(d) CMS \( N \geq 110, 1.0 \text{GeV/c} < p_T < 3.0 \text{GeV/c} \)
NEW FLOW OBSERVABLES: TRIANGULAR FLOW

Predictions:

\( v_3 \)

\( N_{\text{Part}} \)


(Petersen, Qin, Bass, Muller, Phys.Rev. C82 (2010) 041901)

(Schenke, Jeon, Gale, Phys.Rev.Lett. 106 (2011) 042301)

(Xu, Ko, Phys.Rev. C83 (2011) 021903)
**NEW FLOW OBSERVABLES: \( v_3, v_4, v_5, \ldots \)**

Still missing: new \( v_1 \) from fluctuations  

\[
\begin{align*}
  v_1 &= v_1^a + v_1^s \\
  v_1^a(\eta) &= -v_1^a(-\eta) = \text{usual directed flow} \\
  v_1^s(\eta) &= v_1^s(-\eta) = \text{new “directed flow at midrapidity”} \\
  \text{To measure from a 2-particle correlation, must remove} \\
  \text{“momentum conservation” correlation}  \\
&\quad (\text{Luzum, Ollitrault, Phys.Rev.Lett.106:102301,2011})
\end{align*}
\]
**NEW FLOW OBSERVABLES: DIRECTED FLOW AT MIDRAPIDITY**

\[ \langle \cos \Delta \phi \rangle = v_1^{(t)} v_1^{(a)} - \frac{p_t^{(t)} p_t^{(a)}}{\langle \sum p_t^2 \rangle} \]

**Fig. 1:** A schematic of an event with (a) net triangularity and (b) net dipole asymmetry. The triangularity produces a net \( v_3^{(p_T)} \) and the dipole asymmetry produces a net \( v_1^{(p_T)} \).

Here the orientation angles \( \psi_3, \psi_3 \) and \( \psi_1, \psi_3 \) are set to zero. At large enough radius the derivative terms become large and overwhelm the leading term making the distribution negative. This is an unavoidable consequence of truncating a cumulant expansion at any finite order.

As explained in Appendix A we regulate these terms and adjust the overall constant to reproduce the total entropy in a central RHIC collision. Fig. 1a and Fig. 1b illustrate initial conditions with net triangularity and net dipole asymmetry respectively. The distribution with net triangularity leads to a \( v_3^{(p_T)} \) while the dipole asymmetry leads to a \( v_1^{(p_T)} \).

To estimate these parameters and their correlations we have used the PHOBOS monte carlo Glauber code [22]. Fig. 2 shows the distribution of \( \psi_1, \psi_2 \) and \( \psi_3 \) as a function of the number of participants. We see that the dipole asymmetry is about a factor of two smaller than the triangularity but is not negligibly small.

Fig. 3 shows the distribution of \( \psi_1, \psi_3 \) and \( \psi_3, \psi_3 \) with respect to reaction plane at various impact parameters. We see that although \( \psi_3, \psi_3 \) is uncorrelated with respect to the reaction plane, \( \psi_1, \psi_3 \) shows an anti-correlation with respect to the reaction plane, which eventually disappears toward central collisions.

More importantly, the angles \( \psi_1, \psi_3 \) and \( \psi_3, \psi_3 \) are strongly correlated in mid central collisions (a similar observation was made recently by Staig and Shuryak [23]). Fig. 4 shows the conditional probability distribution, i.e.

\[ P(\psi_3, \psi_3 | \psi_1, \psi_3, \Psi_R) \equiv \]

The probability of \( \psi_3, \psi_3 \) given \( \psi_1, \psi_3 \) and \( \Psi_R \).

The strong correlation may be explained physically as follows. When the dipole asymmetry is in plane then the triangular axis is at \( \pi/3 \), i.e. the point of the triangle is aligned with the dipole axis as exhibited in Fig. 5(a). However, when the dipole axis is out of plane then the triangular axis is also out of plane as exhibited in Fig. 5(b).

These correlations are a reflection of the almond shape geometry and their general form can be established by symmetry arguments. First, since the probability of finding a dipole asymmetry in a given quadrant of the ellipse is the same for every quadrant, the probability

\[ (\text{Teaney & Yan, arXiv:1010.1876}) \]

\[ (\text{Gardim, Grassi, Hama, Luzum, Ollitrault, arXiv:1103.4605}) \]