DECONFINEMENT AND CHIRAL SYMMETRY IN EFFECTIVE MODELS WITH FERMIONS IN HIGHER REPRESENTATIONS

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1 Abstract

We study the interaction between the chiral and deconfinement transitions using the Polyakov extended linear sigma model and the Nambu-Jona-Lasinio model. In this work we consider fermions in higher representations of both SU(2) and SU(3) gauge groups. Our results motivate further studies of these theories on the lattice and they are also relevant for models of electroweak symmetry breaking utilizing new strong dynamics, and their cosmological consequences [1]. We find that for different fermion representations the qualitative and quantitative behaviour of the order parameters is compatible with the general expectations based on the global symmetries of the underlying theory [2].

3 Parameter setting

The two models used here approximate QCD equally well, although there are quantitative and also some qualitative differences [3, 4, 5]. To set the model parameters in the chiral sector of the two-color case, we start from the parameter values used to describe QCD and scale them accordingly. In this work we use a simple scaling in which the pion decay constant is assumed proportional to $\sqrt{N_c}$ and the pion and sigma masses are essentially constant. This scaling has been recently used e.g. in [6]. The parameters of the Polyakov potential are fitted to SU(2) lattice data from [7]. The parameters used are shown in the table below.

LSM parameters for SU(2) fundamental			
f_π	m_π	m_σ	g
75.9 MeV	137 MeV	598 MeV	3.9
NJL parameters for SU(2) fundamental			
m_0	Λ	G	
5.0 MeV	678 MeV	13.31 (GeV)$^{-2}$	
Polyakov potential parameters for SU(2)			
a_0	a_1	a_2	b_3
1.19	-1.136	7.94	-2.759

Switching from the fundamental representation to the adjoint will require some additional adjustment of the parameters. For the PNJL model a scaling based on the the Fiertz transformation properties of the current–current interaction was presented in [8]: the adjoint four–fermion coupling G_A is related to the fundamental coupling G through the relation

$$\frac{G_A}{G_F} = \frac{2N_c^3}{(N_c^2-1)^2}.$$

Because the linear sigma model deals with mesons directly, an analogous argument cannot be used. Instead, here we use the PNJL model as a guide and set the PLSM parameters f_π, m_π, m_σ and g to values that we obtain using the PNJL model. Below are the corresponding parameter values for the adjoint fermion case.

LSM parameters for SU(2) adjoint			
f_π	m_π	m_σ	g
96.3 MeV	179 MeV	2811 MeV	14.6
NJL parameters for SU(2) adjoint			
m_0	Λ	G	
5.0 MeV	678 MeV	23.66 (GeV)$^{-2}$	

References

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2 The models

In this work, we consider the PLSM and PNJL models to describe the interplay of deconfinement and chiral symmetry in two-color QCD. We use the same kind of model setup as in our previous work with QCD [3, 4, 5]. The PLSM model consists of the linear sigma model, a Polyakov loop potential and an interaction between the two. The PNJL model is similar with the chiral part of the Lagrangian now corresponding to the NJL model. Here we merely state the form for the grand potential of the models

$$\Omega = -\frac{T\ln\mathcal{L}}{V} = U_{\rm chiral} + U_\ell + \Omega_{\bar{q}q},$$

where

$$U_{\rm chiral} = \frac{\lambda^2}{4}\left(\left(\frac{M}{g}\right)^2 - v^2\right)^2 - \frac{HM}{g}$$

for the PLSM model and

$$U_{\rm chiral} = \frac{(m_q-M)^2}{2G}$$

for the PNJL model.
The Polyakov loop is included to both models through the mean field potential U_ℓ, for which we choose the form

$$U_\ell/T^4 = -\frac{a(T)}{2}|\ell_F|^2 + b(T)\ln(1-|\ell_F|^2),$$

where

$$a(T) = a_0 + a_1\frac{T_0}{T} + a_2(\frac{T_0}{T})^2 \quad \text{and} \quad b(T) = b_3(\frac{T_0}{T})^3,$$

and the constants a_i,b_3 are fitted to reproduce pure gauge lattice data with the a transition temperature $T_0 = 268$ MeV.

Here ℓ_F is the gauge invariant Polyakov loop in the fundamental representation and we consider only the mean field potential of the fundamental loop. Finally there are the interactions between the Polyakov loop and chiral degrees of freedom given by the interaction potential $\Omega_{\bar{q}q}$.
The form of the interaction potential depends on the fermion representation and in this work we consider the fundamental and the adjoint representations. In the fundamental representation

$$\Omega_{\bar{q}q} = -4T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+2\ell_F e^E + e^{2E}\right]$$

and in the adjoint representation

$$\Omega_{\bar{q}q} = -4T\int\frac{d^3p}{(2\pi)^3}\ln\left[(1+e^E)(1+(4\ell_F^2-2)e^E+e^{2E})\right]$$

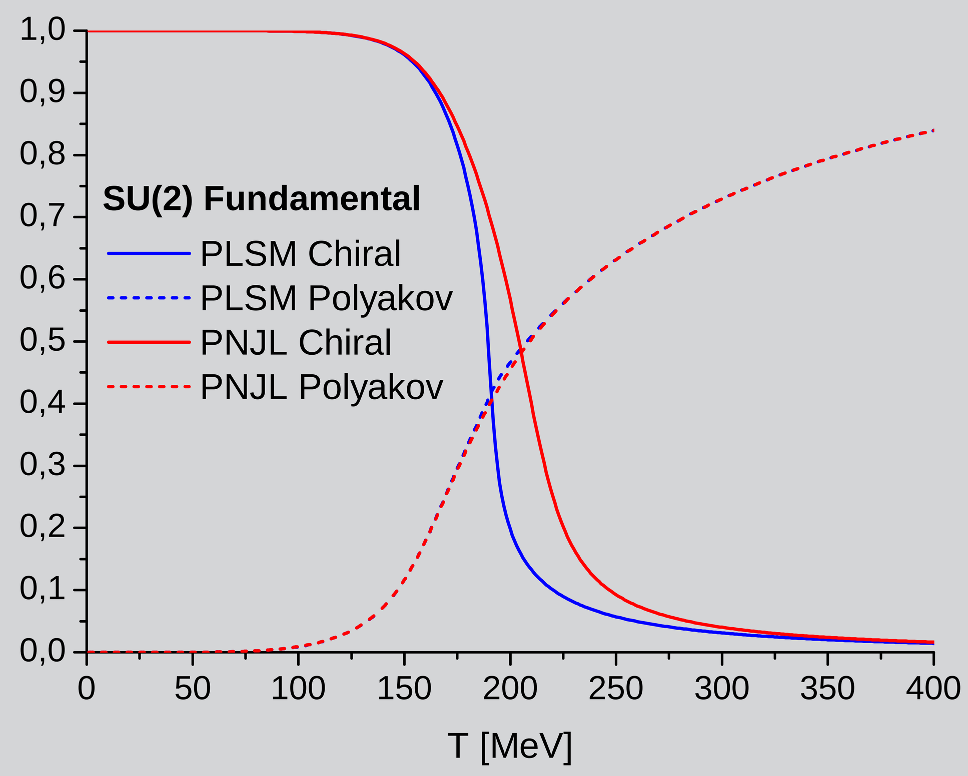
In the adjoint case the naturally arising adjoint loop ℓ_A is written in terms of the fundamental loop ℓ_F, $\ell_A = (4\ell_F^2-1)/3$.
In the PNJL model interaction potential we also include the vacuum contribution regulated by a cutoff

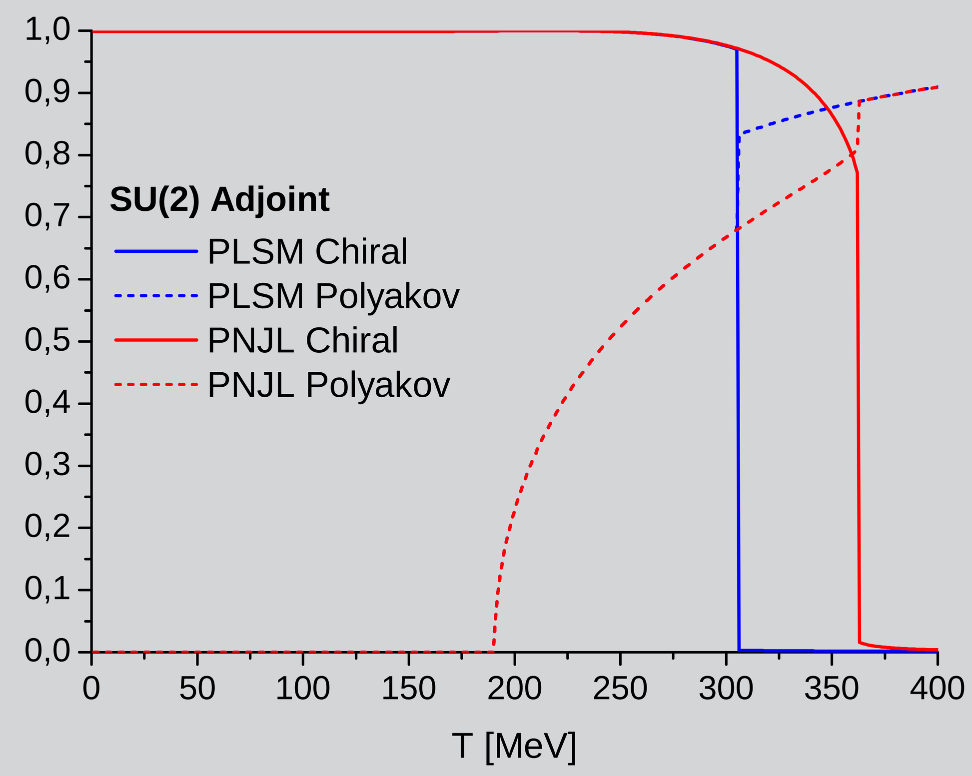
$$-4{\rm Dim}\mathcal{R}\int\frac{d^3p}{(2\pi)^3}E\theta(\Lambda^2-|\vec{p}|^2),$$

where ${\rm Dim}\mathcal{R}$ is the dimension of the fermion representation. The thermodynamics of the models can be determined by minimizing the grand potential Ω with respect to the constituent mass M and the fundamental Polyakov loop ℓ_F.

4 Model comparison

Using the described parameter setting scheme, we compare the PLSM and PNJL model predictions for the temperature dependence of the chiral and deconfinement order parameters in SU(2). In the figures below the normalised chiral condensate and the fundamental Polyakov loop are shown for both models in the two fermion representations. In the fundamental fermion case (left figure) the behaviour of the two models is almost identical with QCD results: The qualitative behaviour of the models is similar with only a slight quantitative difference near the phase transition region, mainly in the chiral sectors of the models. With adjoint fermions (right figure) the situation is somewhat different: The qualitative picture given by both models includes a sharp first order phase transition in the chiral sector which also induces a similar transition to the Polyakov sector, but now the quantitative difference between the models is larger. This may not be suprising since the overall scale of the chiral order parameters increases in switching from the fundamental to the adjoint representation.





Since there are differences between the models and it is not completely clear what is the right prescription to set up the models in the different representations of SU(2), we take a look on how the chiral transition temperature is affected by tuning th PLSM parameters. In the two figure below we have varied the PLSM parameters g and m_σ. The coupling g is basically the ratio of the constituent mass and the chiral condensate, $M = g\langle\bar{q}q\rangle$. In the figures g is plotted on the x–axis and scaled by g_0, which corresponds to a constituent mass given by the PNJL model. The other parameter m_σ is directly related to the PLSM coupling λ and is represented with blue and red data points in the figures. As one can see the transition temperature of the PLSM model can be made to agree with the PNJL model by increasing m_σ or decreasing g. With the current setup this will however lead to different constituent and sigma masses in the models.