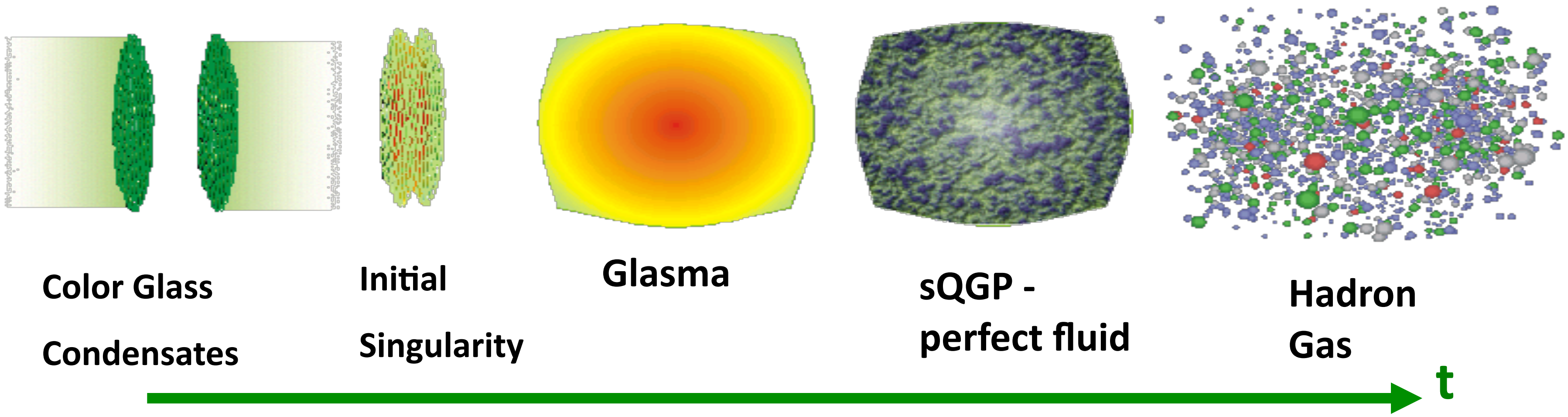
The spectrum of initial fluctuations in the little Bang

Raju Venugopalan
Brookhaven National Laboratory

Quark Matter 2011, Annecy, May 23-28, 2011

Ab initio approach to heavy ion collisions

RV, ICHEP talk, 1012.4699



- Compute properties of relevant degrees of freedom of wave fns. in a systematic framework (as opposed to a “model”)?
- How is matter formed ? What are its non-equilibrium properties & lifetime? Can one “prove” thermalization or is the system “partially” thermal ?
- When is hydrodynamics applicable? How much jet quenching occurs in the Glasma? Are there novel topological effects (sphaleron transitions?)

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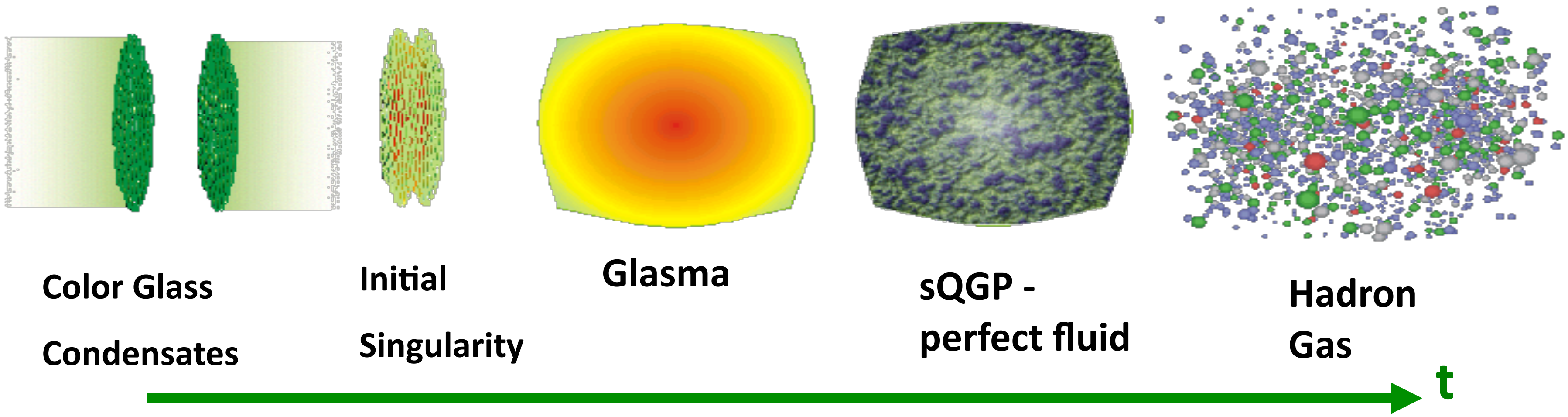


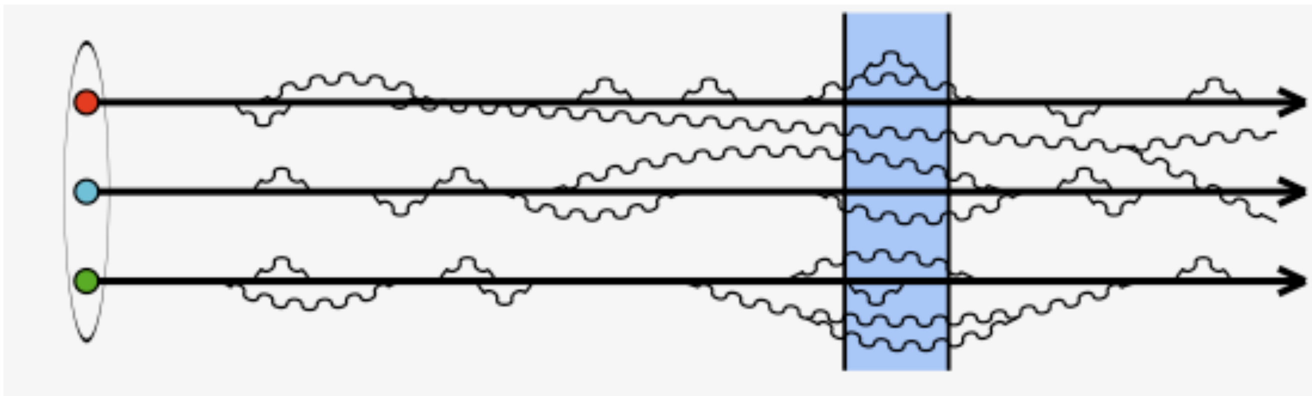
Talk Theme:

i) Even though the paradigm is classical, *quantum (NLO+…) fluctuations* absolutely essential for our understanding

ii) Some quantum modes are essential to understand energy evolution of wave-fns; others grow exponentially after the collision

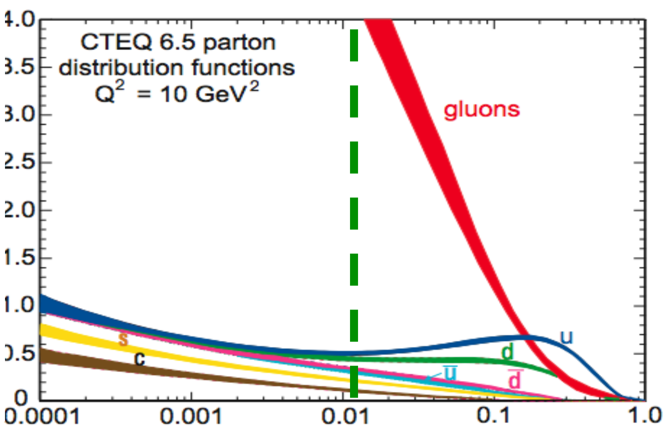
iii) Isolating, factorizing, resumming, quantum fluctuations can be done systematically in a *weak coupling* (albeit non-perturbative) framework

Gluon Saturation in a nucleus: classical coherence from quantum fluctuations



Wee parton fluctuations time dilated on strong interaction time scales

Parton Density



X= Parton momentum / Proton momentum

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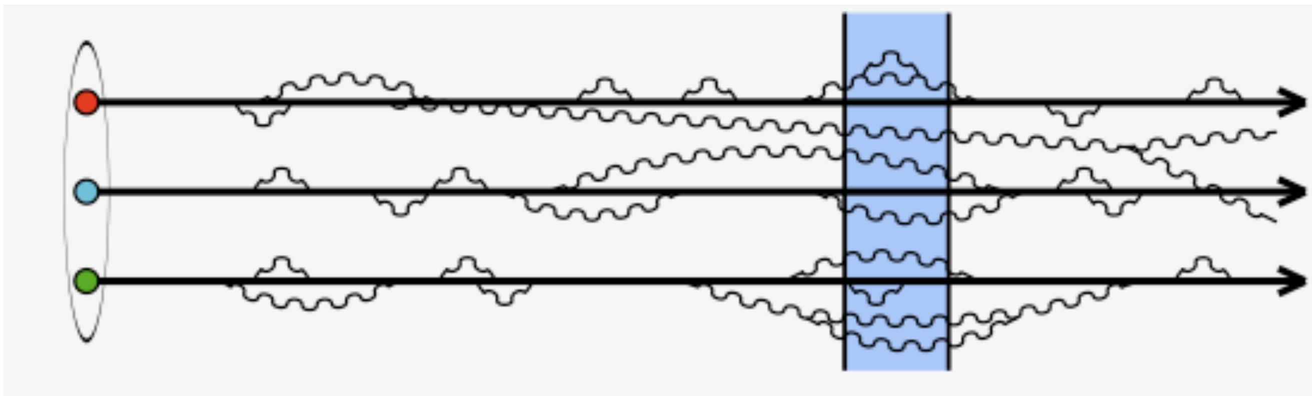

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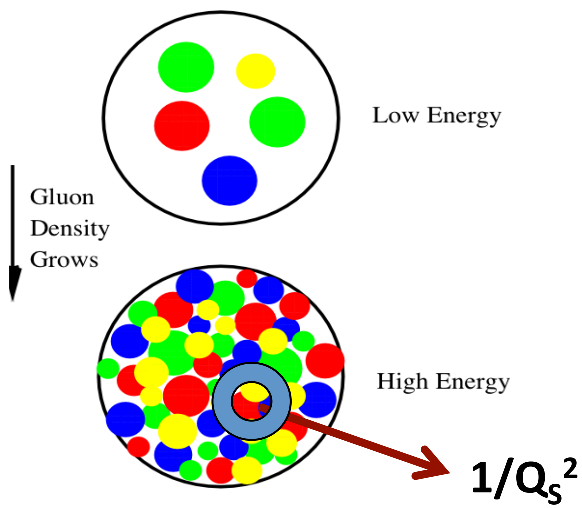


The gluon density saturates at a maximal value of ~ $1/\alpha_S$ ➔ gluon saturation

Large occupation # => classical color fields

|P>_pert ➔ |P>_classical

Many-body high energy QCD:
The Color Glass Condensate

Gelis,Iancu,Jalilian-Marian,RV:
Ann. Rev. Nucl. Part. Sci. (2010), arXiv: 1002.0333

- **QCD light front EFT framework of static light front color sources ρ^a and dynamical gauge fields A^a_μ**

$$\langle \mathcal{O} \rangle_Y = \int [d\rho] W_Y[\rho] \, \mathcal{O}$$

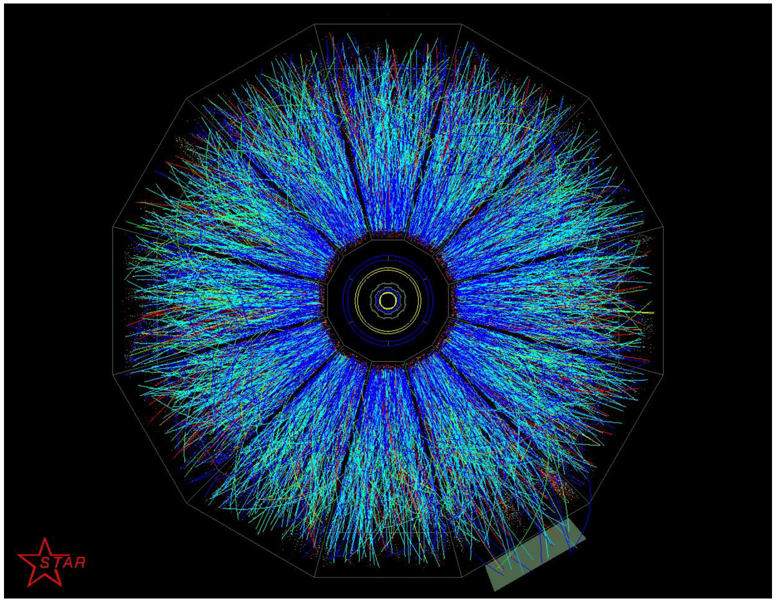
- **Require observables be independent of separation of fast (large x) & slow (small x) modes: functional RG for "density matrices" W[ρ]**

$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho] \, W_Y[\rho]$$

- **JIMWLK Hamiltonian-describes "Fokker-Planck" –like evolution of multi-parton (Wilson line) correlators**
  - can be solved by Langevin techniques on 2+1-D lattices

- **NLL corrections** (see talks by Albacete, Kovchegov and Chirilli)

THE LITTLE BANG



How can we compute multiparticle production *ab initio* in HI collisions ?

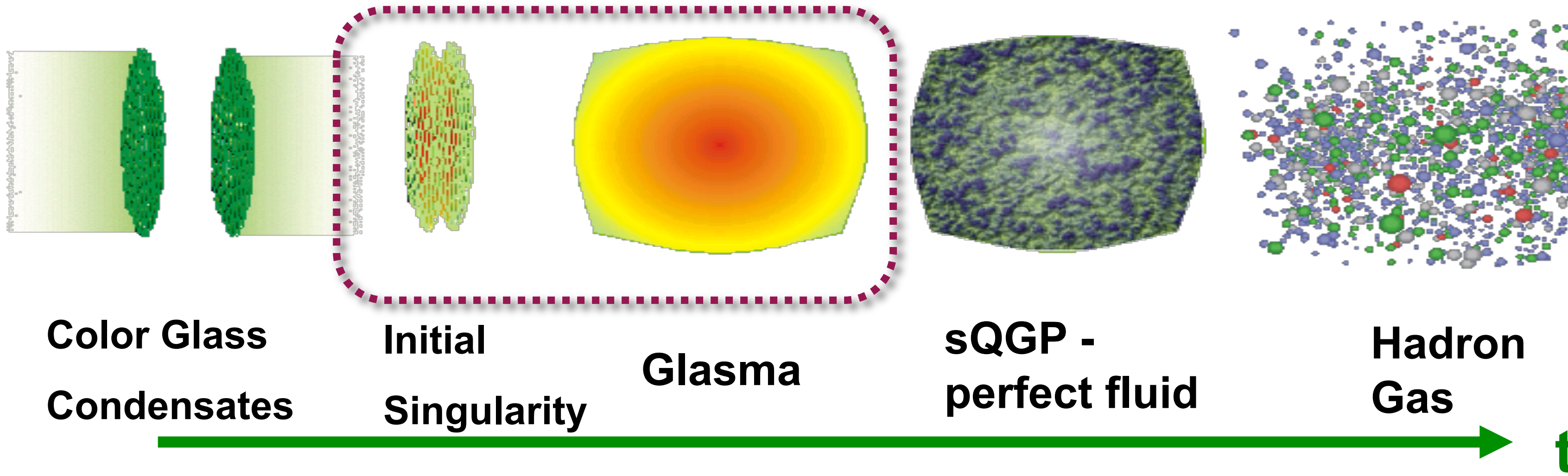
-~~*perturbative VS non-perturbative,*~~

Always non-perturbative for questions of interest in this talk!

strong coupling VS weak coupling

AdS/CFT ? Interesting set of issues… not discussed here

Quantum decoherence from classical coherence



Glasma (\Glahs-maa\): *Noun:* non-equilibrium matter between CGC & QGP

Computational framework

Schwinger-Keldysh formalism: for strong time dependent sources ($\rho \sim 1/g$) , computation of inclusive quantities can be formulated as an *initial value problem*

Gelis,RV NPA (2006)

Power counting:

- **LO: $O(1/g^2)$ but all multiple scatterings $(g\rho)^n$**
- **NLO: $O(1)$ but all multiple scatterings $(g\rho)^n$**

Spoiler alert: divergent contributions in rapidity and proper time modify power counting at NLO: these have to be resummed

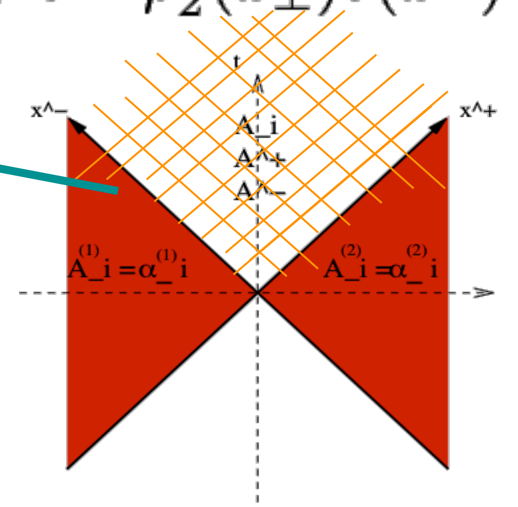
The Glasma at LO:Yang-Mills eqns. for two nuclei

O(1/g²) and all orders in (gρ)ⁿ

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_1^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_2^a(x_\perp)\delta(x^+)$$

Glasma initial conditions from matching classical CGC wave-fns on light cone

Kovner,McLerran,Weigert; Krasnitz, RV; Lappi



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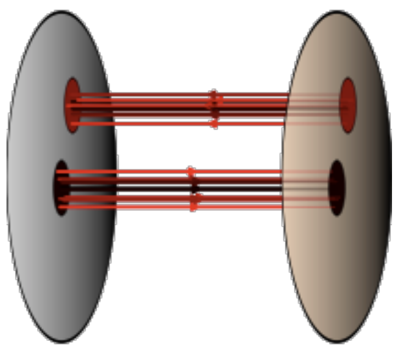
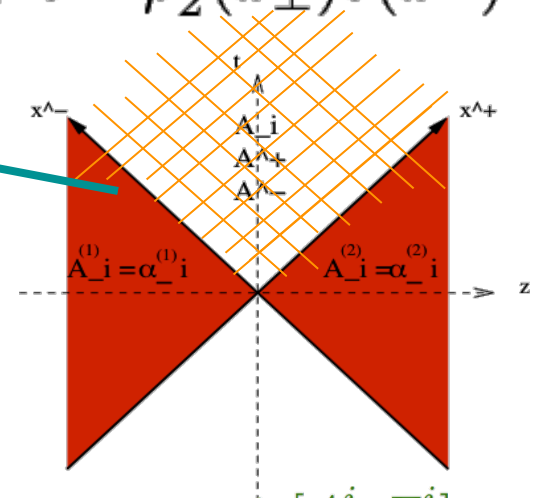


$$\nabla \cdot E = \rho_{\mathrm{electric}}$$
$$\nabla \cdot B = \rho_{\mathrm{magnetic}}$$



$$\rho_{\mathrm{electric}} = ig[A^i, E^i]$$
$$\rho_{\mathrm{magnetic}} = ig[A^i, B^i]$$

The Glasma at LO:Yang-Mills eqns. for two nuclei

$O(1/g^2)$ and all orders in $(g\rho)^n$

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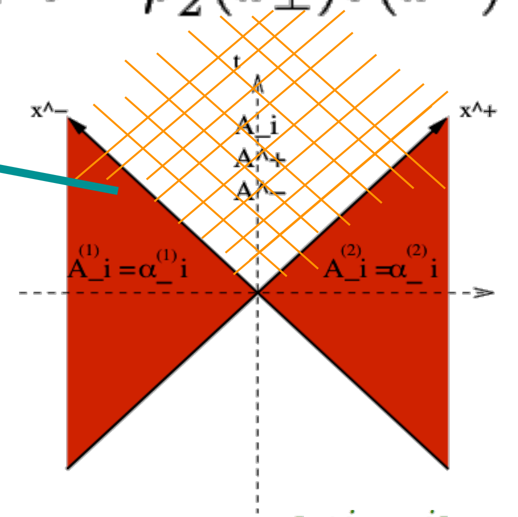
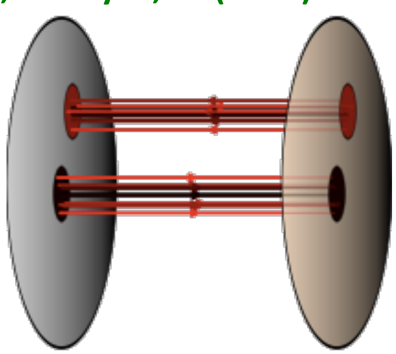


Kovner,McLerran,Weigert; Krasnitz, RV; Lappi
Lappi,Srednyak,RV (2010)



$$\begin{aligned} \nabla \cdot E &= \rho_{\text{electric}} \\ \nabla \cdot B &= \rho_{\text{magnetic}} \end{aligned}$$

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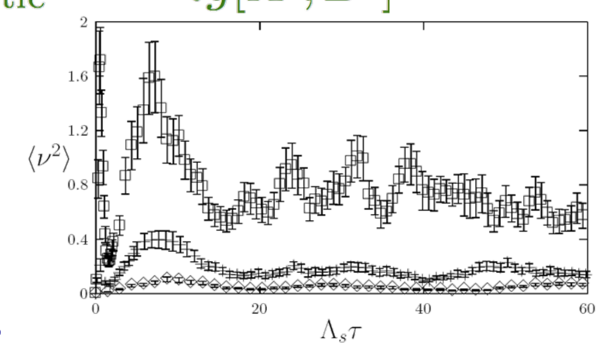


Boost invariant flux tubes of size with || color E & B fields -generate Chern-Simons charge but no large topological transitions

Sphaleron transitions generated by rapidity-dependent quant. fluctuations

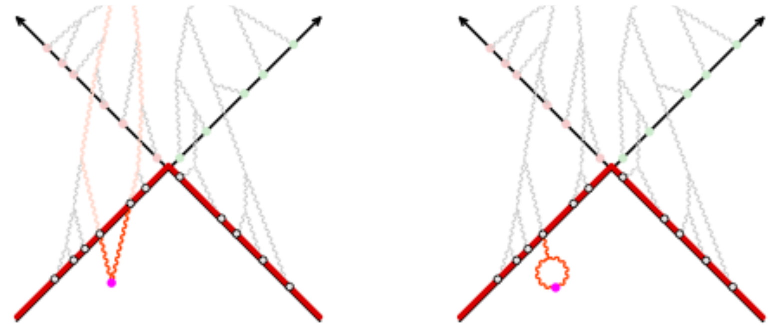
Kharzeev, Krasnitz, RV, Phys. Lett. B545 (2002)
Fukushima,Kharzeev,Warringa (2010)

Factorization of quantum fluctuations

Gelis,Lappi,RV (2008,2009)

O(1) and all orders in $(g\rho)^n$

Divergent contributions at NLO in rapidity in the respective nuclei can be factorized



Initial value problem in Schwinger-Keldysh

$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\,\mathcal{T}_u\mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\,\mathcal{T}_u\right]\mathcal{O}_{\rm LO}$$

$\mathcal{G}(\vec{u},\vec{v})$ and $\beta(\vec{u})$ can be computed on the initial Cauchy surface

$\mathcal{T}_u = \dfrac{\delta}{\delta A(\vec{u})}$ linear operator on initial surface

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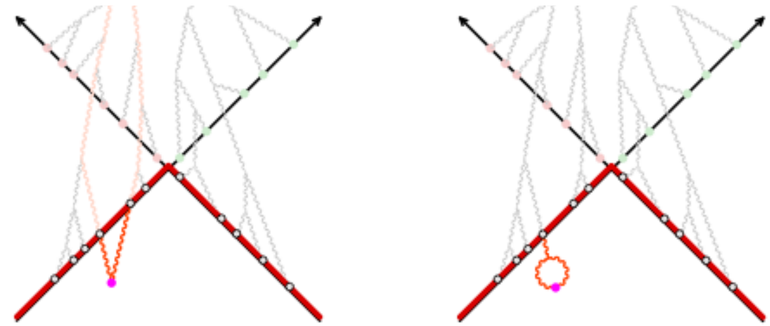

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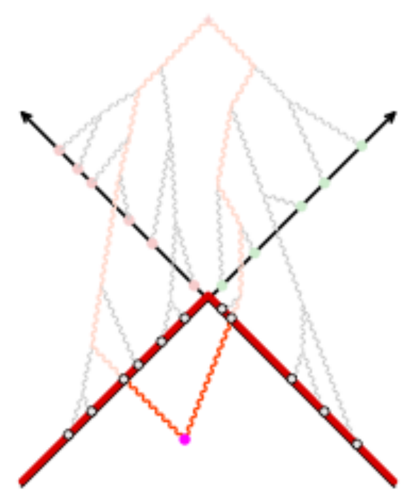
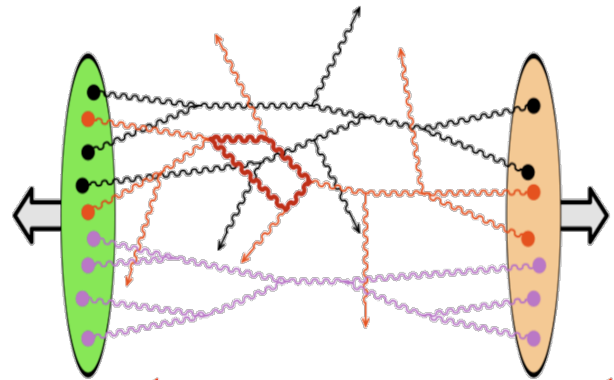


Contributions across both nuclei are finite-no log divergences => JIMWLK factorization

$$\mathcal{O}_{\rm NLO} = \left[\ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\mathcal{H}_2\right] \mathcal{O}_{\rm LO}$$

Factorization in the Glasma

$$T^{\mu\nu}_{\rm LO} = \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta} - F^{\mu\lambda} F^{\nu}_{\lambda} \qquad {\rm O}\left(\frac{Q_S^4}{g^2}\right)$$



$$\rho_1 \sim \frac{1}{g} \qquad \rho_2 \sim \frac{1}{g}$$

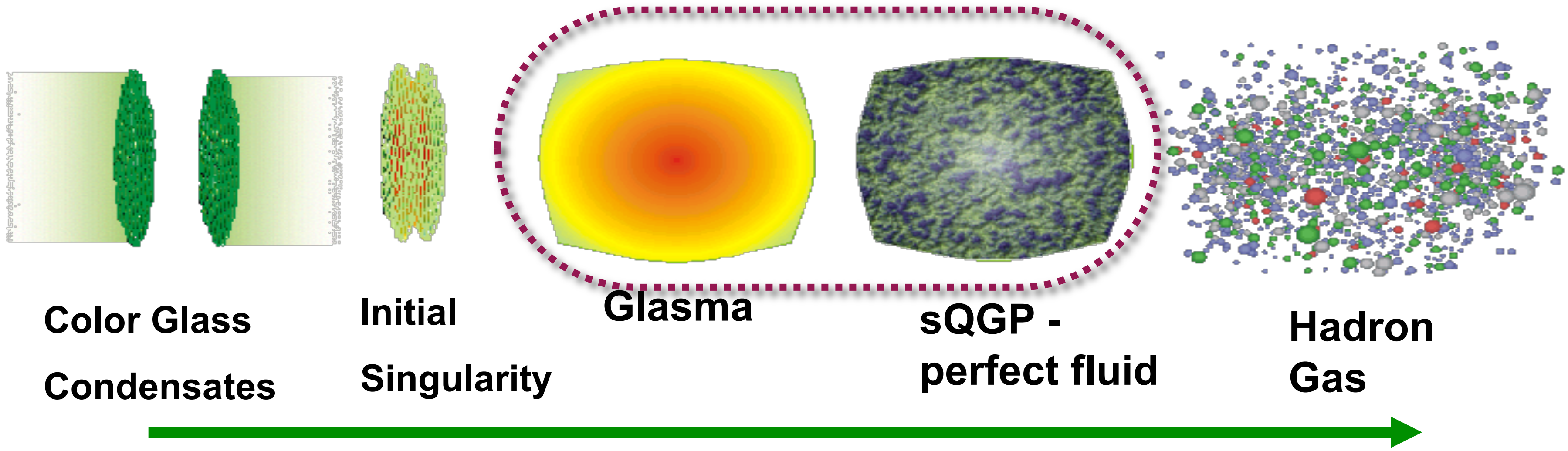
ε=20-40 GeV/fm³ for τ=0.3 fm @ RHIC

$$\langle T^{\mu\nu}(\tau, \underline{\eta}, x_\perp)\rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau, x_\perp)$$

$$Y_1 = Y_{\rm beam} - \eta\,;\; Y_2 = Y_{\rm beam} + \eta$$

Glasma factorization => universal "density matrices W" ⊗ "matrix element"

Evolution of quantum fluctuations in the Glasma



- Rapidity divergent quantum fluctuations are p_η=0 modes
  - integral to coherence of the nuclei before the collisions

- What about $p_\eta \neq 0$ modes that are generated at the instant of the collision?

The problem of thermalization

- **Large v_2 –elliptic flow – appears to require early thermalization/perfect fluidity ($\eta/s \sim 1/4\pi$).**

- **But kinetic theory gives** $$\tau_{\mathrm{therm.}} \sim \frac{1}{\alpha_S^2}\frac{1}{Q_S} >> \frac{1}{Q_S}$$

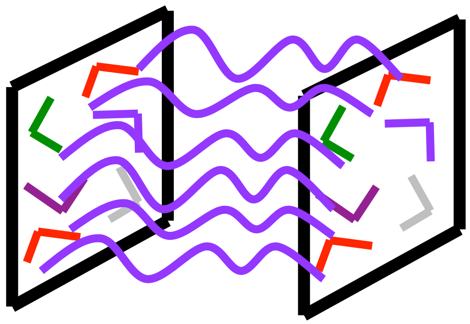
➢ **Our weak coupling (non-perturbative!) framework leads to hydrodynamic behavior outside the framework of kinetic theory.**

➢ **Treatment similar to enhancement of quantum fluctuations due to parametric resonances during inflation**

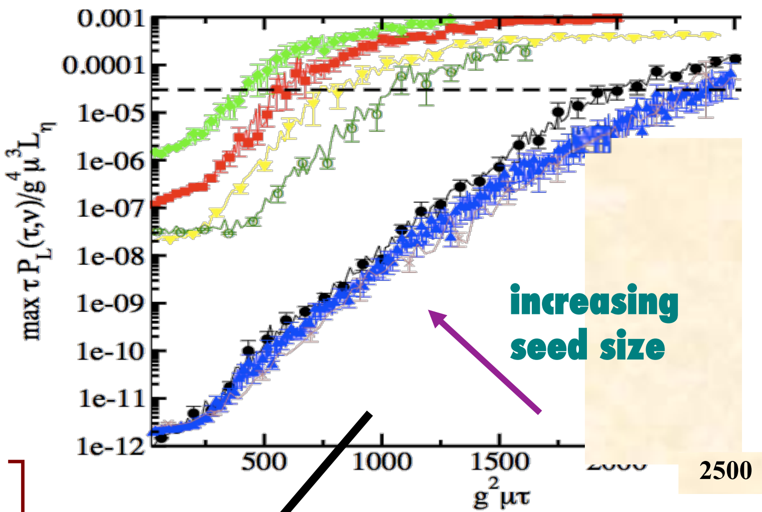
Kofman, Linde, Starobinsky
Micha, Tkachev

From Glasma to Plasma

Romatschke,RV
Fukushima,Gelis,McLerran



Quant. fluct.
grow exponentially
after collision



Recall…

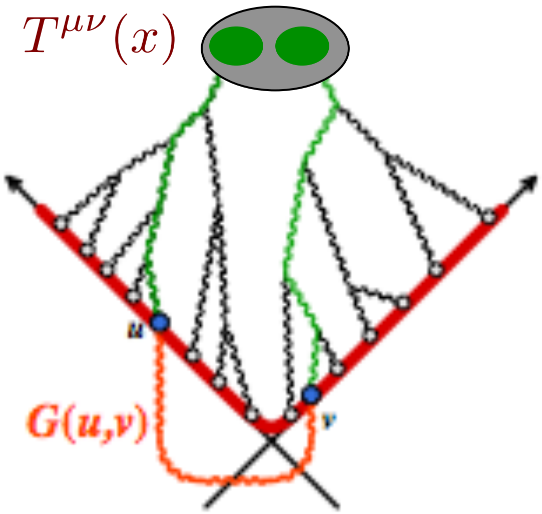
$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\ \mathcal{T}_u\mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\ \mathcal{T}_u\right]\mathcal{O}_{\rm LO}$$

Gelis,Lappi,RV

For $p_\eta \neq 0$ modes: $\mathbf{T}_u\mathcal{A}(x) \sim \dfrac{\delta\mathcal{A}(x)}{\delta\mathcal{A}(0,y)} \sim \exp\left(\sqrt{Q_s\tau}\right)$

- **Resummation of secular divergences** $\left[g\exp\left(\sqrt{Q_S\tau}\right)\right]^n$

Spectrum of initial fluctuations



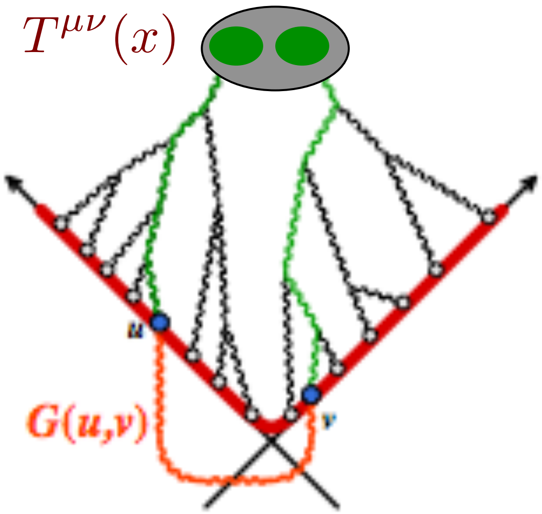
Dusling,Gelis,RV (2011)

$$\mathcal{G}^{\mu\nu} = \int \frac{d^3k}{(2\pi)^3 2E_k} a^{\mu}_{-k}(\vec{u})\ a^{\nu}_{+k}(\vec{v})$$

$$\left[\frac{\delta^2 S_{\mathrm{YM}}}{\delta A^\mu A^\nu}\right]_{\mathrm{A=A_{cl}}} a^{\nu}_{\pm k} = 0$$

$$\lim_{x^0 \to -\infty} a^{\mu}_{\pm k,\lambda a}(x) = \epsilon^\mu(k)\ T^a\ e^{\pm ik\cdot x}$$

Spectrum of initial fluctuations



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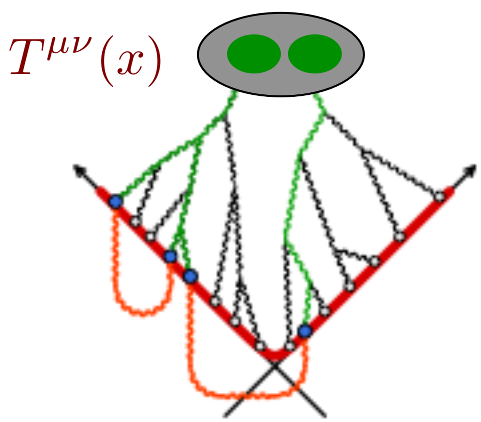
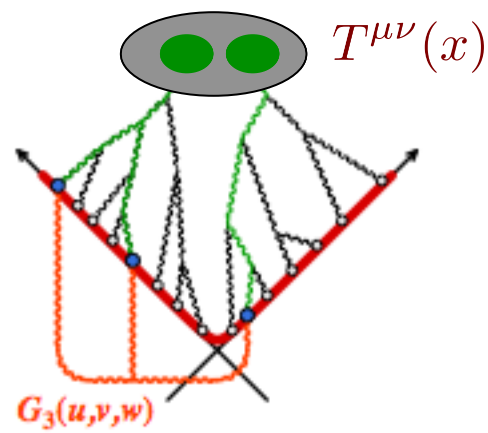
Higher orders:



$$(g\exp(\sqrt{Q_S\tau}))^4 \sim O(1)$$



$$g(g\exp(\sqrt{Q_S\tau}))^3 \sim O(g)$$

Spectrum of initial fluctuations

Dusling,Gelis,RV (2011)

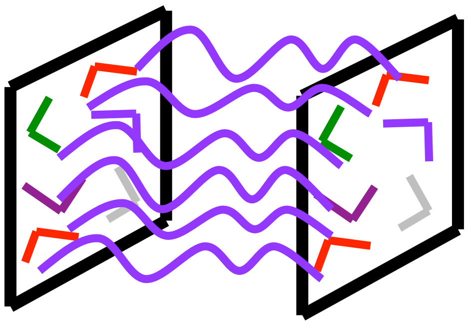
$$T^{\mu\nu}_{\rm resum} = \exp\left[\frac{1}{2}\int_{\tau=0^+} d^3u\; d^3v\; G(u,v)\cdot \mathcal{T}_u\mathcal{T}_v\right] T^{\mu\nu}_{\rm LO}$$

$$= \int [da_0(u)]\; F_{\rm init}[a_0]\; T_{\rm LO}[A_{\rm cl}+a_0]$$

$$\Downarrow$$

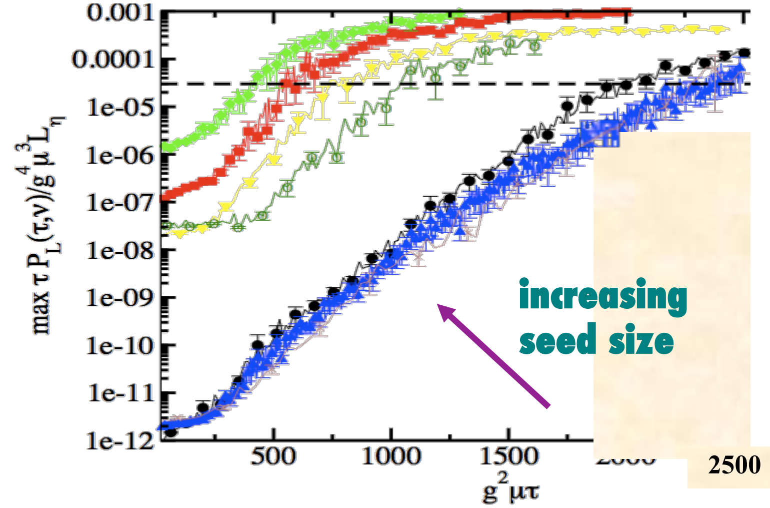
$$\propto \exp\left[-\frac{1}{2}\int_{\tau=0^+} d^3u\; d^3v\; a_0(u)(G^{\mu\nu})^{-1}a_0(v)\right]$$

From Glasma to Plasma



Quant. fluct. grow exponentially after collision

Romatschke,RV
Fukushima,Gelis,McLerran



$$\langle\langle T^{\mu\nu}\rangle\rangle_{\mathrm{LLx+Linst.}} = \int [D\rho_1][D\rho_2]\, W_{\mathrm{Y_{beam}-Y}}[\rho_1]\, W_{\mathrm{Y_{beam}+Y}}[\rho_2]$$
$$\times \int [da(u)]\; F_{\mathrm{init}}[a]\; T^{\mu\nu}_{\mathrm{LO}}[A_{\mathrm{cl}}(\rho_1,\rho_2)+a]$$

Path integral over multiple initializations of classical trajectories in one event can lead to quasi-ergodic "eigenstate thermalization"

Berry; Srednicki; Rigol et al.; …

Hydrodynamics from quantum fluctuations

Dusling,Epelbaum,Gelis,RV (2011)

“Toy” example: scalar Φ^4 theory

Gaussian random variable

$$\langle c_{\nu k} c_{\mu l} \rangle = 0$$
$$\langle c_{\nu k} c^*_{\mu l} \rangle = 2\pi\delta(\nu - \mu)\delta_{kl}$$

$$\phi(\tau, \eta, x_\perp) = \phi_{\text{cl.}}(\tau, x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\; c_{\nu k}\; e^{i\nu\eta}\chi_k(x_\perp) H_{i\nu}(\lambda_k \tau) + c.c$$

Satisfies the equation

$$[-\partial_\perp^2 + V''(\phi_0)]\chi_k = \lambda_k^2\; \chi_k$$

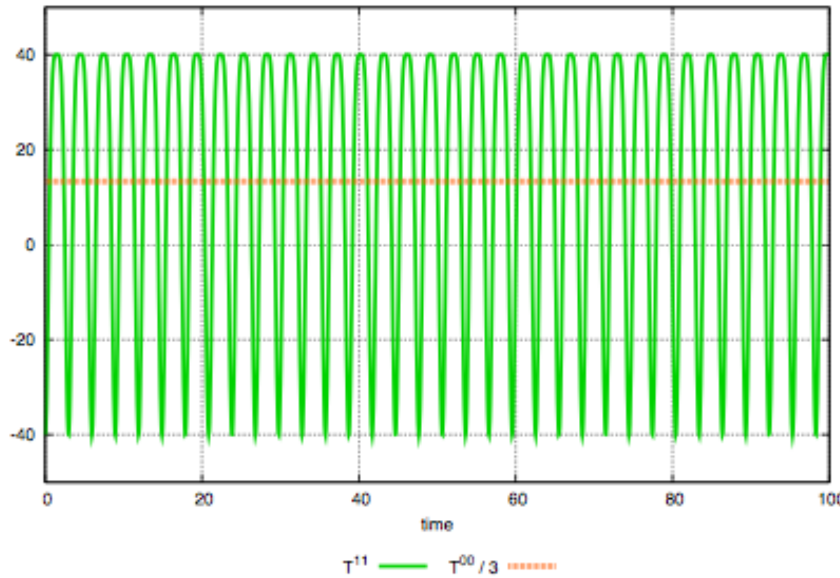
- These quantum modes satisfy the criteria conjectured by Berry (and developed by others) as essential for thermalization of a quantum fluid

Hydrodynamics from quantum fluctuations

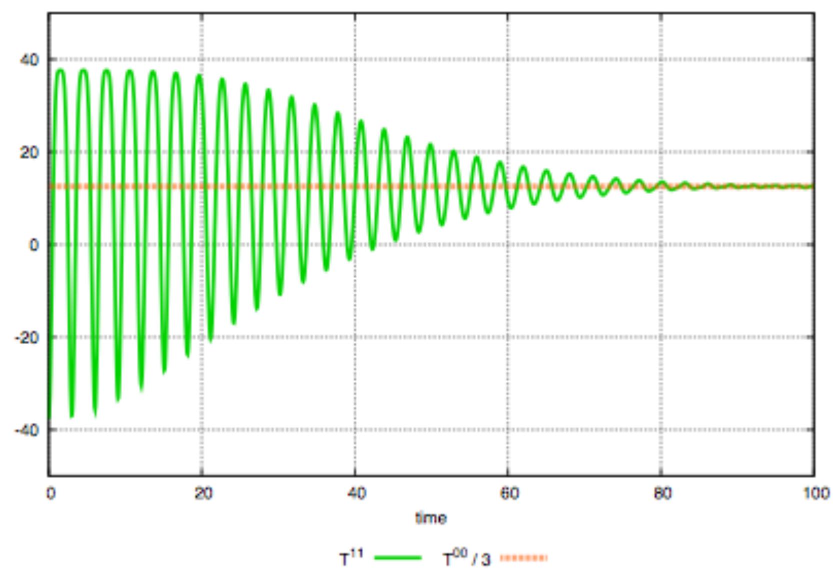
Dusling,Epelbaum,Gelis,RV (2011)

scalar Φ^4 theory:

Energy density and pressure without averaging over fluctuations



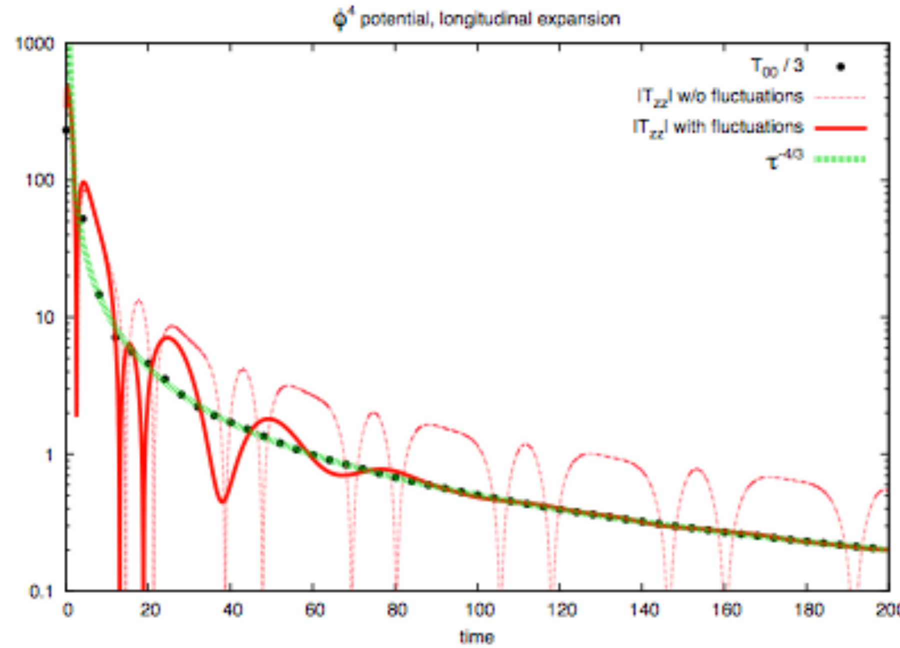
Energy density and pressure after averaging over fluctuations



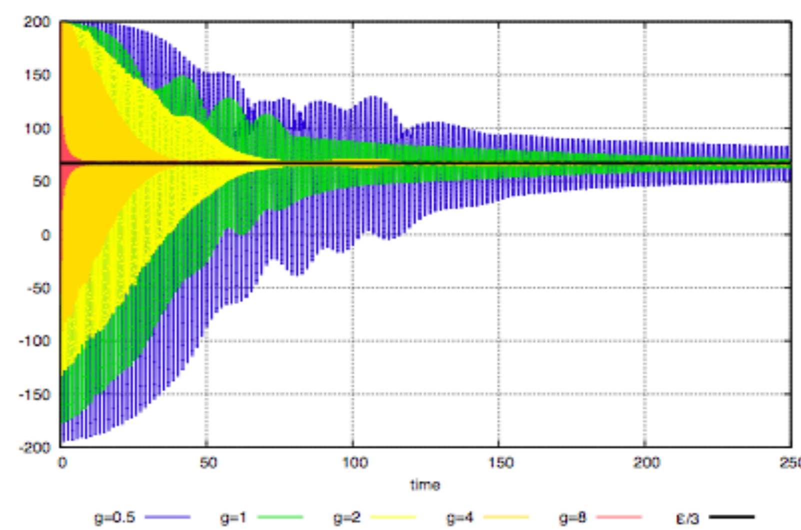
Converges to single valued relation “EOS”

Hydrodynamics from quantum fluctuations

For 1-d expansion recover
$\varepsilon \sim \tau^{-4/3}$ behavior of ideal hydrodynamics

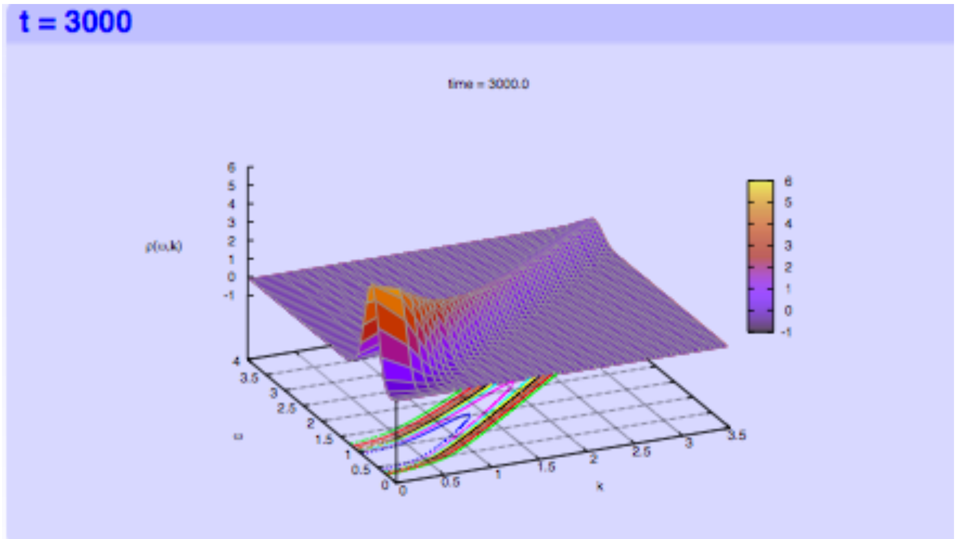


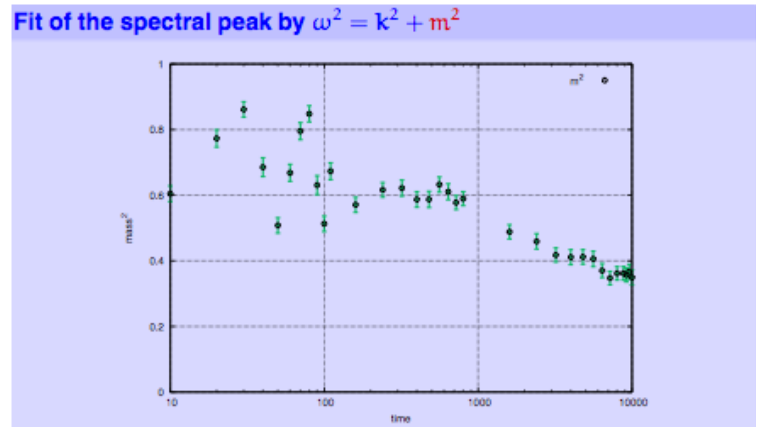
Convergence with increasing g



Quasi-particle description?

Epelbaum,Gelis (2011)

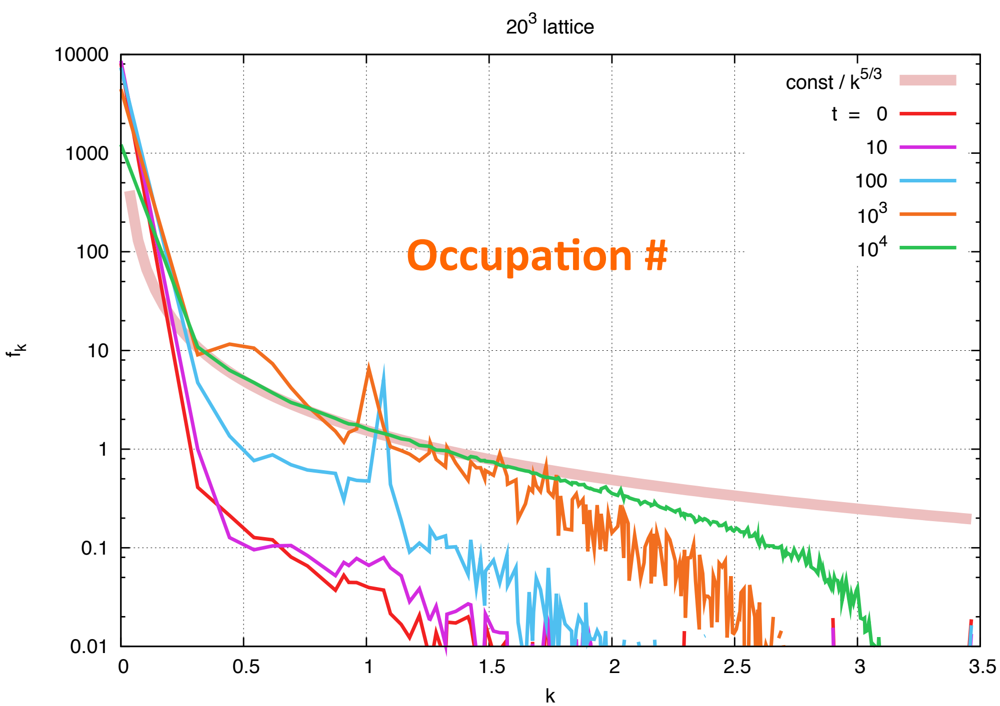


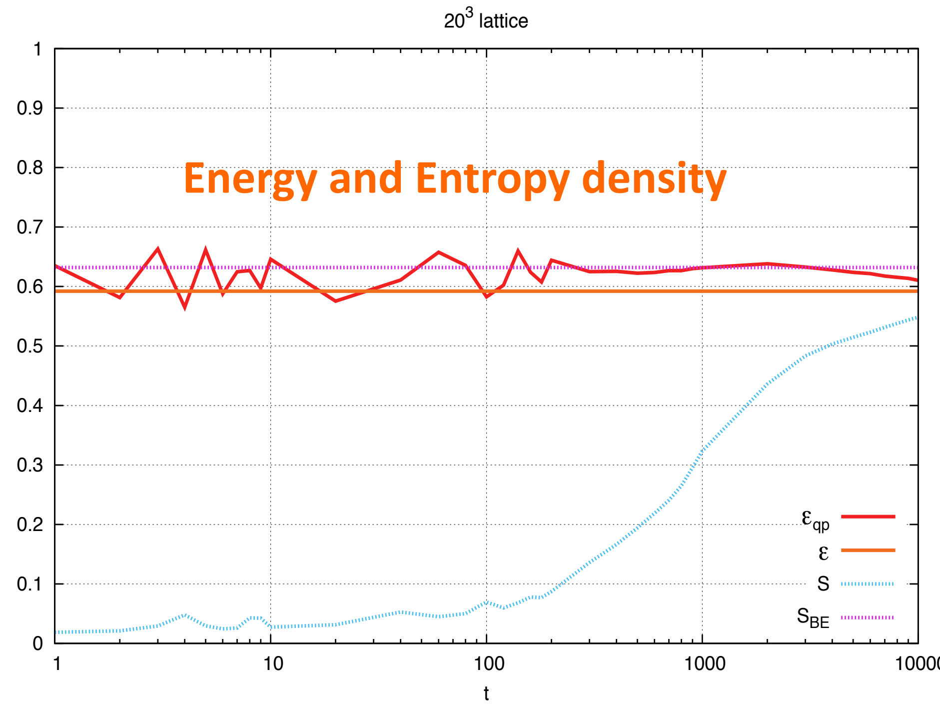


- ❑ **At early times, no quasi-particle description**

- ❑ **May have quasi-particle description at late times.**
Is there an effective kinetic “Boltzmann” description in terms of interacting quasi-particles at late times ?

Thermalization ?





- ***Possible turbulent thermalization? Or "quasi-equilibrium" ?***
- ***Good fit to spectra also for $f = C/\omega\text{-}\mu$ with $\mu=m$***

Summary-I

- ❖ Analogous computation of small fluctuations for QCD – in preparation

- ❖ Similar structure as scalar case - significantly more complex but numerically feasible

- ❖ Eventual result will contain ab initio treatment of

✓ Initial state: energy evolution of inclusive final states including leading logs in x (+ running coupling) + rescattering contributions

✓ ? Final state: resummation of leading instability contributions to all orders

Powerful tool to study i) flow in the Glasma and its properties…

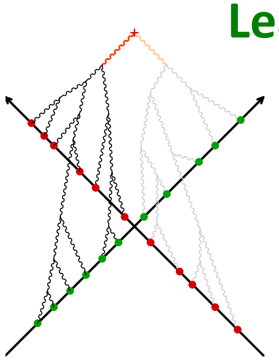
ii) possible onset of thermalization / match to strong coupling approaches

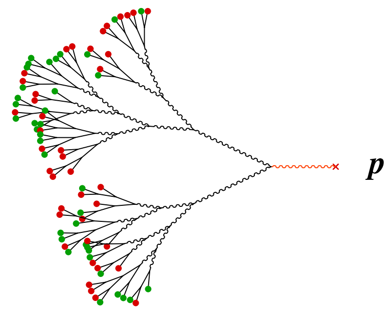
iii) sphaleron transitions – Chiral magnetic effect ?

Summary-II

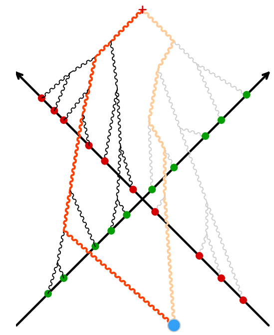
- Initial state/early time energy loss at NLO and beyond

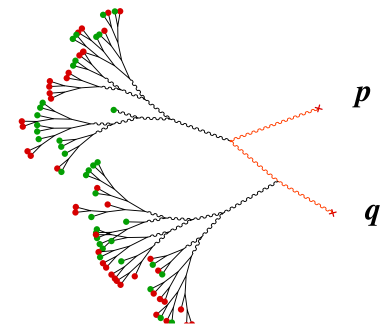
Leading order





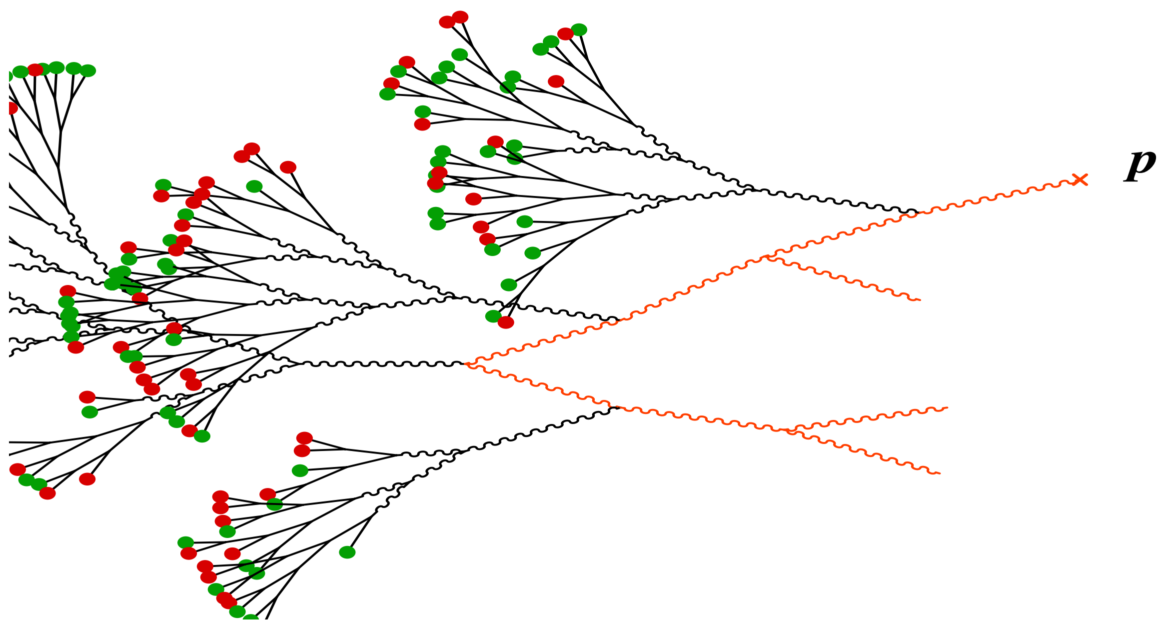
NLO





Path integral over fluctuation spectrum

All order - resummed



Gelis, RV (2006)

Probability of producing n particles in theory with sources:

LSZ: $$\langle p_1 \cdots p_{n\,\mathrm{out}} \big| 0_{\mathrm{in}} \rangle = \frac{1}{Z^{n/2}} \int \left[\prod_{i=1}^{n} d^4x_i e^{ip_i \cdot x_i} \, (\partial_{x_i}^2 + m^2) \frac{\delta}{i\delta\rho(x_i)} \right] e^{i\mathcal{V}[\rho]}$$

$$P_n = \frac{1}{n!} \mathcal{D}^n [j_+, j_-] \, \exp\left(iV[j_+] - iV^\star[j_-]\right) \big|_{j_+ = j_- = j}$$

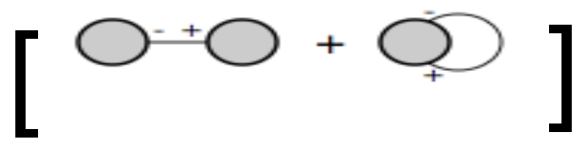
$$D[j_+, j_-] \equiv \frac{1}{Z} \int_{x,y} G^0_{+-}(x,y)(\partial_x^2 + m^2)(\partial_y^2 + m^2) \frac{\delta}{\delta j_+(x)} \frac{\delta}{\delta j_-(y)}$$

$$\int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} \, e^{ip\cdot(x-y)}$$

Inclusive average multiplicity:

$$\langle n \rangle = \sum_n n\, P_n \equiv D\, [\underbrace{e^D e^{iV} e^{-iV}}_{e^{iV_{\rm SK}}}]$$

$$\langle n \rangle = \int_{x,y} Z\, G^0_{+-}(x,y)\, [\Gamma_+(x)\, \Gamma_-(y) + \Gamma_{+-}(x,y)]$$



$$\Gamma_\pm(x) = \frac{\partial_x^2 + m^2}{Z}\, \frac{\delta iV}{\delta j_\pm(x)}|_{j_+=j_-=j}$$

$$\Gamma_{+-}(x,y) = \frac{(\partial_x^2 + m^2)(\partial_y^2 + m^2)}{Z}\, \frac{\delta^2 iV}{\delta j_\pm(x) \delta j_-(y)}|_{j_+=j_-=j}$$