# First results from hybrid HKM for top RHIC and LHC energies


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## Introduction

The work is based on HydroKinetic Model (HKM) for A+A collisions [1]. It combines the advantages of the hydrodynamic approximation, where possible phase transitions are encoded in the corresponding equation of state (EoS), and microscopic approach, accounting for a non-equilibrated process of the spectra formation due to gradual particle liberation. The dynamical decoupling is described by the particle escape probabilities in inhomogeneous hydrodynamically expanding systems in the way consistent with the kinetic equations in the relaxation time approximation for emission function. We present the extension of hydrokinetic model, and focus particularly on the description of HBT radii in A+A collisions for different collision energies.

## Initial conditions

We employ Glauber model for initial conditions. Namely, initial energy density in the transverse plane is proportional to the participant nucleon density

$$\epsilon(\mathbf{b},\mathbf{x}_T)=\epsilon_0\frac{\rho(\mathbf{b},\mathbf{x}_T)}{\rho_0}\quad\text{with}\quad\rho_0\equiv\rho(0,0)\quad\text{and}$$

$$\rho(\mathbf{b},\mathbf{x}_T)=(T(\mathbf{x}_T+\mathbf{b}/2)S(\mathbf{x}_T-\mathbf{b}/2)+T(\mathbf{x}_T-\mathbf{b}/2)S(\mathbf{x}_T+\mathbf{b}/2)),$$

$$S(\mathbf{x}_T)=\left[1-\left(1-\sigma_{NN}\frac{T(\mathbf{x}_T)}{A}\right)^A\right],$$

where $A$ is atomic number, and $\sigma_{NN}=51$ mb(=5.1 fm$^2$) is the nucleon-nucleon cross-section at $\sqrt{s_{NN}}=200$ AGeV. The impact parameter $\mathbf{b}=(b,0)$ is equal to zero, $b$=0, in the considered case of central collision. The parameter $\epsilon_0\equiv\epsilon(b=0,\mathbf{x}_T=0)$ is the maximal energy density at the initial moment of thermalization. The thickness $T(\mathbf{x}_T)$ is expressed through the Woods-Saxon distribution profile for nucleus.

The maximal energy density $\epsilon_0$ at time $\tau_i=1$ fm/c is the first fitting parameter fixed by the charged hadron miltiplicity at mid-rapidity, $dN_{ch}/dy$.

Inspired by the studies of pre-thermal flow development, transverse rapidity profile is chosen to be linear in radius $r_T$:

$$y_T=\alpha\frac{r_T}{R_T},\quad\text{where}\quad R_T=\sqrt{<r_T^2>},$$

here $\alpha$ is the second fitting parameter. Note that the fitting parameter $\alpha$ should include also a positive correction for underestimated resulting transverse flow since in this work we did not account in direct way for the viscosity effects [3] neither at QGP stage nor at hadronic one. In formalism of HKM [1] the viscosity effects at hadronic stage are incorporated in the mechanisms of the back reaction of particle emission on hydrodynamic evolution which we ignore in current calculations. Since the corrections to transverse flows which depend on unknown viscosity coefficients are unknown, we use fitting parameter $\alpha$ to describe the "additional unknown portions" of flows, caused both factors: by a developing of the pre-thermal flows and the viscosity effects in quark-gluon plasma.

## Hydrodynamics

The matter evolution in thermally equilibrated zone is described by the relativistic hydrodynamical equations related to the conservation of energy-momentum:

$$\partial_\nu T^{\mu\nu}=0 \tag{1}$$

and equations associated with the net baryon number, strangeness and isospin conservations

$$\partial_\nu(q_i u^\nu)=0 \tag{2}$$

Here $q_i$ is the density of conserved quantum number. We will use the boost-invariant approach to describe strong longitudinal matter expansion observed at RHIC, as well as at LHC. For such an approach the hyperbolic coordinates in $(t,z)$ directions are more suitable than the Cartesian ones:

$$\tau=\sqrt{t^2-z^2},\ \eta=\frac{1}{2}\ln\frac{t+z}{t-z}$$

## Equation of state

**Equilibrated domain.** At high temperatures corresponding to the QGP phase and crossover transition to hadron phase we use a realistic EoS [4] adjusted to the lattice QCD results for zero barionic chemical potential so that it is matched with an ideal chemically equilibrated multicomponent hadron resonance gas at $T_c=175$ MeV. To take into account a conservation of the net baryon number, electric charge and strangeness in the QGP phase, one has first to make corrections to thermodynamic quantities for nonzero chemical potentials, as it is proposed in [5]:

$$\frac{p(T,\mu_B,\mu_S)}{T^4}=\frac{p(T,0,0)}{T^4}+\frac{1}{2}\frac{\chi_B}{T^2}\left(\frac{\mu_B}{T}\right)^2+\frac{1}{2}\frac{\chi_S}{T^2}\left(\frac{\mu_S}{T}\right)^2+\frac{\chi_{BS}}{T^2}\frac{\mu_B}{T}\frac{\mu_S}{T}$$

The expansion coefficients $\chi_B$ and $\chi_S$ are the baryon number and strangeness susceptibilities, numerically evaluated as a function of the temperature [5].

We use the chemical freeze-out temperature $T_{ch}=165$ MeV, corresponding chemical potentials $\mu_B$ =29 MeV, $\mu_S$ =7 MeV, $\mu_E$ =-1 MeV and also the strangeness suppression factor $\gamma_S=0.935$ which are dictated by 200A GeV RHIC particle number ratios analysis [7, 8].

**Non-equilibrated domain** At the chemical freeze-out temperature $T_{ch}$ the "lattice" EoStaken from [4] and corrected for non-zero chemical potentials is matched with good accuracy with ideal Boltzmann hadronic resonance gas which includes $N=329$ well-established hadron states from the recent PDG table (classes from FASTMC [6]).

After chemical freeze-out, particle numbers are changed due to resonance decays. This change is computed in the proposed scheme for the equation if state, and depend on the evolution of density of each sort of hadron. A covariant relativistic extension of the resonance decay law for a hydrodynamic medium leads to the equation:

$$\partial_\mu(n_i(x)u^\mu(x))=-\Gamma_i n_i(x)+\sum_j b_{ij}\Gamma_j n_j(x) \tag{3}$$

when one neglects a thermal motion of the resonance $j$, that can be justified because post (chemical) freeze-out temperatures are much less than the mass of the lightest known resonance. Also, Eq. (3) for the hydrodynamic evolution is written under supposition of an instant thermalization of the decay products, that is consistent with the ideal fluid approximation (mean free path is zero). In the kinetic part of the HKM we consider the next approximation when the non-equilibrium character of the distribution functions and the kinetics of resonance decays are taken into account. We also can approximately account for a recombination in the processes of resonance decays into expanding medium just by utilizing the effective decay width $\Gamma_{i,eff}=\gamma\Gamma_i$ in Eq. (3). We use $\gamma=0.75$ for the resonances containing $u$ and $d$ quarks supposing thus that about 30% of such resonances are recombining during the evolution.

The equations (3) together with the hydrodynamic equations and the equation of state should give one the energy density and composition of the gas in each space-time points.

## Parameters and results

The model parameters, $\epsilon_0$ and $\alpha$ were fixed from $dN_{\text{charged}}/dy$ for top RHIC and $dN_{\text{charged}}/d\eta$ for LHC 2.76 TeV collision energies at mid-rapidity and from effective temperature of hadron spectra, respectively. This correspond to $\epsilon_0=15$ GeV/fm$^3$ ($\langle\epsilon_0\rangle=10.6$ GeV/fm$^3$) for top RHIC energy and $\epsilon_0=40$ GeV/fm$^3$ ($\langle\epsilon_0\rangle=28.2$ GeV/fm$^3$) for LHC 2.76 TeV. We found that the magnitude of initial transverse flow $\alpha=0.18$ does not change from top RHIC to LHC case.

To demonstrate the difference between hHKM and *hybrid* approaches, three cases were studied:

1. switching from HKM to UrQMD on "isochrone" ($\tau=const$) corresponding to $T(r_T=0,\tau)=T_{ch}$ according to non-equilibrium DFs from hydrokinetic model.
2. switching to UrQMD on the isotherm corresponding to chemical freeze-out temperature (*hybrid model*).
3. switching from HKM to UrQMD on the isotherm $T=130$ MeV.

## Hydrokinetic approach

When the system hadronize (at some hypersurface), hadrons are allowed to escape from the expanding system, and the process of particle liberation and system break-up is described as follows.

We start from the Boltzmann equations for the mixture of hadrons, most of which have finite lifetimes and decay widths compatible with particle masses. The set of such equations for i-components of the hadron resonance gas which account for the only binary interactions (elastic scattering) and resonance decays are:

$$\frac{p_i^\mu}{p_i^0}\frac{\partial f_i(x,p)}{\partial x^\mu}=G_i^{scatt}-L_i^{scatt}(x,p)+G_i^{decay}(x,p)-L_i^{decay}(x,p)$$

The *loss* term is: $L_i^{scatt}(x,p)=f_iR_i$, $L_i^{decay}(x,p)=f_iD_i$ where $R$ is scattering rate, and $D$ is decay rate. For gain-term, the approximations are used: $G_i\approx R_{i,l.eq.}(x,p)f_{i,l.eq.}(x,p)+G_i^{decay}(x,p)$ In the first approximation to hydro-kinetic evolution the parameters of the local equilibrium distribution function $f_{i,l.eq.}(x,p)$, e.g. the temperature $T(x)$, chemical potentials $\mu_i(x)$ are determined by the hydrodynamic evolution.

The formal solutions of BE correspond to the non-equilibrium distribution functions in expanding and decaying multi-hadronic system:

$$f_i(\tau,\theta,\mathbf{r}_T,\mathbf{p}_T)=f_i^{l.eq.}(x^{(\tau_0)}(\tau),\mathbf{p}_T)\exp\left(-\int_{\tau_0}^{\tau}\tilde{R}_i(x^{(s)}(\tau),\mathbf{p}_T)ds\right)+$$

$$\int_{\tau_0}^{\tau}d\lambda\left[f_i^{l.eq.}(x^{(\lambda)}(\tau),\mathbf{p}_T)\tilde{R}_i(x^{(\lambda)}(\tau),\mathbf{p}_T)+\tilde{G}_i^{decay}(x^{(\lambda)}(\tau),\mathbf{p}_T)-\right. \tag{4}$$

$$\left.-L_i^{decay}(x^{(\lambda)}(\tau),\mathbf{p}_T)\right]\exp\left(-\int_{\lambda}^{\tau}\tilde{R}_i(x^{(s)}(\tau),\mathbf{p}_T)ds\right)$$

Here we use the notation $x^{(\tau_0)}(\tau)=\{\tau_0,\theta^{(\tau_0)}(\tau),\mathbf{r}_T^{(\tau_0)}(\tau)\}$

$$\begin{cases}\sinh\theta^{(\tau_0)}(\tau)=\frac{\tau}{\tau_0}\sinh\theta\\ \mathbf{r}_T^{(\tau_0)}(\tau)=\mathbf{r}_T-\frac{\mathbf{p}_T}{m_T}(\tau\cosh\theta-\sqrt{\tau_0^2+\tau^2\sinh^2\theta})\end{cases}$$

and $\tilde{R}_i(\lambda,\theta,\mathbf{r}_T,\mathbf{p}_T)=\frac{\cosh y}{\cosh\theta}R_i(\lambda,\theta,\mathbf{r}_T,\mathbf{p}_T)$, $\tilde{G}_i^{decay}(\lambda,\theta,\mathbf{r}_T,\mathbf{p}_T)=\frac{\cosh y}{\cosh\theta}G_i^{decay}(\lambda,\theta,\mathbf{r}_T,\mathbf{p}_T)$.

The term $G_i^{\text{decay}}$ correspond to the portion of resonance decay products which escape from the system without any interaction:

$$p_i^0G_i^{decay}(x,p_i)=\sum_{j,k}\int\frac{d^3p_j}{p_j^0}\int\frac{d^3p_k}{p_k^0}\Gamma_{j\to ik}f_j(x,p_j)\frac{m_j}{F_{j\to ik}}\delta^{(4)}(p_j-p_k-p_i)$$

and $p_i^0L_i^{decay}(x,p_i)=m_i\Gamma_i f_i(x,p_i)$.

To calculate collision rates, hadron cross-sections are chosen in a form similar to those in UrQMD code.

## Extension: cascade model

Transport code like UrQMD needs a list of particle coordinates and momenta as an input. Given set of (classically defined) particles is then being rescattered by the Monte-Carlo algorithm with given probabilities for each process. To couple transport code to HKM model one thus need, first, the average multplicity for each hadron specie per event, $<N_i>$. This is calculated by integrating the total flow of particles with distribution function (4) through the switching hypersurface $\Sigma_{\text{sw}}$. Then, one proceeds to generate "events", i.e. coordinates and momenta of $N_i$ (Poisson distributed with mean $<N_i>$) particles on switching hypersurface are generated using acception-rejection procedure with distribution function (4).

## Conclusions

We present first results of calculations in hybrid hydro-kinetic approach, where particle interactions at the latest stage of collision are treated with transport code (UrQMD).

Energy dependence of HBT-radii is much improved compared to the previous results from pure hydro-kinetic model [2] due to UrQMD code which is essential for the late, highly nonequilibrium stage of matter expansion. The results indicate notable influence of the last, non-equilibrated stage of evolution to space-time scales at LHC energy.

HHKM results are compared to the *hybrid model* calculations, where the hydrodynamic evolution is switched directly to UrQMD at chemical freeze-out hypersurface. It is found that, whereas $p_T$-dependence of $R_{\text{long}}$ is better reproduced in a case of hybrid model, $R_{\text{out}}/R_{\text{side}}$ ratio favors hHKM procedure.

We conclude that the lack of reproduction of $Ro/Rs$ ratio in hybrid model can be caused by the inconsistenies of hybrid approach: casuality problems at non-space-like sectors of freeze-out hypersurface and application of transport code to a very dense system.

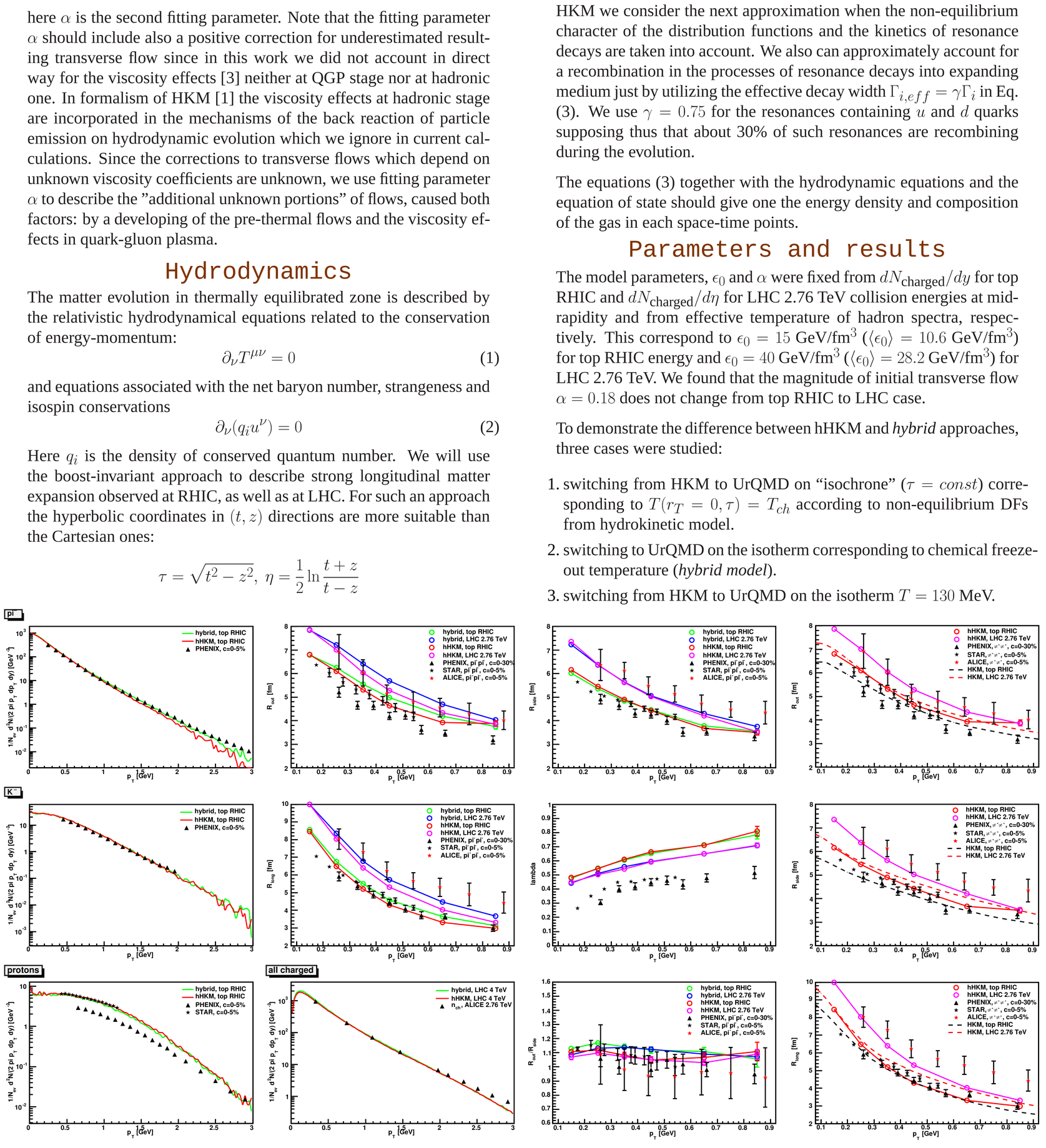


Figure 1: Transverse momentum spectra for $\pi^-$,$K^-$ and $p$, HBT radii for $\pi^-\pi^-$ pairs and intercept $\lambda$ from hHKM and hybrid model calculations, compared to experimental data for $\sqrt{s}=200$A GeV most central Au+Au collisions at RHIC (STAR and PHENIX collaborations) and $\sqrt{s}=2.76$A TeV most central Pb+Pb collisions at LHC (ALICE collaboration). Right panel shows HBT radii in comparison to previous results from HKM model.

## References


[1] S.V. Akkelin, Y. Hama, Iu.A. Karpenko, and Yu.M. Sinyukov, Phys. Rev. C**78**, 034906 (2008); Iu.A. Karpenko, Yu.M. Sinyukov, Phys. Rev. C 81, 054903 (2010).

[2] Iu.A. Karpenko, Yu.M. Sinyukov, Phys. Lett. B 688, 50 (2010).

[3] D. Teaney, Phys. Rev. C **68**, 034913 (2003).

[4] M. Laine, Y. Schroder Phys. Rev. D **73**, 085009 (2006).

[5] F. Karsch, PoS CPOD07:026, 2007.

[6] N.S. Amelin *et al.*, Phys. Rev. C **77**, 014903 (2008).

[7] F. Becattini, J. Manninen J. Phys. G **35**, 104013 (2008); J. Manninen, F. Becattini, Phys. Rev. C **78**, 054901 (2008);

[8] A. Andronic, P. Braun-Munzinger, J. Stachel, Acta Phys.Polon. B **40**, 1005 (2009).