Radiative Energy Loss Reduction in a plasma due to damping

M. Bluhm

with P. B. Gossiaux & J. Aichelin

SUBATECH, Nantes

Outline

- Motivation
- Method to implement damping of radiation
- Results
- Discussion of the formation time

LP(M) effect: Landau+Pomeranchuk 1953

- destructive interference between radiation amplitudes because of multiple scatterings of radiating high-energy charge
- characteristic quantity: formation length l_f ◄——► formation time t_f
- loss of coherence when charge suffers multiple scatterings within t_f

➡ suppression, quantitative change of soft radiation spectrum compared to Bethe-Heitler spectrum

Polarization effect: Ter-Mikaelian 1954

- formation length modified by medium polarization (effects on radiated quanta)
- loss of coherence, i.e. suppression of emission process, by dielectric polarization of medium

investigations of the induced gluon radiation spectrum:

- Kämpfer+Pavlenko 2000 constant thermal mass
- Djordjevic+Gyulassy 2003 colour-dielectric modification of gluon dispersion relation using HTL self-energy

- study influence of damping of radiated (time-like) quanta within an absorptive plasma on energy loss of an energetic charged probe

Method

- study influence of damping of radiated (time-like) quanta within an absorptive plasma on energy loss of an energetic charged probe
- local approach: Total energy loss determined from medium-induced mechanical work (cf. Thoma+Gyulassy 1991)

$$W = 2\,Re\left(\int d^3\vec{r}\,'\int_0^\infty d\omega\,\vec{E}(\vec{r}\,',\omega)\,\vec{j}(\vec{r}\,',\omega)^*\right)$$

$$\frac{1}{\omega\mu(\omega)}\left[k^2\vec{E}_{\vec{k}}(\omega)-\vec{k}(\vec{k}\vec{E}_{\vec{k}}(\omega))\right]-\omega\epsilon(\omega)\vec{E}_{\vec{k}}(\omega)=\frac{iq}{(2\pi)^2}\int dt'\vec{v}(t')e^{i\omega t'-i\vec{k}\vec{r}(t')}$$

for the classical current $\vec{j}(\vec{r}\,',t)=q\vec{v}(t)\delta^{(3)}(\vec{r}\,'-\vec{r}(t))$

- classical calculation: $\omega \ll E$

Method

- study influence of damping of radiated (time-like) quanta within an absorptive plasma on energy loss of an energetic charged probe
- local approach: Total energy loss determined from medium-induced mechanical work (cf. Thoma+Gyulassy 1991)

$$W = 2\,Re\left(\int d^3\vec{r}\,' \int_0^\infty d\omega\,\vec{E}(\vec{r}\,',\omega)\,\vec{j}(\vec{r}\,',\omega)^*\right)$$

$$\frac{1}{\omega\mu(\omega)}\left[k^2\vec{E}_{\vec{k}}(\omega) - \vec{k}(\vec{k}\vec{E}_{\vec{k}}(\omega))\right] - \omega\epsilon(\omega)\vec{E}_{\vec{k}}(\omega) = \frac{iq}{(2\pi)^2}\int dt'\vec{v}(t')e^{i\omega t' - i\vec{k}\vec{r}(t')}$$

for the classical current $\vec{j}(\vec{r}\,',t) = q\vec{v}(t)\delta^{(3)}(\vec{r}\,' - \vec{r}(t))$

- classical calculation: $\omega \ll E$
- missing feature in Thoma+Gyulassy approach: *Multiple scatterings* that change $\vec{v}(t)$ (follow original work of Landau and Pomeranchuk)

Method

- study influence of damping of radiated (time-like) quanta within an absorptive plasma on energy loss of an energetic charged probe
- local approach: Total energy loss determined from medium-induced mechanical work (cf. Thoma+Gyulassy 1991)

$$W = 2\,Re\left(\int d^3\vec{r}\,' \int_0^\infty d\omega\, \vec{E}(\vec{r}\,',\omega)\,\vec{j}(\vec{r}\,',\omega)^*\right)$$

$$\frac{1}{\omega\mu(\omega)}\left[k^2\vec{E}_{\vec{k}}(\omega) - \vec{k}(\vec{k}\vec{E}_{\vec{k}}(\omega))\right] - \omega\epsilon(\omega)\vec{E}_{\vec{k}}(\omega) = \frac{iq}{(2\pi)^2}\int dt'\vec{v}(t')e^{i\omega t' - i\vec{k}\vec{r}(t')}$$

for the classical current $\vec{j}(\vec{r}\,',t) = q\vec{v}(t)\delta^{(3)}(\vec{r}\,' - \vec{r}(t))$

- classical calculation: $\omega \ll E$
- missing feature in Thoma+Gyulassy approach: *Multiple scatterings* that change $\vec{v}(t)$ (follow original work of Landau and Pomeranchuk)
- implement damping mechanisms as small corrections by complex $\epsilon(\omega)$ and $\mu(\omega)$

Important Details

- simplification: $\epsilon(\omega)$ and $\mu(\omega)$ depend on ω only, i.e. no spatial distortions in the plasma $\longrightarrow$ sensitivity of the expressions to poles in the complex momentum plane only
- approach cannot disentangle *detected* from *absorbed* radiation

Important Details

- simplification: $\epsilon(\omega)$ and $\mu(\omega)$ depend on ω only, i.e. no spatial distortions in the plasma $\longrightarrow$ sensitivity of the expressions to poles in the complex momentum plane only
- approach cannot disentangle *detected* from *absorbed* radiation
- decompose *complex index of refraction* $n^2(\omega) = \epsilon(\omega)\mu(\omega)$ into $n = n_r + i n_i$

$$\frac{dW}{d\omega} = Re\left(\frac{iq^2}{4\pi^2}\int dt \int dt' \, \frac{\omega^2 n^3(\omega)}{\epsilon(\omega)} e^{-i\omega(t-t')}\mathcal{A}(t,t')\right)$$

$$\mathcal{A}(t,t') = \left(\vec{v}(t)\vec{v}(t') + (\vec{\nabla}_g \vec{v}(t))(\vec{\nabla}_g \vec{v}(t'))\right)\frac{e^{i\,sgn(n_i)g}}{g} \;,\; \vec{g} = \omega n(\omega)\vec{\Delta r}$$

Important Details

- simplification: $\epsilon(\omega)$ and $\mu(\omega)$ depend on ω only, i.e. no spatial distortions in the plasma $\longrightarrow$ sensitivity of the expressions to poles in the complex momentum plane only
- approach cannot disentangle *detected* from *absorbed* radiation
- decompose *complex index of refraction* $n^2(\omega) = \epsilon(\omega)\mu(\omega)$ into $n = n_r + i n_i$

$$\frac{dW}{d\omega} = Re\left(\frac{iq^2}{4\pi^2}\int dt \int dt' \, \frac{\omega^2 n^3(\omega)}{\epsilon(\omega)} e^{-i\omega(t-t')} \mathcal{A}(t,t')\right)$$

$$\mathcal{A}(t,t') = \left(\vec{v}(t)\vec{v}(t') + (\vec{\nabla}_g \vec{v}(t))(\vec{\nabla}_g \vec{v}(t'))\right) \frac{e^{i\,sgn(n_i) g}}{g}\ , \ \ \vec{g} = \omega n(\omega)\vec{\Delta r}$$

- essential is the **exponential factor**, which always implies damping of medium-induced mechanical work:

$$e^{i\,sgn(n_i) g} = e^{i\,sgn(n_i)\Delta r \omega n_r} e^{-\Delta r \omega |n_i|}$$

Important Details

- simplification: $\epsilon(\omega)$ and $\mu(\omega)$ depend on ω only, i.e. no spatial distortions in the plasma $\longrightarrow$ sensitivity of the expressions to poles in the complex momentum plane only
- approach cannot disentangle *detected* from *absorbed* radiation
- decompose *complex index of refraction* $n^2(\omega) = \epsilon(\omega)\mu(\omega)$ into $n = n_r + in_i$

$$\frac{dW}{d\omega} = Re\left(\frac{iq^2}{4\pi^2}\int dt \int dt' \frac{\omega^2 n^3(\omega)}{\epsilon(\omega)} e^{-i\omega(t-t')}\mathcal{A}(t,t')\right)$$

$$\mathcal{A}(t,t') = \left(\vec{v}(t)\vec{v}(t') + (\vec{\nabla}_g \vec{v}(t))(\vec{\nabla}_g \vec{v}(t'))\right)\frac{e^{i\,sgn(n_i)g}}{g}\,,\ \vec{g} = \omega n(\omega)\vec{\Delta r}$$

- essential is the **exponential factor**, which always implies damping of medium-induced mechanical work:

$$e^{i\,sgn(n_i)g} = e^{i\,sgn(n_i)\Delta r \omega n_r} e^{-\Delta r \omega |n_i|}$$

- for constant $\vec{v}(t)$, $\mathcal{A}(t,t')$ vanishes
- averaging over small deflection angles: $\langle \theta_\tau^2 \rangle \sim \hat{q}\bar{t}/E^2$

Result

$$\frac{d^2W}{dzd\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\right]\right.$$
$$\left.\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

- medium-induced mechanical work depends on $\hat{q}$ in the way known to be important for decoherence; the linear dependence shows its association with radiative energy loss
- expression resembles in vacuum exactly the negative of the radiation intensity derived by Landau and Pomeranchuk
- essential difference in damping medium: **Exponential damping factor**
- set $\mu = 1$ in the following, i.e. concentrating on transerse modes only

Radiation Dispersion Relation

- radiated quanta follow medium-modified dispersion relations of plasma modes
- view emitted hard ($\omega > T$) quanta as time-like excitations, which obey finite *thermal mass* and which are *damped* within the absorptive medium (Pisarski 1989+1993)

Radiation Dispersion Relation

- radiated quanta follow medium-modified dispersion relations of plasma modes
- view emitted hard ($\omega > T$) quanta as time-like excitations, which obey finite *thermal mass* and which are *damped* within the absorptive medium (Pisarski 1989+1993)
- Lorentzian ansatz for spectral function results in retarded propagator (Peshier 2004+2005)

$$-\Delta^{-1}(\omega, \vec{k}) = \omega^2 - \vec{k}^2 - m^2 + 2i\Gamma\omega$$

- Remark: m and Γ in general free parameters of spectral function

Radiation Dispersion Relation

- radiated quanta follow medium-modified dispersion relations of plasma modes
- view emitted hard ($\omega > T$) quanta as time-like excitations, which obey finite *thermal mass* and which are *damped* within the absorptive medium (Pisarski 1989+1993)
- Lorentzian ansatz for spectral function results in retarded propagator (Peshier 2004+2005)

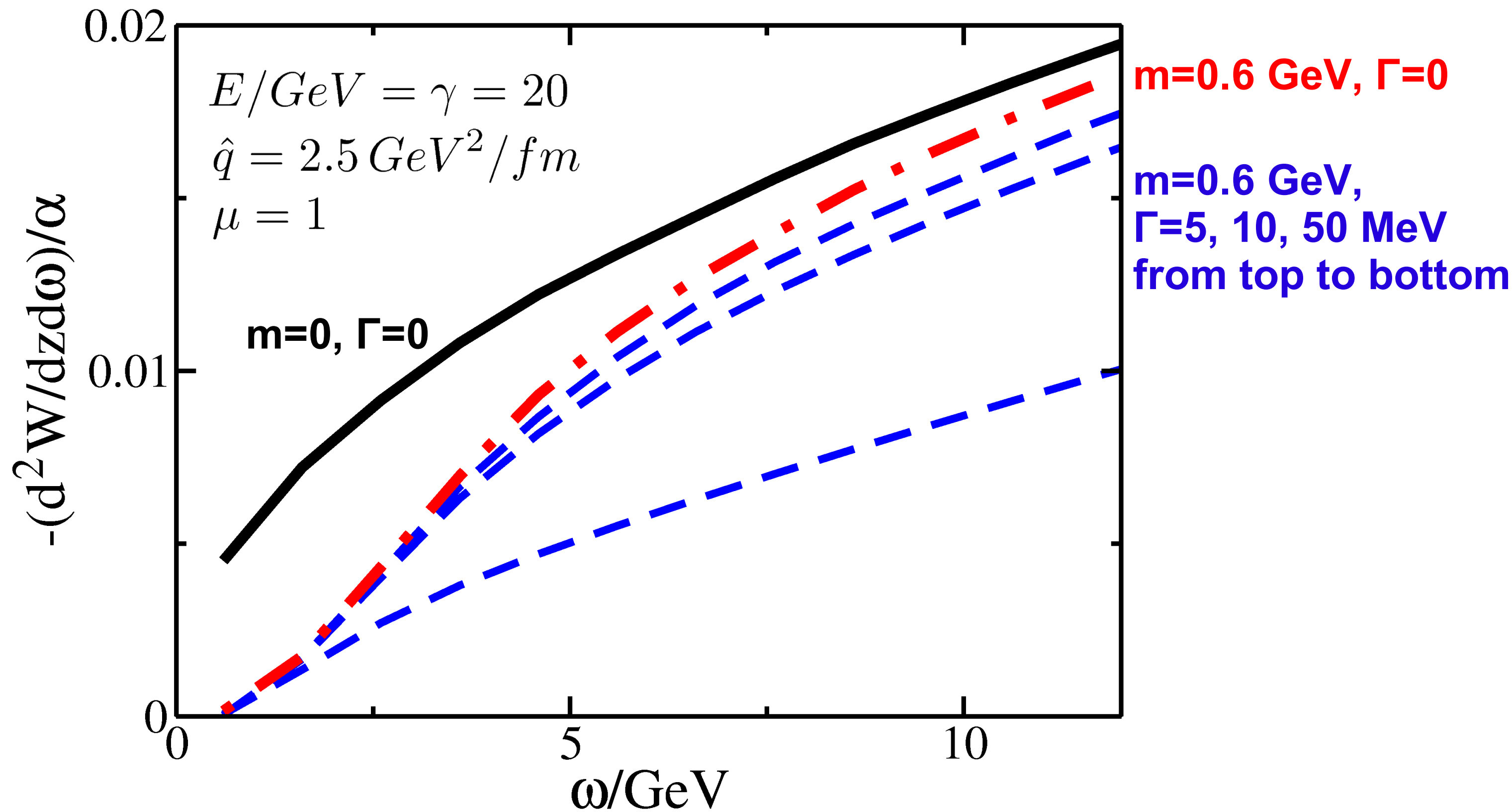
$$-\Delta^{-1}(\omega, \vec{k}) = \omega^2 - \vec{k}^2 - m^2 + 2i\Gamma\omega$$

- Remark: m and Γ in general free parameters of spectral function
- concentration on transverse modes $\Longleftrightarrow$ sensitivity of medium-induced mechanical work on poles in k-integration at $\omega^2\epsilon(\omega) - \vec{k}^2 = 0$
- corresponding *complex index of refraction* follows via $\epsilon = 1 - \Pi/\omega^2$ as

$$n^2(\omega) = 1 - m^2/\omega^2 + 2\,i\,\Gamma/\omega$$

- dispersion relation of emitted quanta: $Re\,\epsilon = \vec{k}^2/\omega^2$
  - absence of radiation for $\omega < m$
  - plasma modes are time-like
  - $Im\,\epsilon = 2\Gamma/\omega$ has support in time-like sector too

Numerical Results



- damping significantly reduces the spectrum
- with increasing E, relative effect of damping compared to non-damping case increases

Formation Time – non-damping case

$$\frac{d^2W}{dzd\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\,\right]\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

- making use of the *phase condition* in the non-damping case, estimate for formation time taking multiple scatterings into account

$$\begin{aligned} 1 &\simeq t(\omega - k(\omega)\beta) + \frac{\beta}{6}\frac{\hat{q}}{E^2}k(\omega)t^2 \\ &\equiv \frac{t}{t_f^{(s)}} + \frac{t^2}{\left(t_f^{(dif)}\right)^2} \end{aligned}$$

where $k(\omega) = \left(\omega^2 - m^2\right)^{1/2}$

$$\Longrightarrow \quad t_f \sim min\left(t_f^{(s)}, t_f^{(dif)}\right)$$

Formation Time – non-damping case

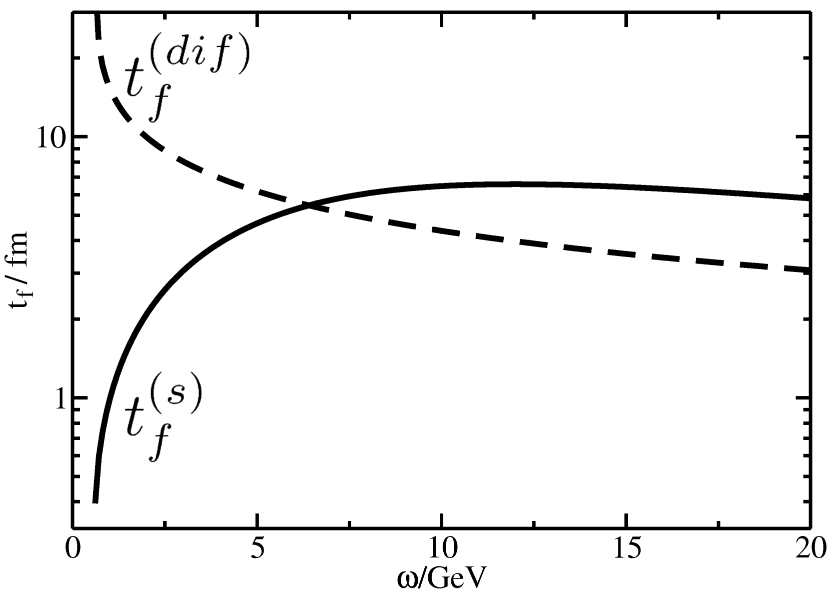
$$\frac{d^2W}{dzd\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\right]\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

- making use of the *phase condition* in the non-damping case, estimate for formation time taking multiple scatterings into account

$$\begin{aligned} 1 &\simeq t(\omega - k(\omega)\beta) + \frac{\beta}{6}\frac{\hat{q}}{E^2}k(\omega)t^2 \\ &\equiv \frac{t}{t_f^{(s)}} + \frac{t^2}{\left(t_f^{(dif)}\right)^2} \end{aligned}$$

where $k(\omega) = \left(\omega^2 - m^2\right)^{1/2}$

$\Rightarrow \quad t_f \sim min\left(t_f^{(s)}, t_f^{(dif)}\right)$



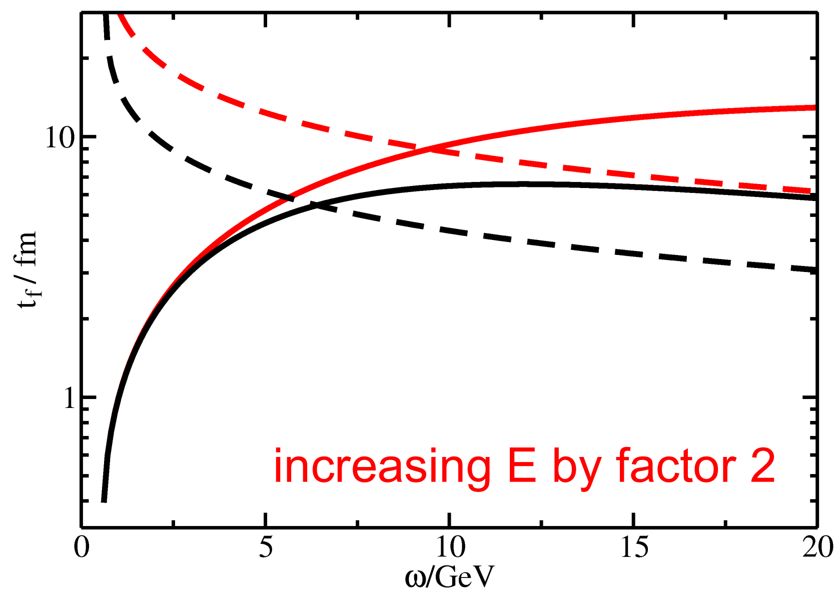
Formation Time – non-damping case

$$\frac{d^2W}{dzd\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\right]\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

- making use of the *phase condition* in the non-damping case, estimate for formation time taking multiple scatterings into account

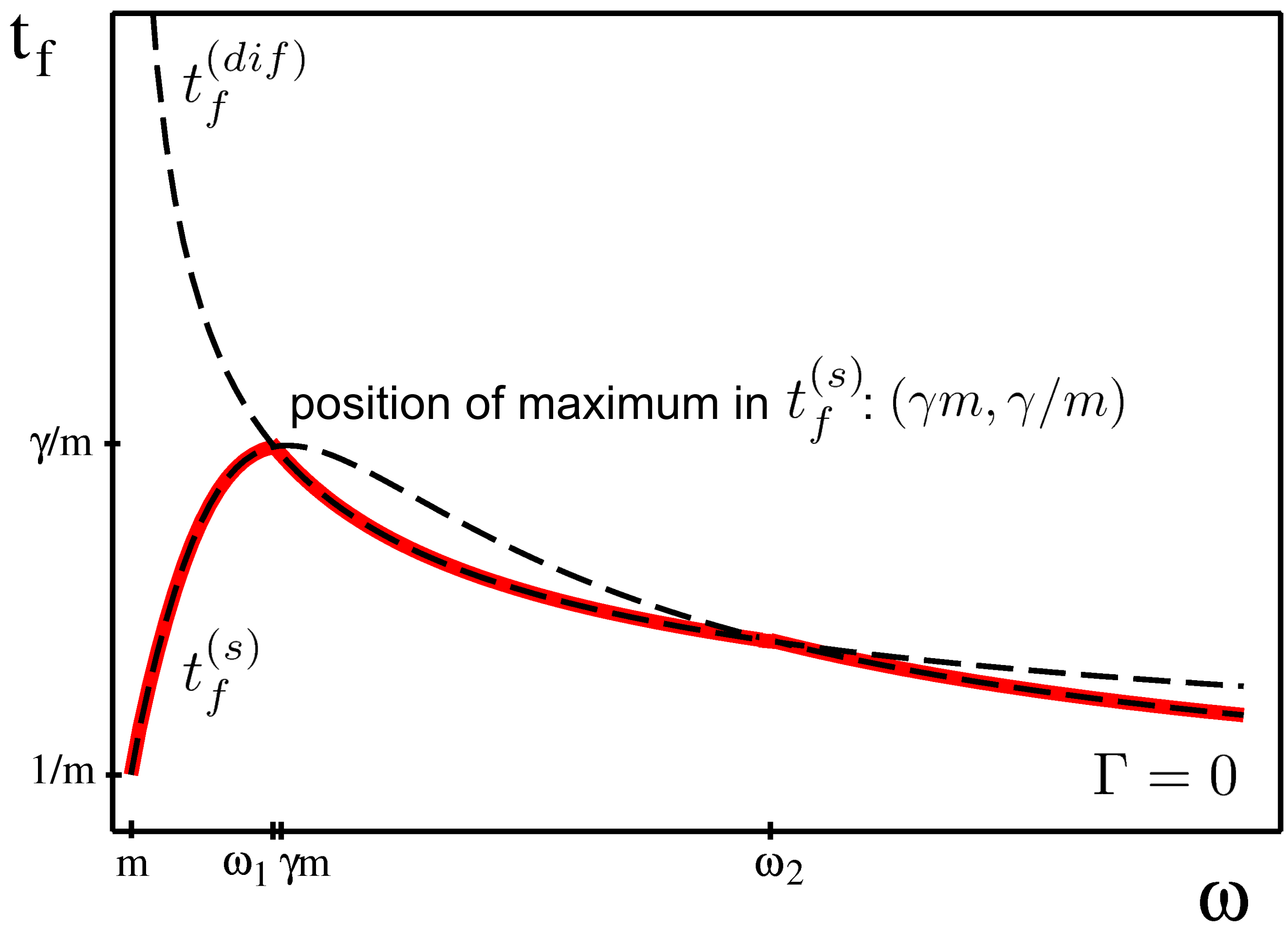
$$\begin{aligned} 1 &\simeq t(\omega - k(\omega)\beta) + \frac{\beta}{6}\frac{\hat{q}}{E^2}k(\omega)t^2 \\ &\equiv \frac{t}{t_f^{(s)}} + \frac{t^2}{\left(t_f^{(dif)}\right)^2} \end{aligned}$$

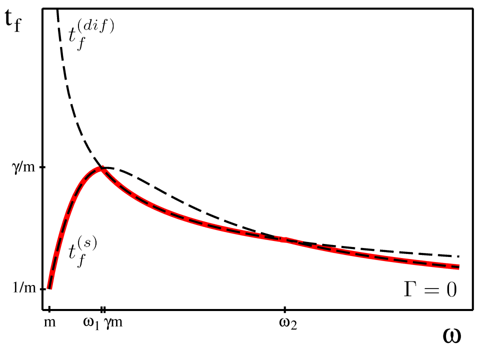
where $k(\omega) = \left(\omega^2 - m^2\right)^{1/2}$



$$\Rightarrow \quad t_f \sim min\left(t_f^{(s)}, t_f^{(dif)}\right)$$

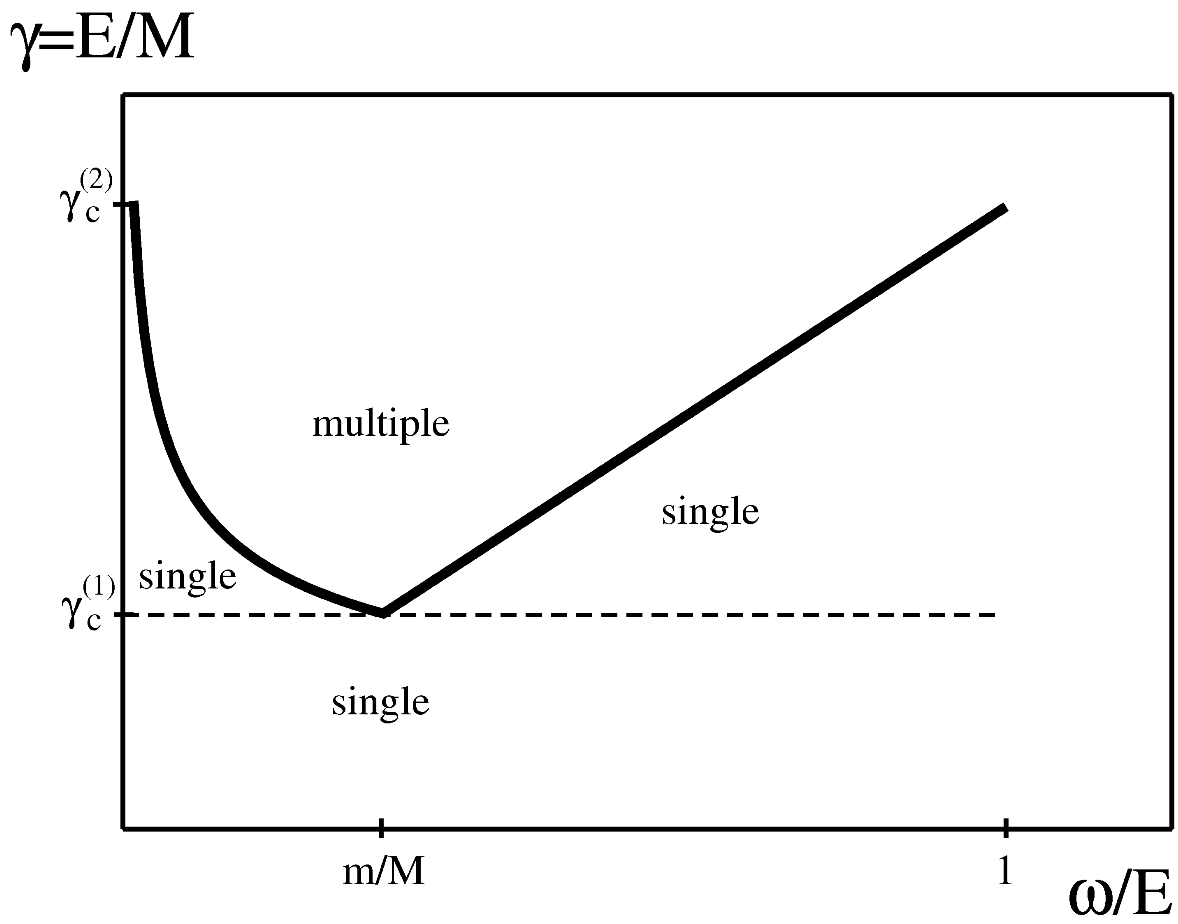
- multiple scatterings dominant for $\omega_1 \sim \gamma m \left(\frac{\gamma_c^{(1)}}{\gamma} \right)^{1/3} < \omega < \omega_2 \sim \gamma M \frac{\gamma}{\gamma_c^{(2)}}$





$$\gamma_c^{(1)} = \frac{mM^2}{\hat{q}}$$

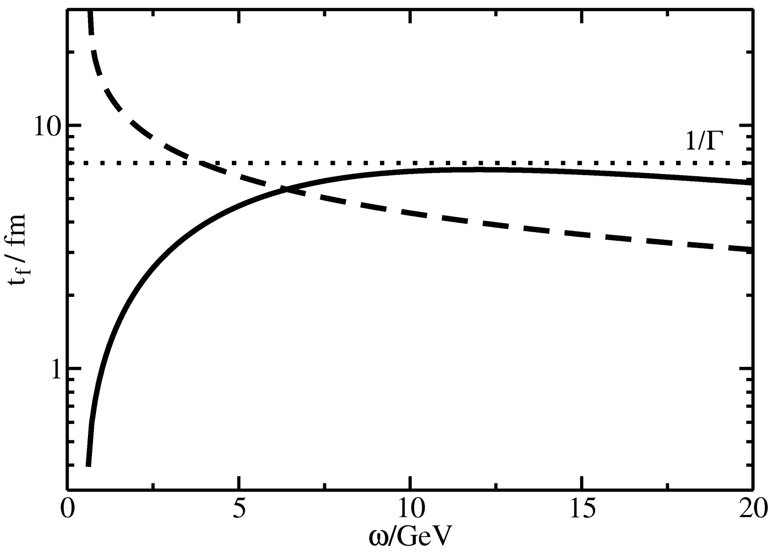
$$\gamma_c^{(2)} = \frac{M}{m}\gamma_c^{(1)}$$



Formation Time – damping case

$$\frac{d^2W}{dzd\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\,\right]\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

- competition with a second time scale, at which the exponential damping factor becomes of order $1/e$: $t_d \sim 1/\Gamma$

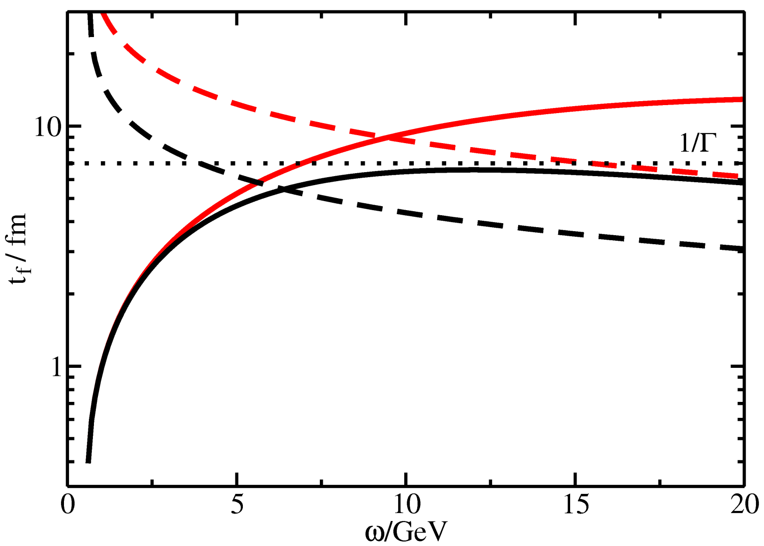


- damping irrelevant for $1/\Gamma \gg t_f(\omega_1)$

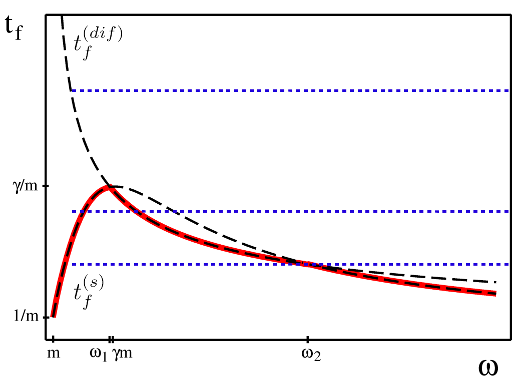
Formation Time – damping case

$$\frac{d^2W}{dzd\omega} \simeq -Re\left(\frac{2i\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\frac{\omega n^2(\omega)}{\epsilon(\omega)}\exp\left[-\omega|n_i(\omega)|\beta\bar{t}\,\right]\cos(\omega\bar{t})\exp\left[isgn(n_i(\omega))\omega n_r(\omega)\beta\bar{t}\left(1-\frac{\hat{q}}{6E^2}\bar{t}\right)\right]\right)$$

- competition with a second time scale, at which the exponential damping factor becomes of order $1/e$: $t_d \sim 1/\Gamma$



- damping irrelevant for $1/\Gamma \gg t_f(\omega_1)$
- damping plays some role in intermediate region $t_f(\omega_1) > 1/\Gamma > t_f(\omega_2)$
- damping becomes dominant for $1/\Gamma \lesssim t_f(\omega_2)$: $\Gamma \gtrsim \hat{q}/M^2$



- if $\Gamma \sim g^2T$, $\hat{q} \sim g^2T^3$ then damping important for $M^2 \gtrsim T^2$

Conclusions

- consideration of the effect of *damping of radiation* on the energy loss of an energetic charged probe
- followed original approach by Landau and Pomeranchuk to implement multiple scatterings in the most simple way
- *classical*, not quantum
- radiated quanta are **time-like** excitations with finite *thermal mass* being *damped* in the absorptive plasma
- find a potential substantial **reduction** of the radiative energy loss per unit distance travelled by energetic charge, which *increases with damping and/or initial energy*
- formation time in non-damping medium already modified due to polarization effects
- in damping-medium a second *competing time scale* evolves that potentially negates the effect of multiple scatterings in a specific ω-region