Quark Matter 2011 - Annecy, France

Continuous Time Monte Carlo for Strong Coupling QCD

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Wolfgang Unger (ETHZ), Philippe de Forcrand (ETHZ, CERN)

Strong Coupling QCD - Motivation and Setup

Why Strong Coupling QCD?

- SC-QCD exhibits **confinement** and **chiral symmetry breaking**.
- Nuclear physics: can derive nuclear interactions between hadrons from (lattice) QCD (see Ref. [1]).
- SC-QCD phase diagram: study nuclear phase transition, possible for arbitrarily large chemical potential: the **sign problem** is **mild** (discrete time) or even absent (continuous time).

1-flavor QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = \bar\psi\left(i\partial\!\!\!/ - gA^a_\mu\gamma^\mu t^a + m_q\right)\psi - \frac{\beta}{2N_c}\sum_P\left({\rm tr}\,U_P + {\rm tr}\,U_P^\dagger\right) + \mathcal{O}\left(a^2\right)$$

- Send the **gauge coupling to infinity:** $g\to\infty \quad\Rightarrow\quad \beta=\frac{2N_c}{g^2}\to 0$.
- Allows to integrate out the gauge fields completely! However, lattice remains coarse.

SC-QCD with staggered fermions:

- First integrate out gauge fields analytically, as the link integration factorizes, then integrate out fermions.
- Fermions become spinless.
- New degrees of freedom (exact rewriting of QCD path integral, once β is set to zero):
  - **Monomers** correspond to mesons, $M(x)=\bar\chi(x)\chi(x)$,
  - **Dimers** correspond to meson hoppings (non-oriented),
  - **Baryons** form self-avoiding oriented loops, $B(x)=\frac{1}{N_c}\epsilon_{i_1\dots i_{N_c}}\chi_{i_1}(x)\dots\chi_{i_{N_c}}(x)$.
- Strong Coupling Partition Function after Grassmann integrals carried out (leading to the constraint):

$$\mathcal{Z}(m_q,\gamma)=\sum_{\{k,n,l\}}\prod_{b=(x,\hat\mu)}\frac{(3-k_b)!}{3!k_b!}\gamma^{2k_b\delta_{\hat 0\hat\mu}}\prod_x\frac{3!}{n_x!}(2am_q)^{n_x}\prod_l w(l),\qquad \sum_{\hat\mu}k_{\hat\mu}+n_x=3$$

$n_x\in\{0,1,\dots,3\}$ $\qquad$ $k_\mu(x)\in\{0,1,\dots,3\}$ $\qquad$ $\bar B(x)B(y)\in\{0,1\}$

Continuous Time Worm Algorithm

Motivation for Continuum Limit and Continous Time:

- Extent of Euclidean time: inverse temperature $\beta=1/T=N_\tau a$, given by lattice extent N_τ and lattice spacing a, can only be adjusted in discrete steps.
- Standard fix: introduce anisotropy parameter γ: $aT\simeq\gamma^2/N_\tau$.
- However: the function $f(\gamma)=\frac{a}{a_t}$ is not known, hence: mandatory to perform the continuum limit for temporal lattice spacing: $a_t\to 0$: $\gamma^2\to\infty,\ N_\tau\to\infty$ at fixed aT.
- Continuum extrapolation turns out to be non-monotonic.
- Problems are bypassed by a **continuous time formulation**.

Basics of Worm Algorithm:

- Worm algorithm samples the monomer Green function $G(x,y)$ by violating the Grassmann constraint.
- **Worm head** and **tail** act as monomer sources.
- Updates proceed in three parts:
  - MDP update (2 monomers $\leftrightarrow$ 1 dimer, see Ref. [2])
  - Mesonic Worm (move dimers around, see Ref. [3])
  - Baryonic Worm (undirected "polymers" $\leftrightarrow$ directed baryon loops, change contour)

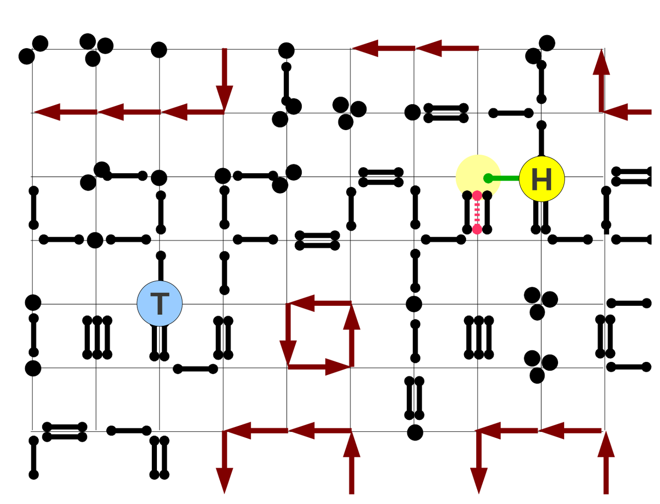

Fig. 1: Typical monomer-dimer-baryon configuration. Mesonic worm update: (T)ail and (H)ead positions indicated, red dimer was removed in the previous step, green dimer is added in the current step.

Continuous Euclidean Time Worm Algorithm:

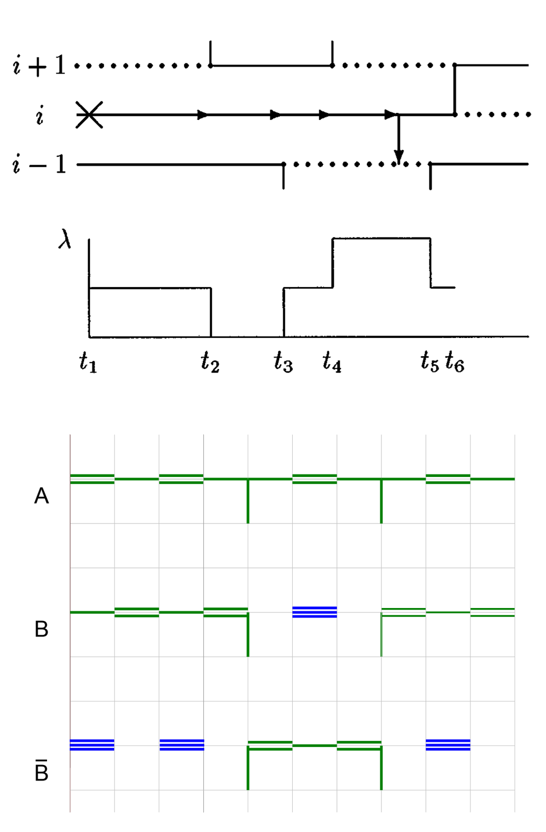

Fig. 2: Top: Transition probabilities in continuous Euclidean time, from [4]. Bottom: identification of temporal dimer sequences solid (green) and dahsed (blue) lines.

- Double and triple spatial dimers are suppressed with γ^{-2} and γ^{-4} $\Rightarrow$ **multiple spatial dimers vanish in continuum limit**.
- Temporal dimer sequences 3-0-3-0... (dashed lines) and 2-1-2-1... (solid lines) become continuous intervals, separated by spatial dimers.
- Rewrite $\mathcal{Z}$ in terms of temporal intervals (here: mesonic part only)

$$\mathcal{Z}(T)=\prod_x v_L^{n_L(x)}v_T^{n_T(x)}\simeq\prod_x 3^{-n_I(x)/2}\prod_i^{n_I(x)}P(\Delta\hat\beta_i)$$

with $v_L=1/\sqrt{6\gamma^2}$, $v_T=2/\sqrt{\gamma^2}$ and $n_L(x)$ and $n_T(x)$ the weights and number of **T-vertices** and **L-vertices**, $n_I(x)$ the number of intervals separated by spatial hoppings and $\Delta\hat\beta_i$ the interval length.
- In d spatial dimensions, solid intervals can emit spatial hoppings with **emission probability:**

$$P(\Delta\hat\beta)=\exp(-d_M\Delta\hat\beta/4)\quad \Delta\hat\beta\in[0,\hat\beta=1/aT]$$

with d_M the number of mesonic neighbors - for U(3) $d_M=2d$, but it is site-dependent for SU(3) (no hoppings to baryon sites allowed).

Results on the SC-QCD Phase Diagram

U(3) results

- Theory is purely mesonic, continuous time extrapolation of the critical temperature of the O(2) 2nd order transition is well described by fit ansatz: $T_{pc}(N_\tau)=T_c+a/N_\tau+b/N_\tau^2$
- Note that a and b have different sign (non-monotonic behaviour).

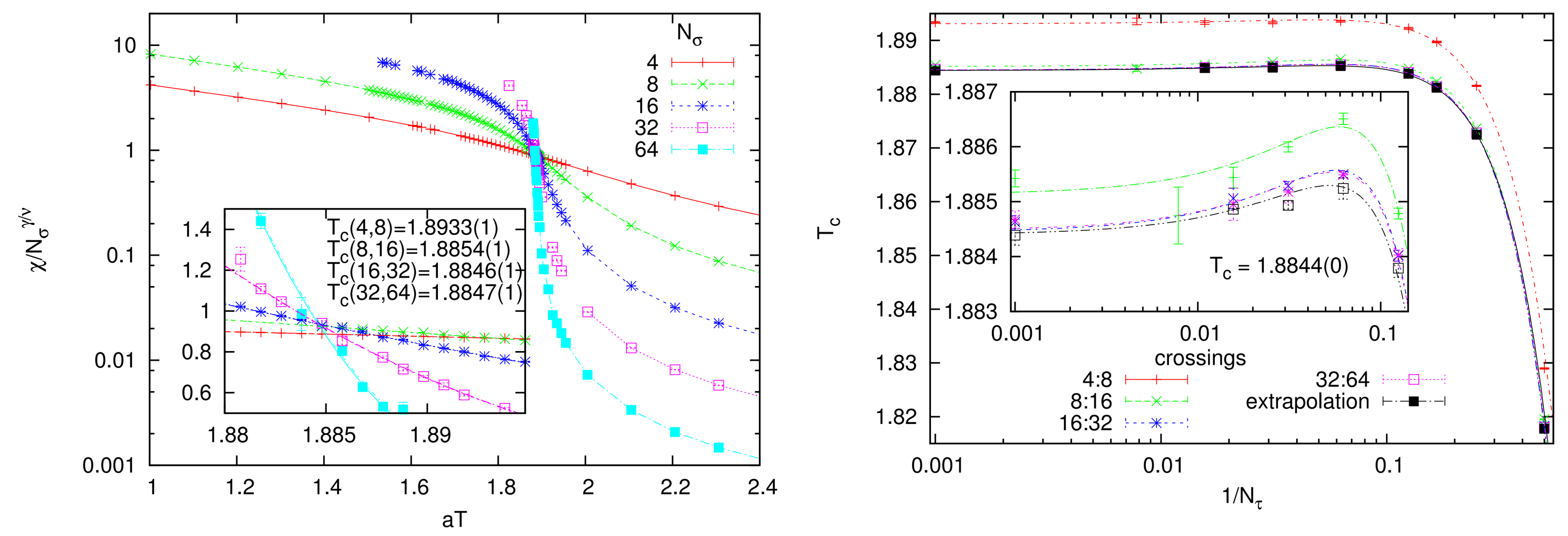

Fig. 4: Chiral susceptibility for U(1) (top) and U(3) bottom, obtained from finite size scaling (left) and continuum extrapolation (right), compared to continuous time results.

SU(3) results

- In the continuum limit: baryon hoppings are suppressed with γ^3 $\Rightarrow$ **baryons become static**.
- Baryonic worm update simplifies in continuous time, positive (negative) oriented baryons are (dis)favored by a factor $\exp(\pm 3\mu/T)$ over dashed lines (mesons).
- **Strong coupling QCD phase diagrams** in the chiral limit:
  - Chiral phase transition measured as a function of μ, nuclear transition additionally measured with baryon density
  - The location of the tricritical point agrees with previous findings.
  - At $T=0$, $\mu^B_{\rm crit}<M_B$: strong nuclear interactions present (see Ref. [1]).
  - Re-entrance seen (the entropy decreases in the high-density phase, due to saturation).

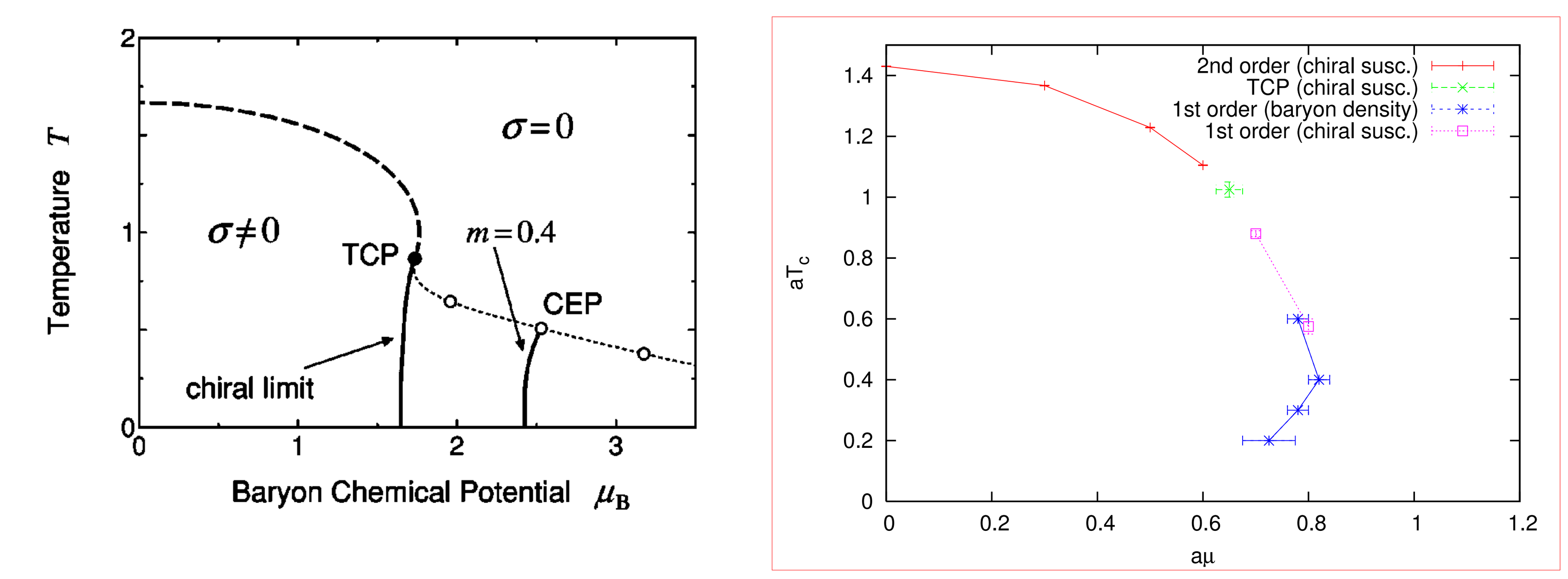

Fig. 5: Left: mean field result, from [5]. Right: new result obtained with continuous time algorithm, with quark chem. potential μ/T

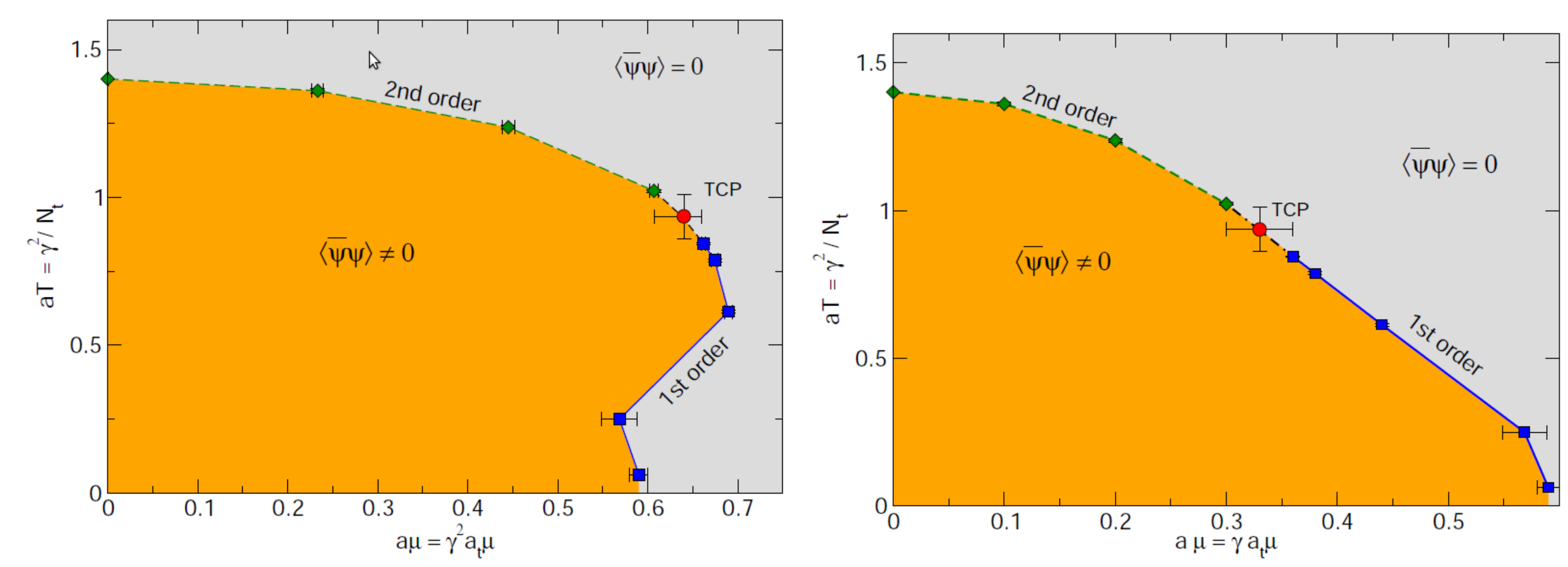

Fig. 6: Discrete algorithm with $N_\tau=4$, from [1]. Note that left and right plot differ in the functional dependence of the chem. potential on γ.

Conclusion & Outlook

- No need to perform continuum extrapolation $N_t\to\infty$. Results consistent with extrapolated discrete results.
- Continuous time algorithm faster than discrete algorithm for $N_t=16$ lattices at T_c.
- The continuum formulation has no sign problem.
- Continuous time correlation functions can be measured and analytically continued.
- Extension to finite quark masses obtained by generating monomers with probability density $\exp(-2m_q\Delta\hat\beta)$.

References

[1] P. de Forcrand, M. Fromm. *Phys. Rev. Lett.* **103** (2010) 112005
[2] F. Karsch, K. H. Mütter. *Nucl. Phys. B* **313** (1989) 541-559
[3] D. H. Adams, S. Chandrasekharan. *Nucl. Phys. B* **662** (1989) 220-246
[4] B. B. Beard, U. J. Wiese. *Phys. Rev. Lett.* **77** (1996) 5130-5133
[5] Y. Nishida. *Phys. Rev. D* **69** (2004) 094501