Towards the phase diagram of QCD

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Introduction

Lattice computations [1, 2, 3] as well ab initio continuum QCD calculations [5, 6, 7] show a broad crossover for both chiral symmetry restoration and the deconfinement transition at vanishing density. Particularly, the change of the order parameter for deconfinement, the Polyakov loop, occurs in a rather broad temperature interval. In contrast, current Polyakov loop extended effective models [8, 9, 10, 11, 12, 13] show steeper slopes in a smaller transition region. Moreover, the critical temperatures show some dependence on the chosen Polyakov loop potential. We qualitatively improve these models towards full QCD by adjusting the Polyakov loop potential to the full glue potential obtained from continuum ab initio computations. We present results for the phase structure of QCD at finite density derived from these improved models.

Polyakov-loop potential

- The Polyakov loop potential cannot be directly extracted from neither lattice computations nor ab initio continuum QCD calculations but has to be motivated by the underlying QCD symmetries in the pure gauge limit.

- In the low-temperature phase $\mathcal{U}(\Phi,\bar\Phi,T)$ has an absolute minimum at $\Phi = 0$. Above the critical temperature for deconfinement the $Z(3)$ center symmetry is spontaneously broken and the minimum of $\mathcal{U}(\Phi,\bar\Phi,T)$ is shifted to a finite value of Φ. In the limit $T\to\infty$ it is $\Phi\to 1$.

→ A possible parametrization is the logarithmic Polyakov loop potential of Ref. [9]:

$$\frac{\mathcal{U}_{\log}}{T^4} = -\frac{1}{2}a\,(T)\,\Phi\bar\Phi + b\,(T)\ln\left[1-6\Phi\bar\Phi+4\left(\Phi^3+\bar\Phi^3\right)-3\left(\Phi\bar\Phi\right)^2\right] \quad (1)$$

with the temperature-dependent coefficients $a\,(T_0/T)$ and $b\,(T_0/T)$.

- The Polyakov loop potential as in Eq. (1) describes the deconfinement phase transition in the pure Yang-Mills sector with a critical temperature $T^0_{\rm YM} = 270\,\mathrm{MeV}$.

→ In the presence of dynamical quarks, the Yang-Mills potential is extended by the matter contributions to a pure glue potential.

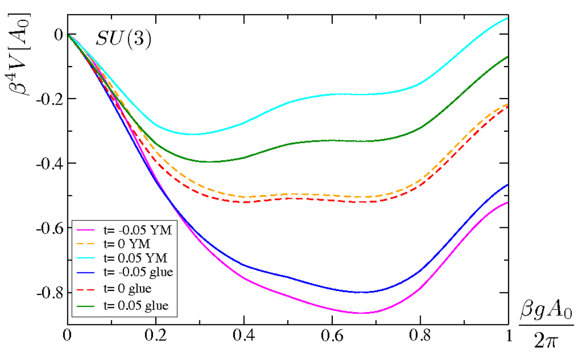

Fig. 1: Yang-Mills potential and pure glue part of the Polyakov loop potential in the two flavour functional renormalization group calculations of Refs. [6, 7]. The form of both is qualitatively the same.

- In Polyakov loop extended effective models, the pure glue potential has been so far just approximated by the Yang-Mills potential.
The qualitative flavour and density dependence in the fully coupled system was constructed only by consistency arguments and the help of HTL/HDL computations in Ref. [10].

→ The QCD functional renormalization group calculations of Refs. [6, 7] allow to compare the Yang-Mills potential to the pure glue potential

→ The form of the pure glue potential is qualitatively unchanged in comparison to the Yang-Mills potential, as can be seen in Fig. 1. Only the transition temperature is significantly reduced, in the two flavour case to $T^0_{\rm glue}\,(N_f=2)\approx 208\,\mathrm{MeV}$.

→ This allows us to map the Yang-Mills potential (1) to the pure glue potential with a polynomial fit of corresponding temperatures: $t_{\rm YM}\,(t_{\rm glue})$, where $t_{\rm YM} = (T_{\rm YM}-T^0_{\rm YM})/T^0_{\rm YM}$ and $t_{\rm glue} = (T_{\rm glue}-T^0_{\rm glue})/T^0_{\rm glue}$ are the respective reduced temperatures.

Polyakov-Quark-Meson model

Thermodynamic Potential

In mean-field approximation the thermodynamic potential consists of the mesonic $U\,(\sigma_{\rm x},\sigma_{\rm y})$, the quark/antiquark $\Omega_{q\bar q}$ and the Polyakov loop contributions:

$$\Omega = U\,(\sigma_{\rm x},\sigma_{\rm y}) + \Omega_{q\bar q}\left(\sigma_{\rm x},\sigma_{\rm y},\Phi,\bar\Phi\right) + \mathcal{U}\left(\Phi,\bar\Phi\right)\ .$$

The mesonic contribution describes the chiral symmetry aspects of QCD with in a 2+1 flavour linear sigma model that depends on the non-strange condensate $\sigma_{\rm x}$ and the strange condensate $\sigma_{\rm y}$:

$$U\,(\sigma_{\rm x},\sigma_{\rm y}) = \frac{m^2}{2}\left(\sigma_{\rm x}^2+\sigma_{\rm y}^2\right) - h_{\rm x}\sigma_{\rm x} - h_{\rm y}\sigma_{\rm y} - \frac{c}{2\sqrt{2}}\sigma_{\rm x}^2\sigma_{\rm y} + \frac{\lambda_1}{2}\sigma_{\rm x}^2\sigma_{\rm y}^2 + \frac{1}{8}\left(2\lambda_1+\lambda_2\right)\sigma_{\rm x}^4 + \frac{1}{8}\left(2\lambda_1+2\lambda_2\right)\sigma_{\rm y}^4\ .$$

The parameters are fitted to the pseudoscalar meson masses m_π, $m_{\rm K}$, $m_\eta^2+m_{\eta'}^2$ and the weak-decay constants f_π, $f_{\rm K}$. The scalar meson mass m_σ is used as a free parameter.

The Polyakov loop variables and the fermionic part are coupled via

$$\Omega_{q\bar q}\left(\sigma_{\rm x},\sigma_{\rm y},\Phi,\bar\Phi\right) = -2T\sum_{f={\rm u,d,s}}\int\frac{d^3p}{(2\pi)^3}\Big\{ \ln\left[1+3\left(\Phi+\bar\Phi e^{-(E_f-\mu_f)/T}\right)e^{-(E_f-\mu_f)/T}+e^{-3(E_f-\mu_f)/T}\right] + \ln\left[1+3\left(\bar\Phi+\Phi e^{-(E_f+\mu_f)/T}\right)e^{-(E_f+\mu_f)/T}+e^{-3(E_f+\mu_f)/T}\right]\Big\}\ .$$

The flavour-dependent single-particle energies are given by $E_f = \sqrt{k^2+m_f^2}$ with the flavour-dependent quark masses $m_l = g\,\sigma_{\rm x}/2$ and $m_{\rm s} = g\,\sigma_{\rm y}/\sqrt{2}$ for the light and strange quarks, respectively. The Yukawa coupling g is fixed to a light constituent quark mass of $m_{\rm q} = 300\,\mathrm{MeV}$ which produces a strange constituent quark mass of $m_{\rm s}\simeq 433\,\mathrm{MeV}$.

The dependence on the temperature and quark chemical potentials of the order parameters for the chiral and deconfinement transition are determined as solutions of the coupled equations of motion with respect to the constant mean-fields $\langle\sigma_{\rm x}\rangle$, $\langle\sigma_{\rm y}\rangle$, $\langle\Phi\rangle$, $\langle\bar\Phi\rangle$:

$$\frac{\partial\Omega}{\partial\sigma_{\rm x}} = \frac{\partial\Omega}{\partial\sigma_{\rm y}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar\Phi}\bigg|_{\sigma_{\rm x}=\langle\sigma_{\rm x}\rangle,\sigma_{\rm y}=\langle\sigma_{\rm y}\rangle,\Phi=\langle\Phi\rangle,\bar\Phi=\langle\bar\Phi\rangle} = 0\ .$$

Results

In the following, we compare the results of the improved Polyakov-Quark-Meson model (full lines) to the Polyakov-Quark-Meson model calculations of Ref. [12] (dashed lines) and also to the latest lattice results of the Wuppertal-Budapest group (grey bands and lines, [3, 4]) and the hotQCD collaboration (data points, [2]) for vanishing density.
In the PQM model calculations $m_\sigma = 600\,\mathrm{MeV}$ is used.

As chiral order parameter we use the subtracted condensate

$$\Delta_{l,s} = \left[\langle\sigma_{\rm x}\rangle(T) - (h_{\rm x}/h_{\rm y})\langle\sigma_{\rm y}\rangle(T)\right] / \left[\langle\sigma_{\rm x}\rangle(0) - (h_{\rm x}/h_{\rm y})\langle\sigma_{\rm y}\rangle(0)\right]\ .$$

Thermodynamics observables as pressure, entropy, particle and energy density are derived from the grand canonical potential as follows:

$$p\,(T,\mu_{\rm q}) = -\Omega\,(T,\mu_{\rm q})\ ,\quad s=-\partial\Omega/\partial T\ ,\quad n=-\partial\Omega/\partial\mu$$
$$\text{and}\qquad \epsilon = -p+Ts+\mu n\ ,$$

with the vacuum normalizations $p\,(0,0) = 0$, $s\,(0,0) = 0$, $n\,(0,0) = 0$.

Vanishing density

Phase structure

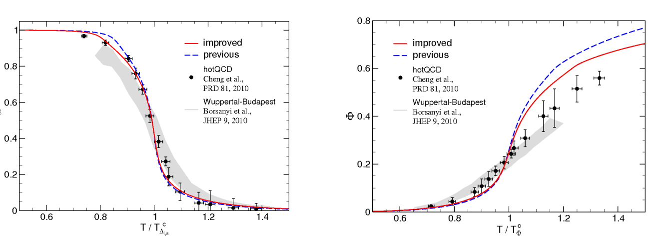

Fig. 2: The subtracted condensate $\Delta_{\rm l,s}$ and the Polyakov loop variable Φ as functions of temperature.

The slope of the Polyakov loop is significantly reduced.

Thermodynamics

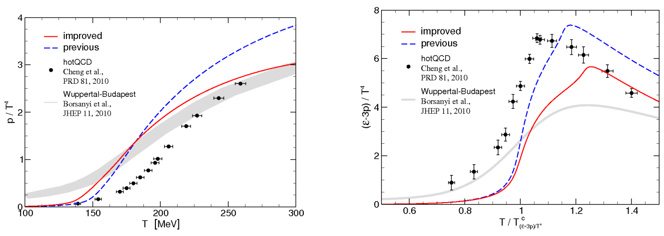

Fig. 3: Normalized pressure and trace anomaly as functions of temperature.

Pressure as well as trace anomaly are closer to the continuum extrapolation of Ref. [3].

Non-vanishing density

Phase structure

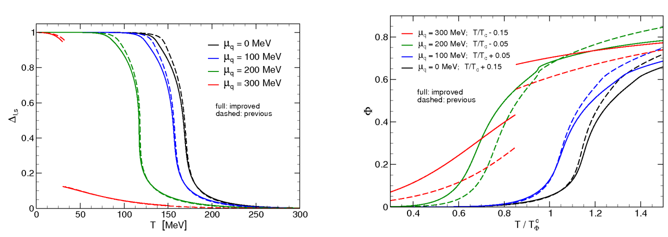

Fig. 4: The subtracted condensate $\Delta_{\rm l,s}$ and the Polyakov loop variable Φ as functions of temperature for different quark chemical potentials.

Smaller slopes for the chiral condensate and Polyakov loop persist at finite chemical potential. At $\mu_{\rm q} = 200\,\mathrm{MeV}$ we are close to a first order phase transition.

Thermodynamics

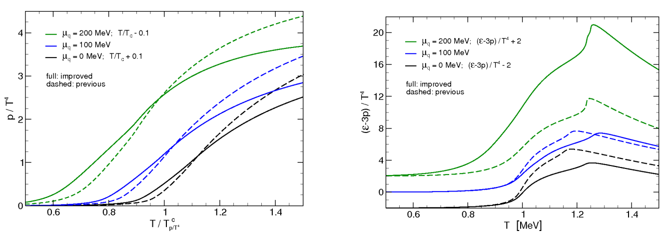

Fig. 5: Normalized pressure and trace anomaly as functions of temperature for different quark chemical potentials.

Conclusions & Outlook

We have shown that one can improve Polyakov loop extended effective models by a temperature mapping of the pure Yang-Mills Polyakov loop potential to the full glue potential obtained from continuum ab initio computations. The change of the order parameter for deconfinement, the Polyakov loop, is reduced and is therefore closer to the lattice results. To further reduce the persisting deviation, we will adjust the form of the effective Polyakov loop potential to characteristic points of the Yang-Mills potential of QCD functional renormalization group calculations. Moreover, including quark-meson fluctuations like in Ref. [13] will further reduce the stiffness of the phase transition.

References

[1] A. Bazavov et al., *Phys. Rev. D* 80, 2009.
[2] M. Cheng et al., *Phys. Rev. D* 81, 2010.
[3] S. Borsányi et al., *JHEP* 9, 2010.
[4] S. Borsányi et al., *JHEP* 11, 2010.
[5] J. Braun, L. M. Haas, F. Marhauser, J. M. Pawlowski, *Phys. Rev. Lett.* 106, 2011.
[6] J. Braun, H. Gies, J. M. Pawlowski, *Physics Letters B* 684, 2010.
[7] J. M. Pawlowski, arXiv:1012.5075 [hep-ph].
[8] C. Ratti, M. A. Thaler, W. Weise, *Phys. Rev. D* 73, 2006.
[9] S. Rößner, C. Ratti, W. Weise, *Phys. Rev. D* 75, 2007.
[10] B. J. Schaefer, J. M. Pawlowski, J. Wambach, *Phys. Rev. D* 76, 2007.
[11] K. Fukushima, *Phys. Rev. D* 77, 2008.
[12] B. J. Schaefer, M. Wagner, J. Wambach, *Phys. Rev. D* 81, 2010.
[13] T. K. Herbst, J. M. Pawlowski, B. J. Schaefer, *Physics Letters B* 696, 2011.