What is the surface tension of quark gluon bags?

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Abstract

We discuss a novel view at the colour confinement which, on the one hand, allows us to find out the surface tension coefficient of quark gluon bags and, under the plausible assumptions, to determine the endpoint temperature of the QCD phase diagram, on the other hand. The developed model considers the confining colour tube as the cylindrical quark gluon bag with non-zero surface tension. A close inspection of the free energies of elongated cylindrical bag and the confining colour tube that connects the static quark-antiquark pair allows us to find out the string tension in terms of the surface tension, thermal pressure and the bag radius. Using the derived relation it is possible to estimate the bag surface tension at zero temperature directly from the lattice QCD data. The requirement of positive entropy density of such bags leads to negative values of the surface tension coefficient of quark gluon bags at the cross-over region, i.e. at the continuous transition to deconfined phase. It is shown that such an approach naturally accounts for an existence of a very pronounced surprising maximum of the string entropy observed in the lattice QCD simulations, which, as we argue, signals about the fractional surface formation of the confining tube.

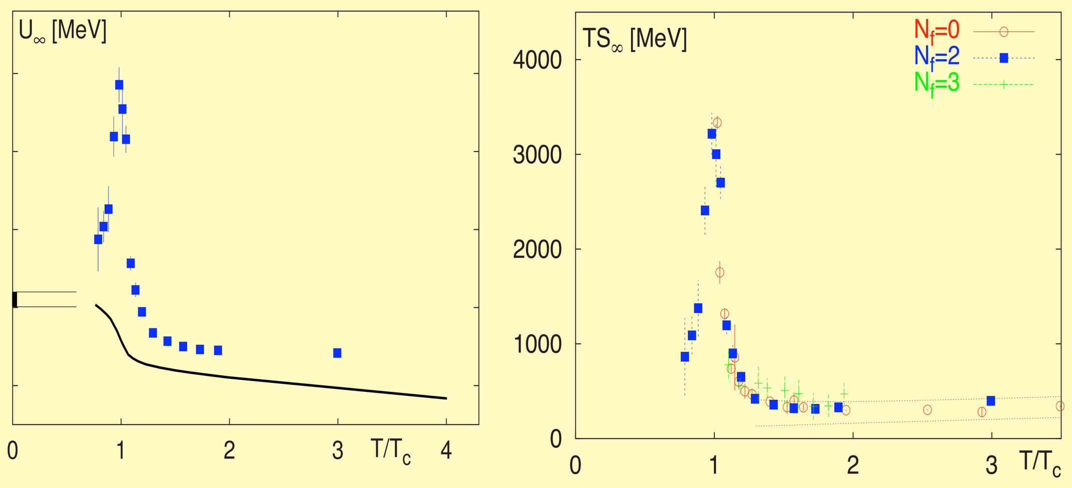
I. Colour confining tube and sQGP

Colour confinement, i.e. an absence of free colour charges, is usually described by the free energy of heavy (static) quark-antiquark pair $F_{q\bar{q}}(T,L) = \sigma_{str} \cdot L$.

In lattice QCD $F_{q\bar{q}}(T,L)$ as a function of temperature T and separation distance L can be extracted from the Polyakov line correlation in a colour singlet channel.

- **confinement:** $\sigma_{str} > 0$
- **deconfinement:** $\sigma_{str} \rightarrow 0$ at $T \rightarrow T_{co}$, but there is no colour charge separation up to $T \geq 1.3\,T_{co}$ values of the cross-over temperature T_{co} [1].

Explanation: although at large distances $L \rightarrow \infty$ the potential energy of static $q\bar{q}$ pair is finite $U_{q\bar{q}}(T,L) = F_{q\bar{q}} - T\frac{\partial F_{q\bar{q}}}{\partial T} = F_{q\bar{q}} + TS_{q\bar{q}}$, near T_{co} the values of $U(T,\infty)$ are very large (see figure below for 2 flavor QCD [O. Kaczmarek and F. Zantow, PoS LAT2005, 192 (2006)])



Conclusion N 1: Near T_{co} region QGP is a strongly interacting plasma (sQGP) which is similar to a liquid [1].

Conclusion N 2: At $T = T_{co}$ the entropy of static $q\bar{q}$ pair is very large $S_{q\bar{q}}(T_{co},\infty) \approx 20$. Such a value signals that really a **huge number of degrees of freedom** $\sim \exp(20)$ is involved (see figure above for different number of flavors [O. Kaczmarek and F. Zantow, PoS LAT2005, 192 (2006)]).

Problem N 1: There are several estimates for the surface tension coefficient σ_{surf} of QGP bags, but can we determine σ_{surf} from lattice QCD?

Problem N 2: The origin of large energy $U_{q\bar{q}}(T,\infty)$ and entropy $S_{q\bar{q}}(T,\infty)$ values near T_{co} remains **mysterious** [1] despite many attempts to explain it.

Here we resolve Problems N 1 & N 2

II. Cylindrical bag model for vanishing baryonic density

Consider very long unbroken confining tube (string) between static $q\bar{q}$ pair of length L, radius $R \ll L$ and free energy $F_{str} \approx \sigma_{str} L$. Equating F_{str} to a free energy of elongated cylinder of same length and radius ($V_0 = 1 \mathrm{fm}^3$)

$$F_{cyl}(T,L,R) \equiv -\underbrace{p_v(T)\pi R^2 L}_{bulk} + \underbrace{\sigma_{surf}(T) 2\pi R L}_{surface} + \underbrace{T\tau \ln \frac{\pi R^2 L}{V_0}}_{small} \qquad (1)$$

in the limit $L \gg R$ we obtain the relation between the string tension and surface tension [2]

$$\sigma_{surf}(T) = \frac{\sigma_{str}(T)}{2\pi R} + \frac{1}{2} p_v(T) R \,. \qquad (2)$$

In doing so we match an ensemble of all string shapes of fixed L to a mean elongated cylinder, which according to a well known M. E. Fisher conjecture and the Hills and Dales Model estimates [3], represents a sum of all surface deformations of a given bag.

Eq. (2) opens a principal possibility to determine the T-dependence of bag surface tension directly from the lattice QCD, if $R(T)$, $\sigma_{str}(T)$ and $p_v(T)$ are known. Unfortunately, at the moment there are no lattice QCD simulations for which all functions $R(T)$, $\sigma_{str}(T)$ and $p_v(T)$ were found simultaneously. Therefore, to find $\sigma_{surf}(T=0)$ we use the bag model pressure $p_v(T=0) = -(0.25)^4$ GeV^4 and the lattice QCD results $R = 0.5$ fm and $\sigma_{str}(T=0) = (0.42)^2$ GeV^2 and obtain a conservative estimate

$$\sigma_{surf}(T=0) = (0.2229 \text{ GeV})^3 + \frac{1}{2} p_v\, R \approx (0.183 \text{ GeV})^3 \approx 157.4 \text{ MeV fm}^{-2} \,. \qquad (3)$$

Note that for larger values of the bulk pressure $p_v(T=0)$ the value of $\sigma_{surf}(T=0)$ in (3) gets larger.

Using this result and the usual assumptions made for ordinary liquids [3] we can determine the (tri)critical endpoint temperature as $T_{cep} = \sigma_{surf}(T=0)\, V_0^{\frac{2}{3}} \cdot \lambda^{-1} = 152.9 \pm 4.5$ MeV [4] since $\lambda \in [1; 1.06009]$ [3].

III. Surface tension at cross-over temperature

The above results allow us to tune the interrelation with the colour tube model and to study the bag surface tension near the cross-over to QGP. The lattice QCD data indicate that at large R the string tension behaves as $\sigma_{str}^{LQCD} \approx \frac{\ln(L/L_0)}{R^2} C$, where L_0 and C are some positive constants. Assuming that in the infinite available volume $\sigma_{str}(T) \rightarrow +0$ which means that the string radius diverges, i.e. $R \rightarrow \infty$, in such a way that ($\omega_k = const \sim \ln(L/L_0)$)

$$\sigma_{str}(T)\, R^k \rightarrow \omega_k > 0 \quad \text{with} \quad k > 0 \,, \qquad (4)$$

which extends a range for the power k. Parameterizing $\sigma_{str} = \sigma_{str}^0\, t^\nu$, where $t \equiv \frac{T_{co}-T}{T_{co}} \rightarrow +0$ and $\sigma_{str}^0 \sim \ln(L/L_0)$, we can find the total pressure p_{tot} and total entropy density s_{tot} of the cylinder

$$p_{tot} = p_v(T) - \frac{\sigma_{surf}(T)}{R} \equiv \frac{\sigma_{surf}(T)}{R} - \frac{\sigma_{str}}{\pi R^2} \rightarrow \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \left[\sigma_{surf} - \frac{\omega_k}{\pi}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{k+1}{k}}\right] , \qquad (5)$$

$$s_{tot} = \left(\frac{\partial p_{tot}}{\partial T}\right)_\mu \rightarrow \underbrace{\frac{1}{k\,\sigma_{str}}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial \sigma_{str}}{\partial T}\,\sigma_{surf}}_{dominant \;\; since \;\; \sigma_{str}\rightarrow 0} + \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial \sigma_{surf}}{\partial T} - \frac{k+2}{\pi k}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{2}{k}} \frac{\partial \sigma_{str}}{\partial T} > 0 \,. \qquad (6)$$

Since $s_{tot} > 0$ and $\frac{\partial \sigma_{str}}{\partial T} < 0$, one finds that **at the cross-over the surface tension coefficient is negative** $\sigma_{surf}(T \rightarrow T_{co}) < 0$. We stress that there is nothing wrong or unphysical with the negative values of surface tension coefficient, since $\sigma\, 2\pi R L$ in (1) is **the surface free energy** and, hence, as any free energy, it contains the energy part E_{surf} and the entropy part S_{surf} multiplied by temperature T, i.e. $F_{surf} = E_{surf} - TS_{surf}$ [3]. Therefore, at low temperatures the energy part dominates and the surface free energy is positive, whereas at high temperatures the number of configurations of a cylinder with large surface drastically increases and the surface free energy becomes negative since $S_{surf} > \frac{E_{surf}}{T}$.

The results (1)–(6) are valid for nonzero baryonic chemical potential μ up to the (tri)critical endpoint. The main modification in (1)–(6) is an appearance of μ-dependences of $p_v(T,\mu)$ and $T_{co}(\mu)$ [2]. In the (tri)critical endpoint vicinity the behavior of p_{tot} and s_{tot} is defined by the t-dependence of the surface tension coefficient.

To explain a mysterious maximum of the lattice entropy we assume that the probability of the unbroken confining tube among other configurations measured by lattice QCD is $W(L) \sim [L \ln(L/L_0)]^{-1}$. Then the contribution of the unbroken confining tube into the lattice free energy is small, since $W(L)F_{str}(L) \sim R^{-k}$ for $t \rightarrow +0$ and $R \rightarrow R_{lat} - 0$ (R_{lat} denotes the lattice size), but its contribution to the tube entropy

$$W(L)S_{str} = -W\frac{dF_{str}}{dT} = W L \frac{\sigma_{str}^0 \nu}{T_{co}} t^{\nu-1} \rightarrow W L \frac{\nu}{T_{co}} \left[\frac{\sigma_{str}^0}{\omega_k^{1-\nu}}\right]^{\frac{1}{\nu}} R^{\frac{k(1-\nu)}{\nu}} \sim R^{\frac{k(1-\nu)}{\nu}} \qquad (7)$$

is an increasing function of the tube radius R for $\nu < 1$.

The origin of a singular behavior of the tube entropy (7) encoded in $\nu < 1$ is rooted in the formation of fractal surfaces of the confining tube in the cross-over temperature vicinity [2]. This can be seen from the power $\frac{k(1-\nu)}{\nu}$ of R on the right hand side of (7) which is fractal for any $\nu \neq \frac{k}{k+n}$ where $n = 1, 2, 3, \ldots$ Moreover, the appearance of fractal structures at $T = T_{co}$ can be easily understood within our model, if one recalls that only at this temperature the fractal surfaces can emerge at almost no energy costs due to almost zero total pressure (5).

An explanation of the tube entropy decrease for $t < 0$ is similar [2, 4]. The fractal surfaces gradually disappear since for $T > T_{co}$ the tube gradually occupies the whole available volume.

IV. Conclusions

- **A relation between the surface tension of QGP bags and the string tension is derived.**
- **The surface tension coefficient of large QGP bags $\sigma_{surf} \approx 157.4$ MeV fm^{-2} at $T = 0$ and the value of (tri)critical temperature $T_{cep} = 152.9 \pm 4.5$ MeV are found under the plausible assumptions.**
- **It is shown that at the cross-over temperature the surface tension coefficient of large QGP bags should be negative.**

References

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