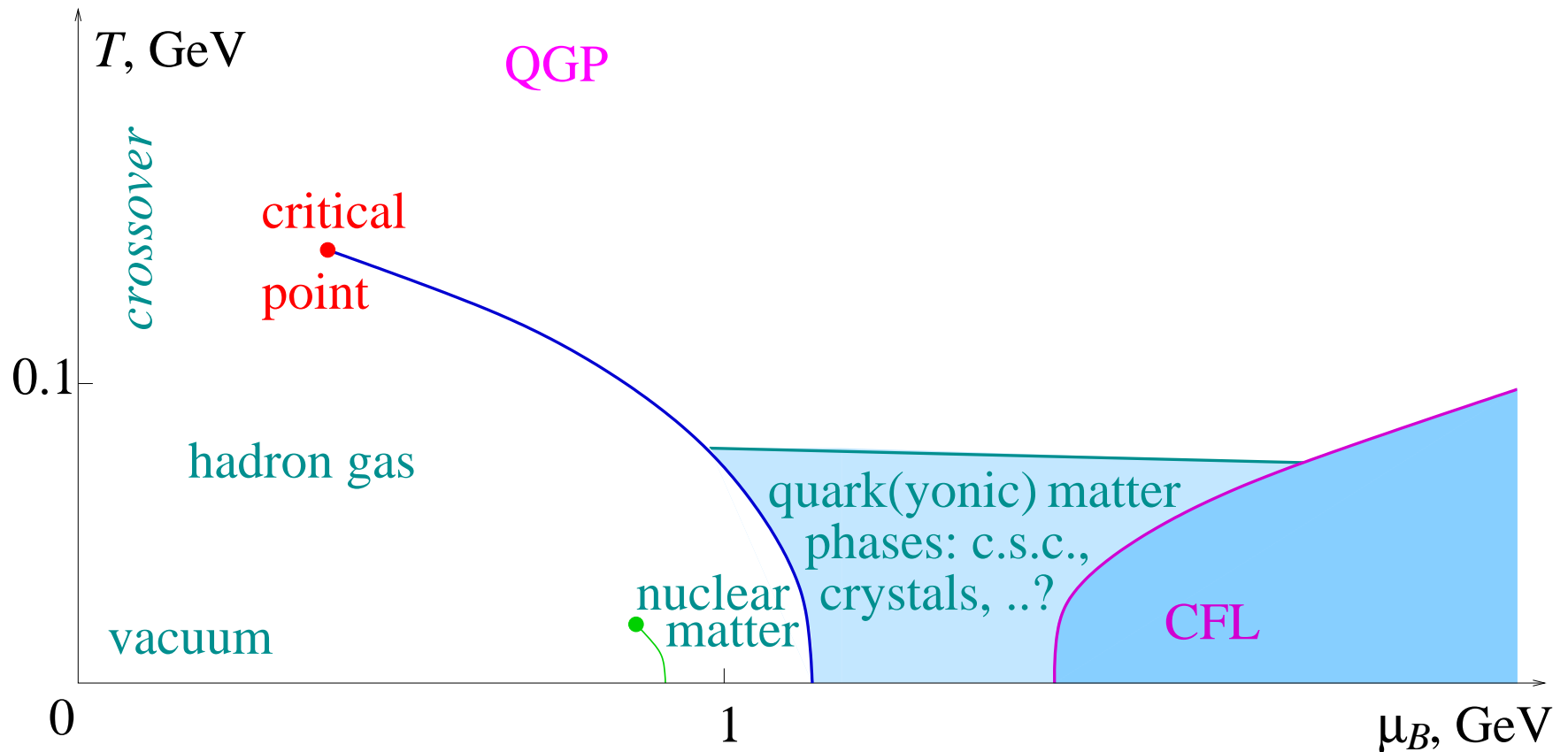


QCD critical point and event-by-event fluctuations

M. Stephanov

U. of Illinois at Chicago

QCD phase diagram (a sketch)

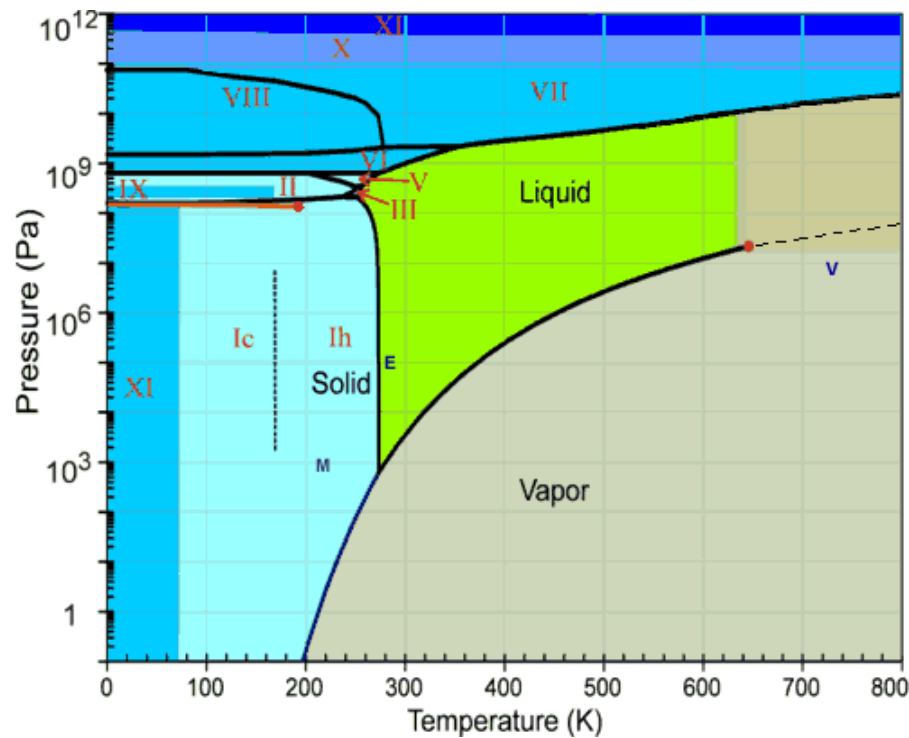


- Models (and lattice) suggest the transition becomes 1st order at some μ_B .
- Can we observe the **critical point** in heavy ion collisions, and how?

Critical point(s) in known liquids

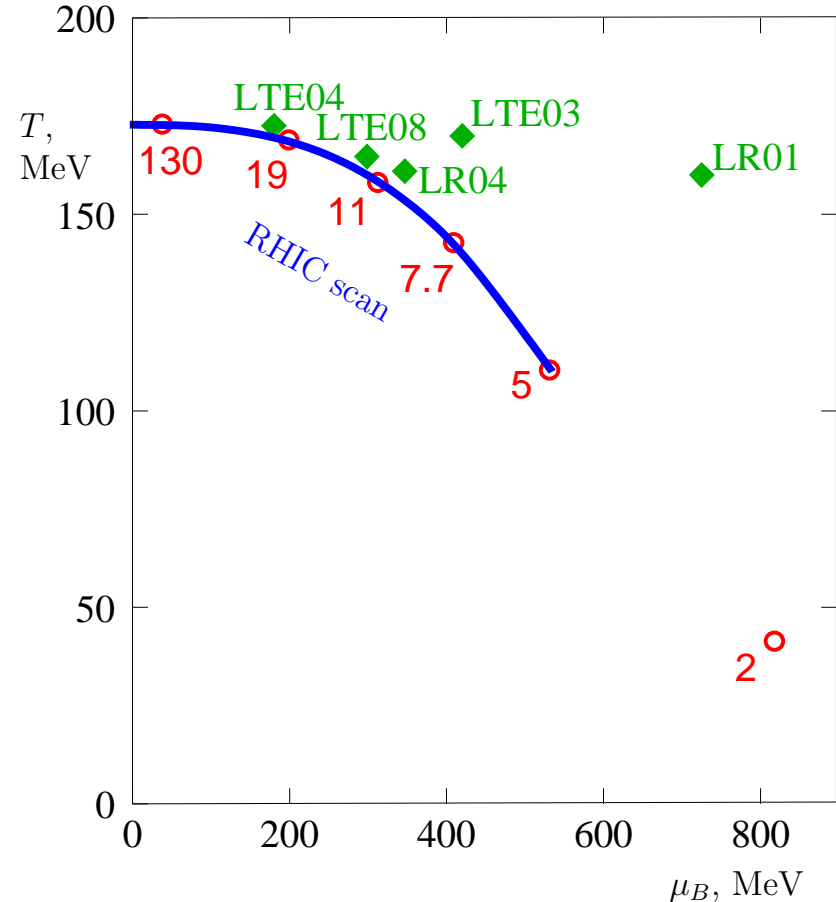
Most liquids have a critical point (seen, e.g., by critical opalescence).

Water:



Does QCD “perfect liquid” have one?

What do we need to discover the critical point?



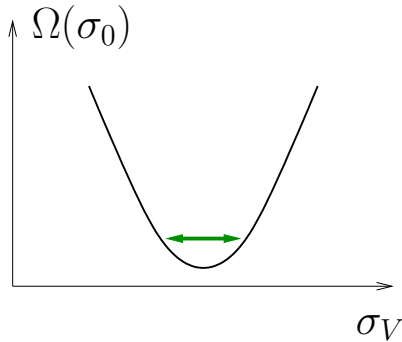
- Experiments: RHIC, NA61(SHINE), FAIR/GSI, NICA
- Better lattice predictions, with known systematic uncertainties.
- Good experimental signatures.

Critical fluctuations: theory



1

$\mu < \mu_{\text{CP}}$

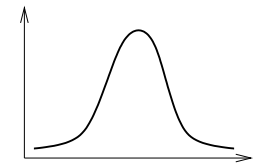


Consider an observable such as,
e.g., $\sigma_V = \int_V \sigma$, where $\sigma \sim \bar{\psi}\psi$.

$$\langle \sigma_V^2 \rangle \sim (\Omega'')^{-1}$$

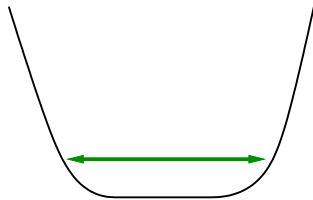
Einstein, 1910:

$P(\sigma_V) \sim$ number
of states with that σ_V
i.e., e^S , or $e^{-\Omega/T}$



2

$\mu = \mu_{\text{CP}}$

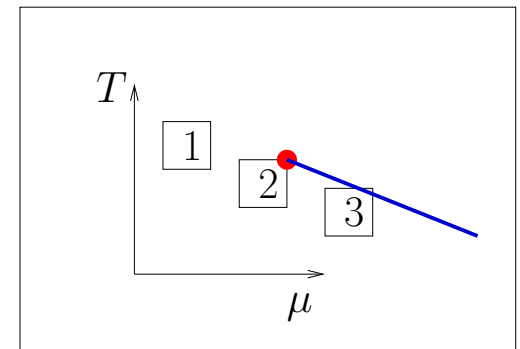
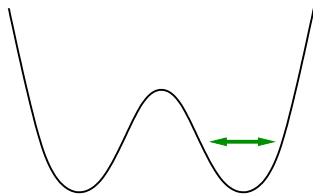


$$(\Omega'')^{-1} \rightarrow \infty$$

large **equilibrium** fluctuations

3

$\mu > \mu_{\text{CP}}$



Why does CP seem to defy the central limit theorem?

Because, correlation length $\xi \rightarrow \infty$. This is a collective phenomenon.

Indeed, the magnitude of fluctuations $\langle \sigma_V^2 \rangle \sim \xi^2$.

Fluctuation signatures

- Experiments measure multiplicities N_π , N_p , ..., mean p_T , etc.

These quantities fluctuate event-by-event.

- Fluctuation magnitude is quantified by e.g., $\langle(\delta N)^2\rangle, \langle(\delta p_T)^2\rangle$.

- What is the magnitude of these fluctuations near the QCD C.P.? (Rajagopal-Shuryak-MS, 1998)

- Universality tells us how it grows at the critical point: $\langle(\delta N)^2\rangle \sim \xi^2$.

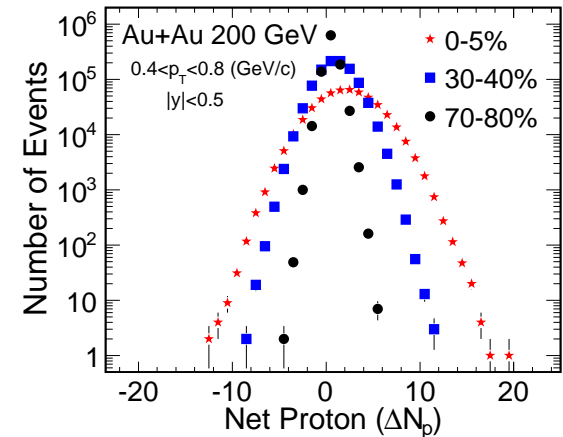
- Magnitude of ξ is limited $< \mathcal{O}(2-3 \text{ fm})$ (Berdnikov-Rajagopal).

- “Shape” of the fluctuations can be measured: non-Gaussian moments. As $\xi \rightarrow \infty$ fluctuations become less Gaussian.

- Higher cumulants show even stronger dependence on ξ (PRL 102:032301,2009):

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}, \quad \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \sim \xi^7$$

which makes them more sensitive signatures of the critical point.



Higher moments (cumulants) and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp \{ -\Omega[\sigma]/T \},$$

$$\Omega = \int d^3x \left[\frac{1}{2} (\nabla \sigma)^2 + \frac{m_\sigma^2}{2} \sigma^2 + \frac{\lambda_3}{3} \sigma^3 + \frac{\lambda_4}{4} \sigma^4 + \dots \right]. \quad \Rightarrow \quad \xi = m_\sigma^{-1}$$

- Moments (connected) of $q = 0$ mode $\sigma_V \equiv \int d^3x \sigma(x)$:

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT \xi^2; \quad \kappa_3 = \langle \sigma_V^3 \rangle = 2VT^2 \lambda_3 \xi^6;$$

$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3 \langle \sigma_V^2 \rangle^2 = 6VT^3 [2(\lambda_3 \xi)^2 - \lambda_4] \xi^8.$$

- Tree graphs. Each propagator gives ξ^2 .



- Scaling requires “running”: $\lambda_3 = \tilde{\lambda}_3 T (T\xi)^{-3/2}$ and $\lambda_4 = \tilde{\lambda}_4 (T\xi)^{-1}$, i.e.,

$$\kappa_3 = \langle \sigma_V^3 \rangle = 2VT^{3/2} \tilde{\lambda}_3 \xi^{4.5}; \quad \kappa_4 = 6VT^2 [2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4] \xi^7.$$

Moments of observables

- Example: Fluctuation of multiplicity is the fluctuation of occup. numbers,

$$\delta N = \sum_{\mathbf{p}} \delta n_{\mathbf{p}}.$$

Any moment of the multiplicity distribution is related to a correlator of $\delta n_{\mathbf{p}}$:

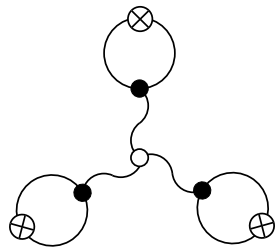
$$\kappa_{3\pi} = \langle (\delta N)^3 \rangle = \sum_{\mathbf{p}_1} \sum_{\mathbf{p}_2} \sum_{\mathbf{p}_3} \langle \delta n_{\mathbf{p}_1} \delta n_{\mathbf{p}_2} \delta n_{\mathbf{p}_3} \rangle, \quad \text{where } \sum_{\mathbf{p}} = V \int \frac{d^3 \mathbf{p}}{(2\pi)^3}.$$



$n_{\mathbf{p}}$ fluctuates around $\bar{n}_{\mathbf{p}}(m)$,

which also fluctuates: $\delta m = g \delta \sigma$, i.e.,

$$\delta n_{\mathbf{p}} = \delta n_{\mathbf{p}}^0 + \frac{\partial \bar{n}_{\mathbf{p}}}{\partial m} g \delta \sigma.$$



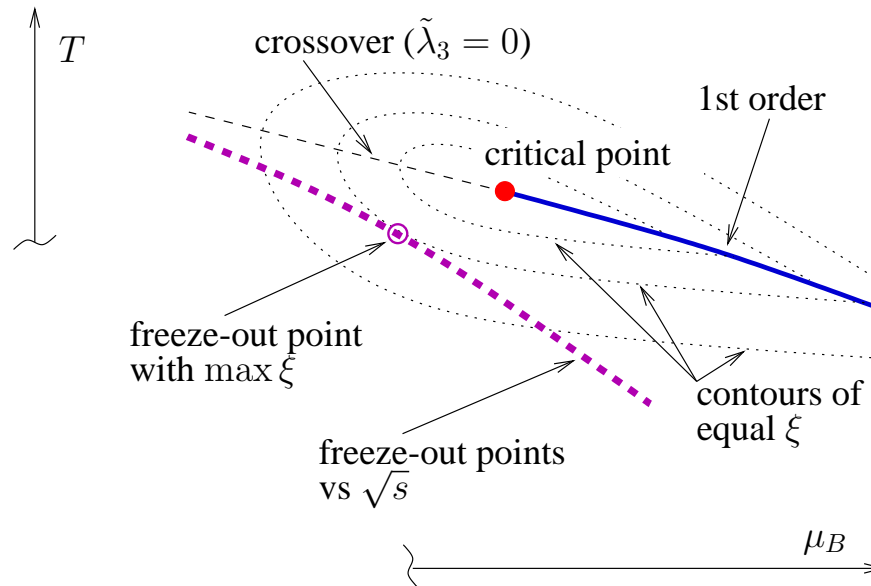
$$\langle \delta n_{\mathbf{p}_1} \delta n_{\mathbf{p}_2} \delta n_{\mathbf{p}_3} \rangle_{\sigma} = \frac{2\lambda_3}{V^2 T} \left(\frac{g}{m_{\sigma}^2} \right)^3 \frac{v_{\mathbf{p}_1}^2}{\gamma_{\mathbf{p}_1}} \frac{v_{\mathbf{p}_2}^2}{\gamma_{\mathbf{p}_2}} \frac{v_{\mathbf{p}_3}^2}{\gamma_{\mathbf{p}_3}}$$

$$v_{\mathbf{p}}^2 = \bar{n}_{\mathbf{p}}(1 \pm \bar{n}_{\mathbf{p}}), \quad \gamma_{\mathbf{p}} = (dE_{\mathbf{p}}/dm)^{-1}$$

Similarly for $\langle (\delta N)^4 \rangle_c$.

- Since $\langle (\delta N)^3 \rangle$ scales as V^1 we suggest $\omega_3(N) \equiv \frac{\langle (\delta N)^3 \rangle}{\bar{N}}$ which is V^0 .

Energy scan and fluctuation signatures: notes



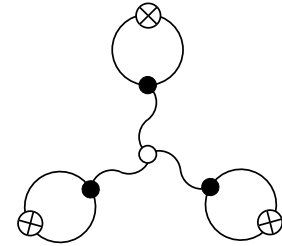
- Higher moments provide more sensitive signatures.
- However, this comes at a price:
 - Harder to predict – more theoretical uncertainties.
 - Signal/noise lower for higher moments.
 - Sensitivity could mean narrower critical region:
if it rises faster with rising ξ , it also falls faster with falling ξ .
- One could, e.g., try combining various higher moments to eliminate uncertainties.

Using ratios and mixed moments

Athanasίου, Rajagopal, MS (2010)

- The dominant dependence on μ_B (i.e., on \sqrt{s}) is from two sources ξ and n_p , e.g., $\kappa_{3p} \sim \tilde{\lambda}_3 g_p^3 \xi^{4.5} n_p^3$.

- $\xi(\mu_B)$ has a peak at $\mu_B = \mu_B^{\text{critical}}$;
- $n_B \sim e^{\mu_B^{\text{critical}}/T}$ determines the height of the peak;
- other factors: g_p^3 and $\tilde{\lambda}_3$ depend on μ_B weaker.



- Leading dependence on μ_B^{critical} can be cancelled in ratios. E.g.,

$$\frac{\kappa_{3p}}{N_p} \left(\frac{N_\pi}{N_p} \right)^2 \sim \tilde{\lambda}_3 g_p^3 \xi^{4.5}$$

- Unknown/poorly known coupling parameters g_p or g_π can be also cancelled in ratios. E.g.,

$$\frac{\kappa_{4p}}{\kappa_{2p}^2} \frac{\kappa_{2\pi}^2}{\kappa_{4\pi}}, \quad \text{or} \quad \frac{\kappa_{4p}^3}{\kappa_{3p}^4} \frac{\kappa_{3\pi}^4}{\kappa_{4\pi}^3}.$$

These ratios are 1 when critical point fluctuations dominate.

- Mixed moments allow more possibilities. E.g.,

$$\frac{\kappa_{2p2\pi}^2}{\kappa_{4p}\kappa_{4\pi}}.$$

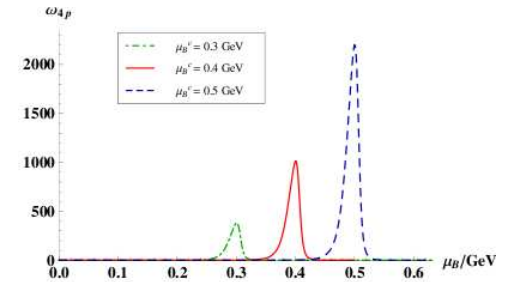
Mixed moments have no trivial Poisson contribution.

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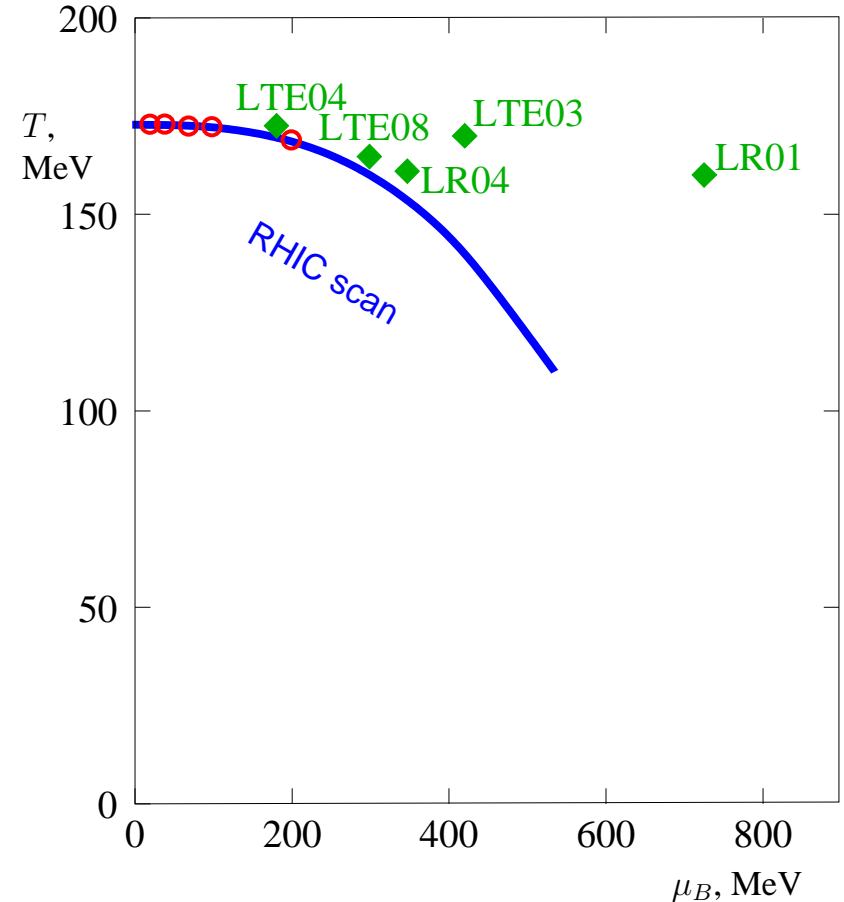
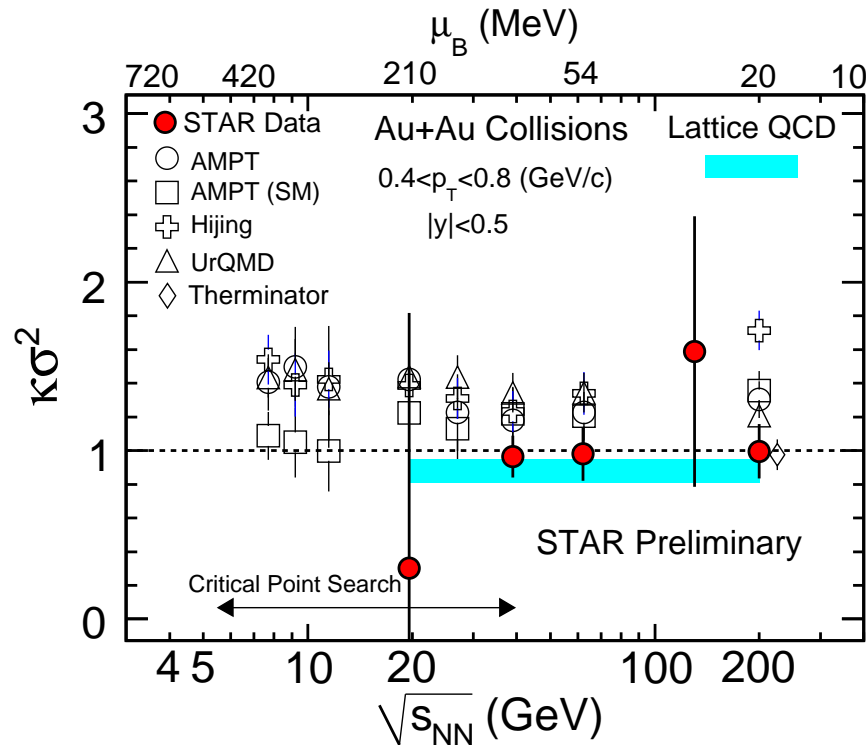
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Mixed moments have no trivial Poisson contribution.

Experiment (pre-QM)



$$(\kappa\sigma^2 = \kappa_4/\kappa_2 \approx \omega_4 \quad \text{if } \kappa_2 \approx N).$$

No critical signatures seen at those values of μ_B .

Consistent with expectations that $\mu_B^{\text{critical}} > 200$ MeV.

What is happening at $\sqrt{s} = 19.6$ GeV? Low statistics.

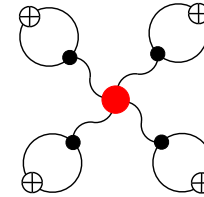
Large positive contribution to Poisson is excluded, but large negative — is not.

Negative kurtosis?

- Could the critical contribution to kurtosis be negative? (MS, arxiv:1104.1627)

$$\langle (\delta N)^4 \rangle_c = \langle N \rangle + \langle \sigma_V^4 \rangle_c \left(\frac{g}{T} \int_p \frac{v_p^2}{\gamma_p} \right)^4 + \dots,$$

$$\langle \sigma_V^4 \rangle_c = 6VT^2 [2\tilde{\lambda}_3^2 - \tilde{\lambda}_4] \xi^7.$$

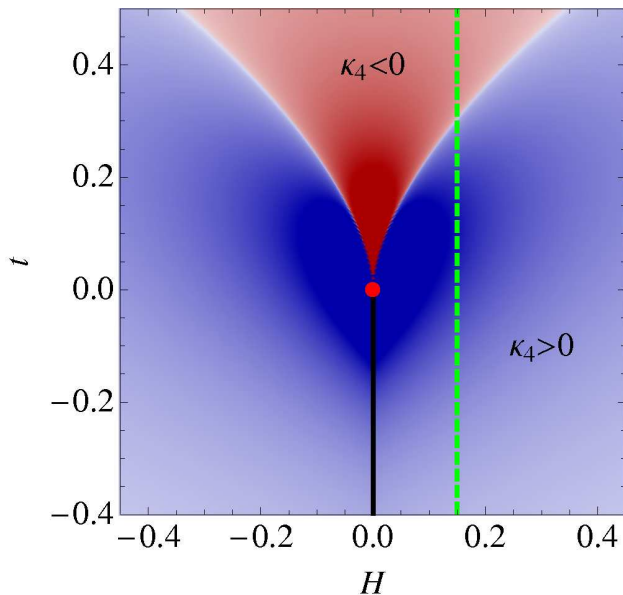


- On the crossover line $\tilde{\lambda}_3 = 0$ by symmetry, while $\tilde{\lambda}_4 \approx 4. > 0$.

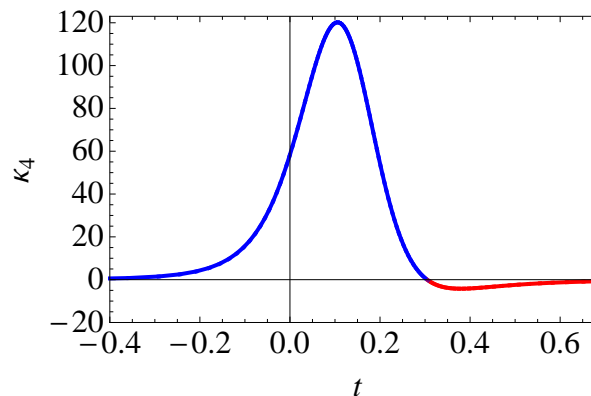


Thus $\langle \sigma_V^4 \rangle_c < 0$ and $\omega_4(N) < 1$ on the crossover line. And around it.

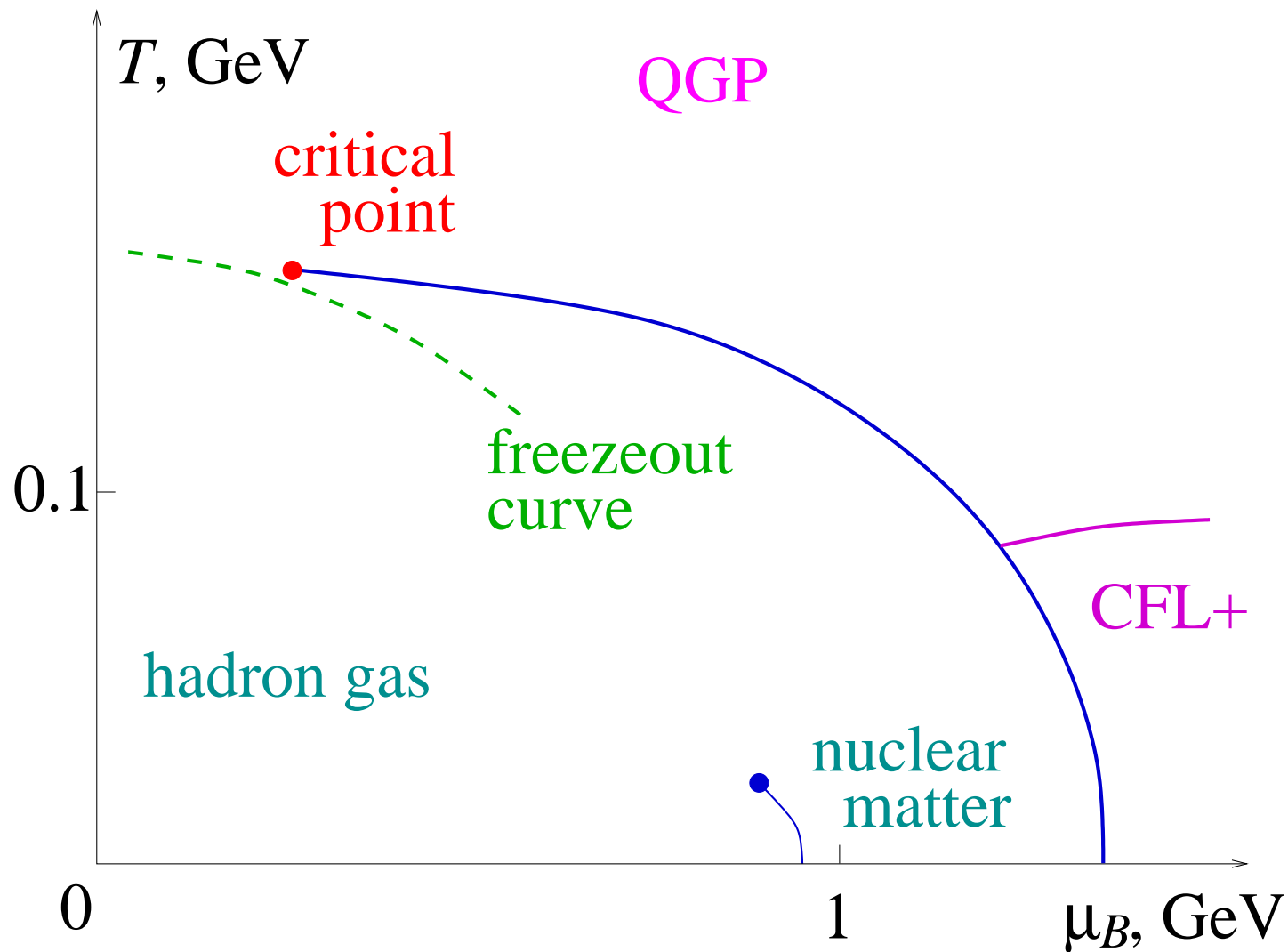
- Universal Ising eq. of state: $M = R^\beta \theta$, $t = R(1 - \theta^2)$, $H = R^{\beta\delta} h(\theta)$



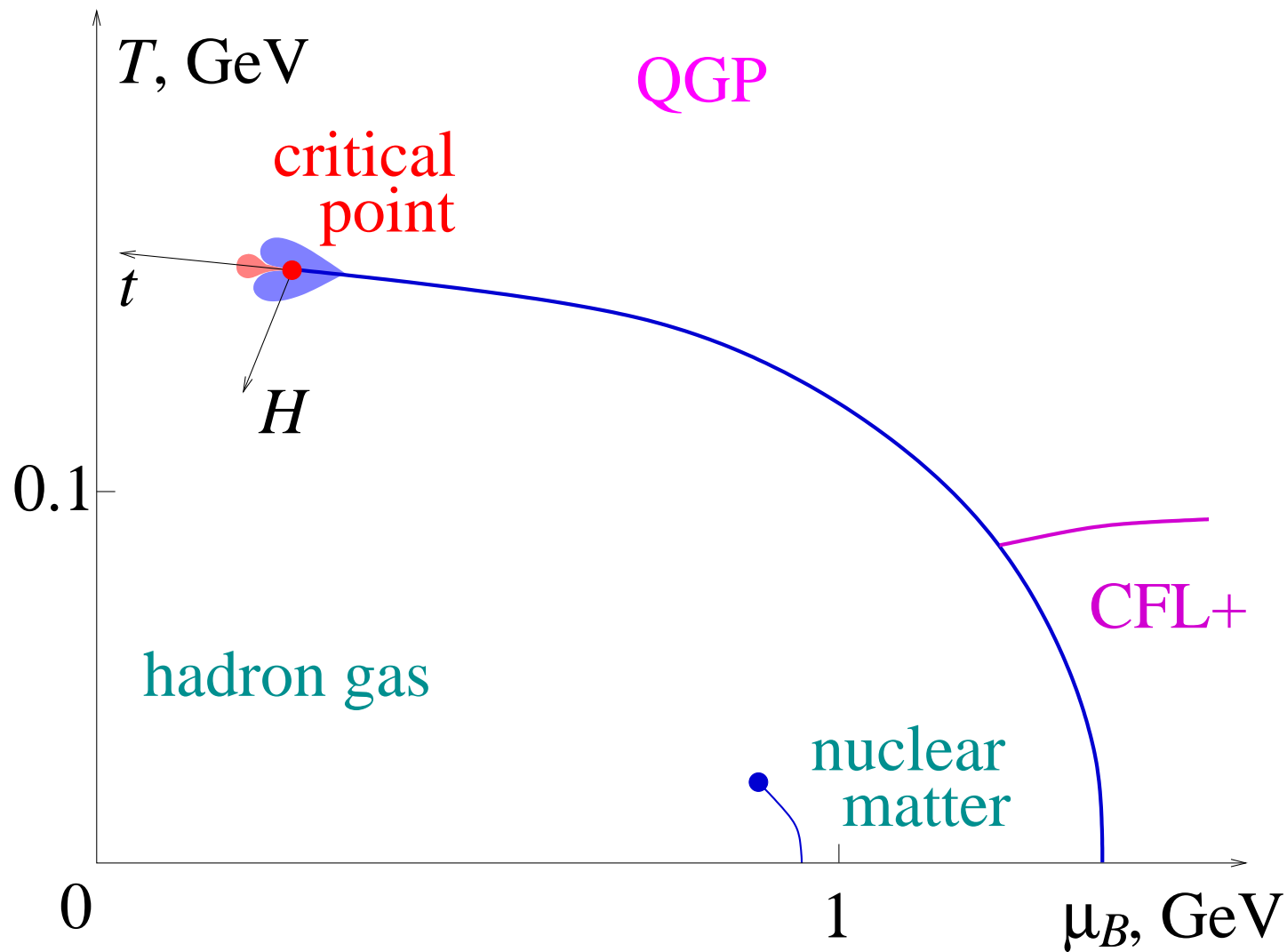
here κ_4 is $\kappa_4(M) \equiv \langle M^4 \rangle_c$



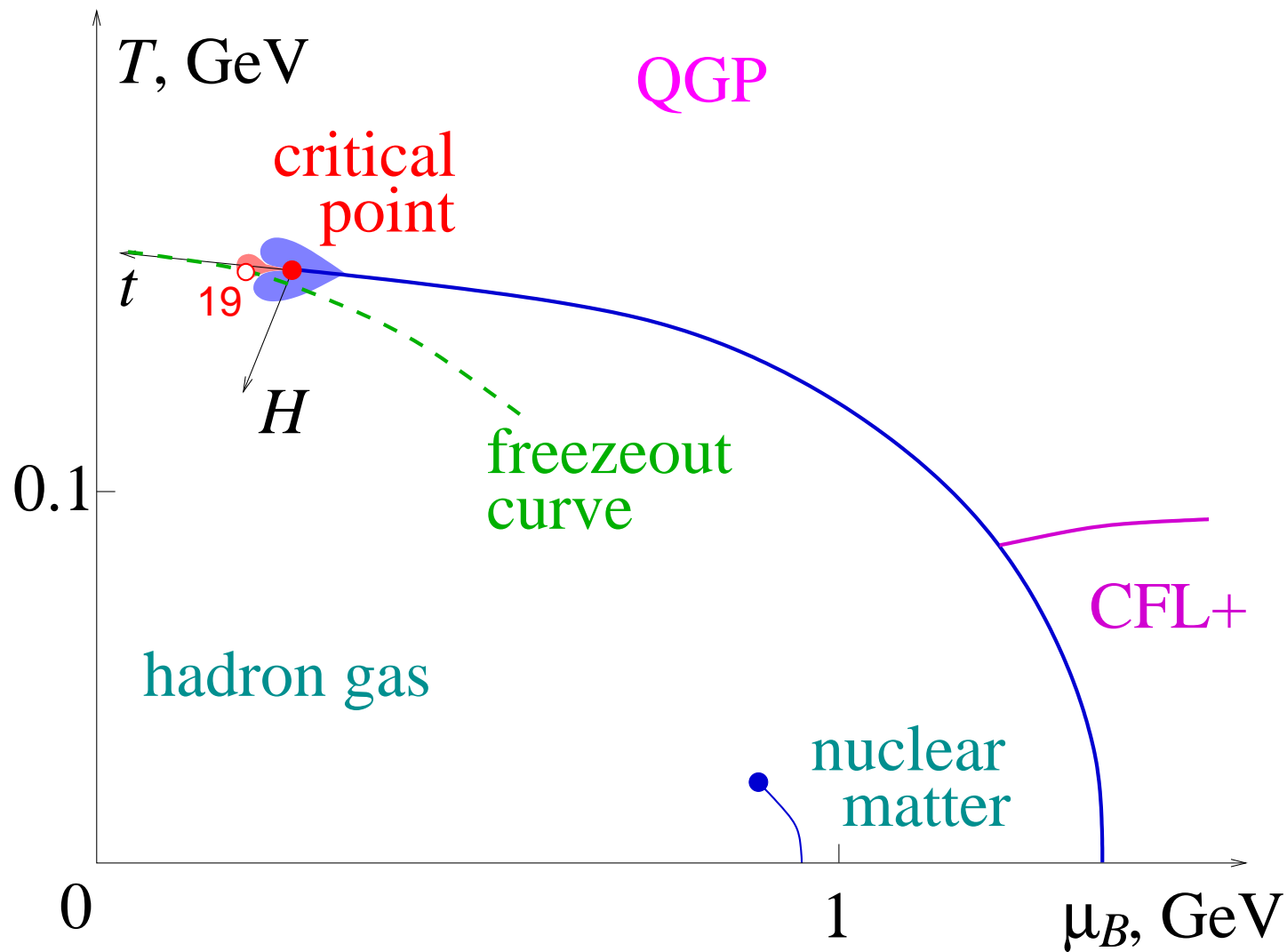
Implications for the energy scan



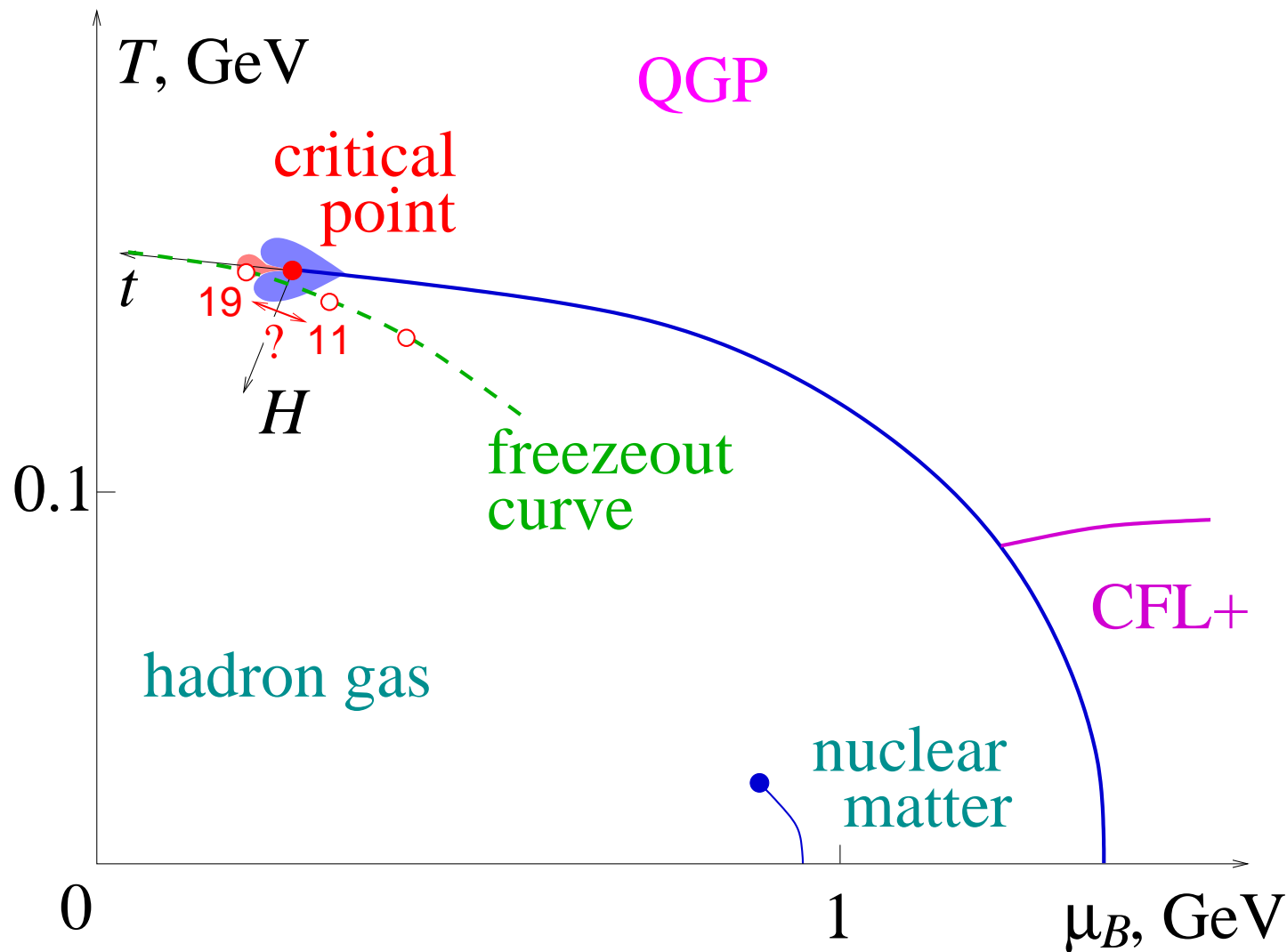
Implications for the energy scan



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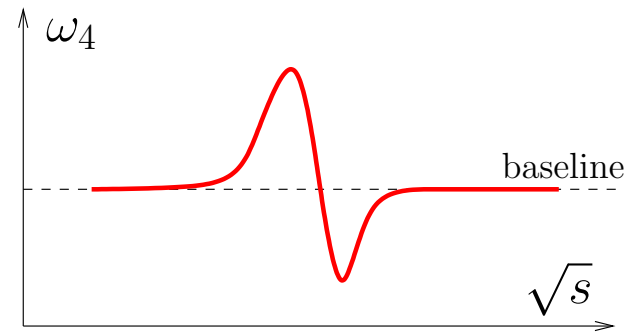
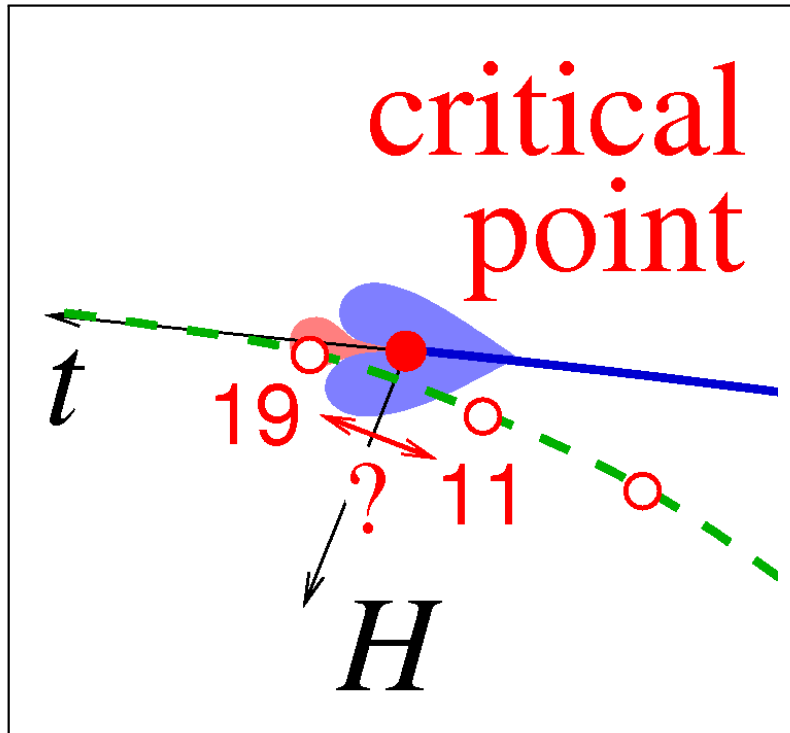


Implications for the energy scan



- If the kurtosis stays significantly below Poisson value in 19 GeV data, the logical place to take a closer look is between 19 and 11 GeV.

Implications for the energy scan



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Concluding remarks

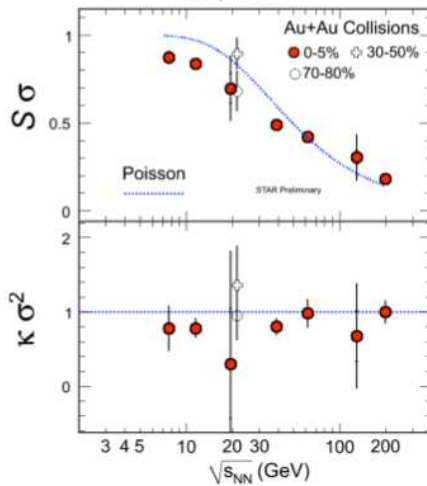
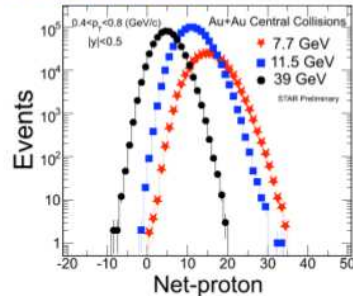
- Critical point is a special singular point on the phase diagram, with unique signatures. This makes its experimental discovery possible.
- Locating the point is still a challenge for theory.
- The search for the critical point is on. ~~We are waiting for~~ RHIC results at $\mu_B > 200$ MeV are just in!
- If kurtosis stays significantly below Poisson value at $\sqrt{s} = 19$ GeV, then the critical point could be close, to the right, on the phase diagram.
Then: $\sqrt{s} = 15$ GeV?

QM notes



Higher Moments of Net-Protons

Observations:



QM2011

Critical point:

Correlation length and Susceptibilities diverge
Long wavelength or low momentum number
fluctuations. Distributions are non-Gaussian

Higher moments:

M. A. Stephanov, PRL 102 (2009) 032301

Measure of non-Gaussian nature
Proportional to higher powers of ξ
Kurtosis x Variance $\sim \chi^{(4)} / [\chi^{(2)}]$
Skewness x Sigma $\sim [\chi^{(3)}] / [\chi^{(2)}]$
Product of moment - Volume effect cancels

Net-protons:

Y. Hatta et al., PRL 91 (2003) 102003

\sim reflects net-baryons - conserved quantity
Neutrons immaterial due to isospin
blindness of σ field

Deviation from Poissionian expectations
from 39 GeV and below

Bedanga Mohanty

17

- Potential sources of negative contribution at high baryon density:
 - Fermi statistics: $\omega_4 \approx 1 - 7\langle n_p \rangle_p$ (small, but increases with μ_B).
 - Baryon number conservation (talk by M. Nahrgang).