Role of Finite Size Baryons in QCD Phase Transition and Critical Point

P. K. Srivastava and C. P. Singh
Department of Physics, Banaras Hindu University, Varanasi-221005, India

Abstract

The physics regarding the existence of the critical point on the QCD phase boundary still remains unclear and its precise location is quite uncertain. We follow the suggestion of T. D. Lee et. al. [Phys. Rev. D9, 2291 (1974)] that a phase transition at high baryon density in a bulk matter can be realized in which nucleon loses part of its mass and thus baryons play a significant role in the phase transition. We suggest that the hadron gas consists of pointlike mesons and each baryon having a geometrical hard-core size. It means that mesons can fuse into one another while baryons experience a repulsive force with other baryons when densely packed. We formulate an excluded volume model existing for the equation of state (EOS) of hot, dense hadron gas and for the quark gluon plasma we use a thermodynamically consistent quasiparticle model (QPM). We construct a first order phase transition using Gibbs' equilibrium criteria. This leads to an interesting and surprising finding that a critical point exists in such a formulation beyond which a cross-over region appears. We find that such a picture always appears in all excluded volume models considered in the literature. For ideal hadron gas model, there is no critical point in the diagram. In the mean field model also, we do not get a critical point unless we incorporate an excluded potential effect. Our analysis strongly suggests that the existence of a critical point and a cross-over region owes its explanation arising due to the finite size baryons in the hadron gas. We find an interesting result that the ratio of the baryons to the total hadrons at the critical point is around 0.2 in all types of models and thus a cross-over region starts as soon as this ratio becomes smaller than 0.2.

Introduction

Precise mapping of the QCD phase boundary existing between two distinct phases of hot, dense hadron gas (HG) and weakly interacting plasma of quarks and gluons (QGP) and the location of hypothesized critical point (CP) have recently emerged as very interesting and challenging problems before the experimental and theoretical heavy-ion physicists today [1-3] . The discovery of QCD critical end point is bound to clear the mist surrounding our understanding of the conjectured QCD phase diagram and hence it would help us to ascertain various properties and signals of QGP to some extent. The possible existence of CEP in the temperature (T) and baryon chemical potential (μ_B) plane of the QCD phase boundary was proposed a decade ago and it represents a second-order transition point where the first-order transition boundary terminates as T increases and μ_B decreases . Its separation from the temperature axis ($\mu_B = 0$) spans the region of a cross-over transition . We still do not have any intuitive picture for understanding the circumstances under which a cross-over transition can occur around $\mu_B = 0$ and it finally culminates into a CEP as μ_B increases. This is qualitatively supported by some lattice QCD findings . In this work, we take the help of a phenomenological model to emphasize the dominant role played by the finite-size baryons as constituents of a hot, dense HG in the determination of a cross-over as well as CP on the QCD phase boundary.

Equation of States

(a) HG EOS (Our Excluded Volume Model) [4-7]:

Features :

- We have used **full quantum statistics** (in earlier version by Mishra et al., Boltzmann statistics has been used) to explore the entire T-μ_B plane.
- The excluded volume model is **thermodynamically consistent** as we start from the partition function and we get number density from it.
- We have used all the **hadrons and their resonances upto mass of 2 GeV** (to include the effect of attractive interactions) in the HG.
- We have considered a **hard-core size for each baryon** to include the effect of repulsive interactions at high density.
- **Mesons are treated as a pointlike particles** because they don't have any hard - core size and they overlap and fuse into each other.

The Grand canonical partition function using full statistics and including excluded volume correction in a thermodynamically consistent manner is :

$$\ln Z_i^{ex} = \frac{g_i}{6\pi^2 T}\int_{V_i^0}^{V-\sum_j N_j V_j} dV \int_0^\infty \frac{k^4\,dk}{\sqrt{k^2+m_i^2}}\frac{1}{[\exp(\frac{E_i-\mu_i}{T})+1]}$$

Where g_i is the degeneracy factor of ith species of baryon, E is the energy of the particle V^0_i is the eigen volume of one ith species of baryon and $\sum_j N_j V_j^0$ is the total volume occupied .

We can write above equation as $\ln Z_i^{ex} = V(1-\sum_j n_j^{ex}V_j^0)I_i\lambda_i$

where $$I_i = \frac{g_i}{6\pi^2 T}\int_0^\infty \frac{k^2\,dk}{\sqrt{k^2+m_i^2}}\frac{1}{[\exp(\frac{E_i}{T})+\lambda_i]}$$

$\lambda_i = \exp(\pm\frac{\mu_i}{T})$ is the fugacity of the ith particle. (+) sign is used for particles and (-) sign is used for anti- particles.

n^{ex}_j is the number density of jth type of baryons after excluded volume correction.

The total baryonic pressure after excluded volume correction is : $P_B^{ex} = T(1-R)\sum_i I_i\lambda_i$

$R = \sum_j n_j^{ex}V_j^0$ is the fractional occupied volume by the baryons.

Thus the total hadronic pressure is given as follows : $P_{HG} = P_B^{ex} + P_i^{meson}$

(b) HG EOS (Mean-Field Model) [9]:

The attractive and repulsive interactions in the mean-fields are incorporated in HG by scalar σ and vector ω - exchange, respectively [9].
The main drawback of this model is that the Yukawa potential due to ω - exchange generates a mean potential energy in HG which vanishes when net baryon density n_B-> 0.
To cure this problem by adding a Vander-Waals repulsive interaction term which depends on T and n and has its origin in the excluded volume correction.

The expression of the total pressure of the HG in this mean-field model,with excluded volume correction, can be written as :

$$p = \frac{1}{3}\sum_j g_j\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{(M_j^{*2}+k^2)^{1/2}}[f_j+f_{\bar{j}}] + p_{vdw}(n,T) + p_B(n_B) + p_\sigma(\sigma) + \sum_m p_m(T)$$

where $$f_{j(\bar{j})} = \left[\exp\left(\frac{(M_j^{*2}+k^2)^{1/2}+U_{vdw}(n,T)\pm U_{Bj}(n_B)\mp\mu_j}{T}\right)+1\right]^{-1}$$

The expressions for Vander-Waal hard-core repulsion terms are: $p_{vdw}(n,T) = nT\frac{V_0 n}{1-V_0 n}$,

$$U_{vdw}(n,T) = T\frac{V_0 n}{1-V_0 n} - T\ln(1-V_0 n)$$

$p_B(n_B)$ is the pressure arises due to vector ω-field

$P_\sigma(\sigma)$ is the pressure arises due to scalar σ -field.

The last term in the right hand side is due to the contribution of pointlike mesons.

The HG consists of p, n, π, K, Δ........ Unfortunately the calculation becomes more involved if we increase the number of particles.

(c) QGP Equation Of State in Bag Model :

$$P_{QGP} = \frac{37}{90}\pi^2T^4 + \frac{1}{9}\mu_B^2T^2 + \frac{\mu_B^4}{162\pi^2} - \alpha_s[\frac{11}{9}\pi T^4 + \frac{2}{9\pi^2}\mu_B^2T^2 + \frac{1}{81\pi^3}\mu_B^4] + \frac{8\alpha_s^{3/2}T}{3\pi^2\sqrt{2\pi}}[\frac{8\pi^2T^2}{3}+\frac{2}{9}\mu_B^2]^{3/2} - B$$

~~where~~ $$\alpha_s = \frac{12\pi}{29}[\ln(\frac{0.089\mu_B^2+15.622T^2}{\Lambda^2})]^{-1}$$

Here we have used $B^{1/4}$=216 MeV and Λ =100 MeV.

Reference :QCD Phase Boundary and Critical Point in a Bag Model Calculation; C. P. Singh, P. K. Srivastava, S. K. Tiwari, Phys. Rev. D 80,114508 (2009)

The main drawback of this model is that the energy density and entropy density is in contradiction with lattice QCD results at and near critical point T_C. Therefore, we used a quasiparticle model in our further study.

(d) QGP EOS (Quasiparticle Model) [10]:

System of interacting massless particles $\xrightarrow{\text{effectively}}$ Ideal gas of "massive" non-interacting particles

Dispersion relation for these massive particles is assumed as :

Energy $$\omega^2(k,m) = k^2 + m^2(T)$$

Where m(T) → temperature - dependent mass
and k → momentum of the particle

Effective masses : (Using Finite temperature field theory)

For Gluons $$m_g^2(T) = \frac{N_C}{6}g^2(T)T^2(1+\frac{N_f'}{6})$$ Where N_C is the no. of colours

$$N_f' = N_f + \frac{3}{\pi^2}\sum_f\frac{\mu_f}{T^2}$$ Where N_f is no. of flavours of quarks and μ_f is chemical potential

For quarks $$m_{qf}^2(T) = \frac{g^2(T)T^2}{6}\left(1+\frac{\mu_f^2}{\pi^2T^2}\right)$$

Energy density $$\varepsilon = \frac{T^4}{\pi^2}\sum_{l=1}^\infty\frac{1}{l^4}\left[\frac{d_g}{2}\varepsilon_g(x_g l) + (-1)^{l-1}d_q\cosh\left(\frac{\mu_q}{T}\right)\varepsilon_q(x_q l) + (-1)^{l-1}\frac{d_s}{2}\varepsilon_s(x_s l)\right]$$

Quark density $$n_q = \frac{d_qT^3}{\pi^2}\sum_{l=1}^\infty(-1)^{l-1}\frac{1}{l^3}\sinh(\mu_q/T)I_i(x_i l)$$

with $$\varepsilon_i(x_i l) = (x_i l)^3K_1(x_i l) + 3(x_i l)^2K_2(x_i l)$$

Where K_1 and K_2 are the modified bessel's functions.

Pressure at μ_q =0, $$\frac{P(T,0)}{T} = \frac{P_0}{T_0} + \int_{T_0}^{T} dT\frac{\varepsilon(T,\mu=0)}{T^2}$$

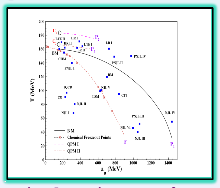
Pressure at finite μ_q $$P(T,\mu_q) = P(T,0) + \int_0^{\mu_q} n_q\,d\mu_q$$

This model is thermodynamically consistent and the results from this model compare well with the lattice results.

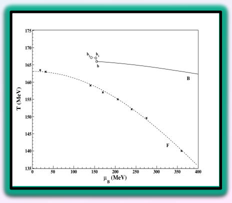
Results

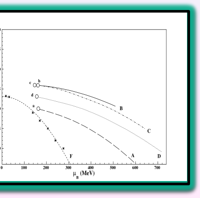
QCD Phase diagram using Quasiparticle Model EOS for QGP

P_1 --> Ist order phase transition curve in Bag Model
BM--> End point in Bag Model
P_2--> Ist order phase transition curve in QPM I
C_1--> End point in QPM I
P_3--> Ist order phase transition curve in QPM II
C_2--> End Point in QPM II
F --> Chemical Freeze-out curve
All other labels from--> [7]



Role of Finite- size of baryon in the existence of critical point and cross-over region





A --> Ist order phase transition curve in Bag Model
a--> End point in Bag Model
D--> Ist order phase transition curve in mean-field model
d--> End point in mean field model
C--> Ist order phase transition curve in Cleymans & Suhonen model
c--> End point in Cleymans & Suhonen model
B--> Ist order phase transition curve in our HG model
b --> End point in our HG model

In all the excluded Volume models, we have taken the baryon hard- core radius r = 0.8 fm.

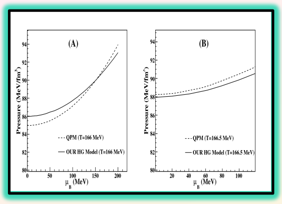
The existence of CEP is not sensitive to the model details .

The existence and location of CEP is less sensitive to the baryonic size down to 0.4 fm but strongly sensitive below that.

Gibbs' Construction at and near CEP

We find that the Gibbs' equilibrium condition fails to hold after the CEP, if we increase the temperature by 0.5 MeV more than the critical end point temperature and the deconfining transition does not occur beyond CEP.

It also defines the beginning of a cross-over region lying beyond the critical end point where the mesons dominant HG pressure is approximately equal (but no intersection) to the QGP pressure.



Ratio of total baryons to total hadrons in the system at Critical End Point

HG Models	Coordinates of CEP (T,μ_B)	r (in fm)	No. of Baryons n_B (fm^{-3})	Total No. Den. n (fm^{-3})	n_B/n
Our Model	(166,155)	0.8	0.104	0.54	0.192
Mean field	(163,157)	0.8	0.0981	0.492	0.199
Cleymans & Suhonen	(166,149)	0.8	0.163	0.86	0.190

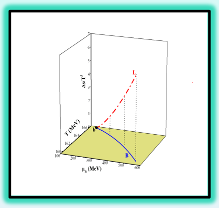
A surprising result that the ratio **x= n_B/n = 0.195±0.005** at CEP in all the models.

It means that the ratio n_m: n_B = 4:1 at CEP and is independent of HG models used in the calculations.

It is interesting to investigate why and how n_m: n_B = 4:1 yields the precise location of CEP.

Entropy – Order Parameter

The variation of difference of normalized entropy density between QGP phase and HG phase with respect to the coordinates of the phase transition points lying at the phase boundary between HG and QGP.



We find that difference of normalized entropy density ~ 0 at CEP.

It clearly indicates that CEP obtained in our calculation can either give an isentropic or a second order phase transition point.

Cross-over Mechanism in our Model

(a) Our model falls in line with the ideas proposed recently [10-11] where it was shown that under circumstances, hot and dense HG consisting of extended hadrons could produce phase transition of the first or second order and also a smooth cross-over.

(b) We propose that although each baryon possesses a hard-core size, mesons are also extended particles but they lack a hard-core size. So they can overlap, fuse and interpenetrate.

(c) At CEP, mesons and baryons saturate the volume of the hot fireball. In meson dominated region (i.e., $T>T_{CEP}$), mesons have a far larger density than that of baryons. When they start overlapping on each other, they fuse into one another and cluster formation starts where colour can flow and only the cluster as a whole is colour-singlet [12]. As the clusters merge together resulting into an infinitely sized cluster, analytic cross-over into a new phase occurs.

(d) But why does the ratio n_B/n_m=0.25 (a fixed value) at the CEP? How does the presence of baryon density affect the cluster formation? These questions need a thorough investigation before we make a clear picture.

Conclusions

- Searching for the precise location of the critical end point (CEP) in the QCD phase diagram still poses a challenging problem.
- Although various calculations have predicted its existence but the quantitative predictions regarding its location wildly differ.
- Experiments face an uphill task in probing the CEP in QCD phase diagram because a clarity in theoretical prediction is missing.
- Moreover, many unstudied problems such as short lifetime and the reduced volume of the QGP formed at colliders also affect the location of CEP and its verification.
- In these circumstances, our results arising due to baryon size, will be helpful in understanding the origin of CEP and determining its location on the phase diagram.

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