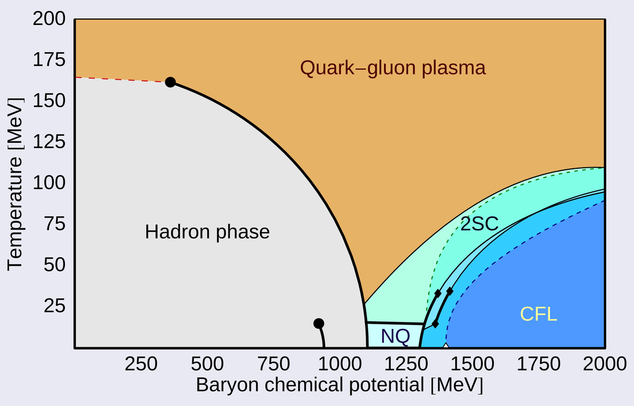
Chiral transition in a magnetic field and at finite baryon density

NTNU – Trondheim
Norwegian University of Science and Technology

Jens O. Andersen and Rashid Khan
Department of Physics, Norwegian Institute of Science and Technology, N-7491 Trondheim, Norway

Introduction



The QCD phase diagram as function of temperature and baryon density has received a lot of attention in recent years, in particular the position of the critical endpoint where the curve of first-order chiral transitions terminates in a second-order transition, and the color-superconducting phases at large baryon density and low temerature.
The global symmetry of QCD is $SU(N_f)_V \times SU(N_f)_A$ in the chiral limit and $SU(N_f)_V$ if the quark masses are equal.With $N_f = 2$ in the chiral limit, we use isomorphism

$$SU(2) \times SU(2) \sim O(4) \tag{1}$$

and use the $O(4)$ linear sigma model (LSM) as a low-energy effective theory for QCD. Coupling the linear sigma model to quarks we obtain the quark-meson (QM) model.

The effects of strong magnetic fields on the QCD-phase diagram are also considred, which is relevent to compact stars since strong magnetic fields exist inside ordinay neutron stars as well as magnetars [1]. It has been suggested that strong magnetic fields are created in heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) and play an important role [2]. The magnetic field strength has been estimated to be up to $|qB| \sim 6m_\pi^2$, $|q|$ is the charge of pions.

The Model

The quark-meson effective model with $N_f = 2$ flavors is

$$\mathcal{L} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \mathcal{L}_{Yukawa} \ , \tag{2}$$

where the various terms are

$$\mathcal{L}_{meson} = Tr[\partial_\mu \Phi^\dagger \partial_\mu \Phi] + m^2 Tr[\Phi^\dagger \Phi] + \frac{\lambda}{3} Tr[\Phi^\dagger \Phi]^2 - \frac{1}{2} H Tr[\Phi^\dagger + \Phi] \ , \tag{3}$$

$$\mathcal{L}_{quark} = \bar\psi[\gamma_\mu \partial_\mu - \mu\gamma_4]\psi \ , \tag{4}$$

$$\mathcal{L}_{Yukawa} = g\bar\psi[\sigma - i\gamma_5 \tau.\pi]\psi \ , \tag{5}$$

$$\Phi = \frac{1}{2}(\sigma + \tau.\pi) \ . \tag{6}$$

σ is the sigma field, π denotes the neutral and charged pions, τ are the Pauli matrices, μ is the quark chemical potential, related to the baryon chemical potential by $\mu = \mu_B/3$.
If $H = 0$, the first three terms in Eq. (3) are invariant under $U(2)_L \times U(2)_R \sim SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_A$.
If $H \neq 0$, chiral symmetry is explicity broken, otherwise it is spontaneously broken in the vacuum. In the broken phase, Φ acquires a nonzero expectation value v and we write

$$\sigma \rightarrow v + \tilde\sigma \ , \tag{7}$$

where $\tilde\sigma$ is a quantum fluctuating field with vanishing expectation value.

The coupling of charged pions $\pi^\pm$ and quarks to the external magnetic field is implemented by substitution $\partial_\mu \rightarrow \partial_\mu + iqA_\mu$, where A_μ is the four-vector potential. For a constant magnetic field in the z-direction, one can choose $(A_0, \mathbf{A}) = (0, 0, \mathbf{B}x, 0)$. The classical solutions to the Klein-Gordon equation in a constant magnetic field are given by

$$(E^\pm_{n,p_z})^2 = p_z^2 + m^2 + \frac{1}{6}\lambda v^2 + (2n+1)|qB| \ , \tag{8}$$

where n is non-negative integer, p_z is spatial momentum in the z-direction. The subscript $\pm$ denotes $\pi^\pm$. The solutions to the Dirac equation in a constant magnetic field are

$$E^2_{n,p_z} = p_z^2 + m_q^2 + (2n + 1 - s)|q_f B| \ , \tag{9}$$

where $m_q = gv$ is the quark mass after symmetry breaking, and $s = \pm 1$ denotes the spin up/down, respectively.

Effective Potential

At tree level, the free energy is given by,

$$\mathcal{F}_0 = \frac{1}{2}m^2 v^2 + \frac{\lambda}{24}v^4 - Hv \ . \tag{10}$$

At one-loop, the contributions to the free energy $\mathcal{F}_1$ are,

Effective Potential

$$\mathcal{F}_1 = \mathcal{F}_\sigma + \mathcal{F}_{\pi^0} + \mathcal{F}_{\pi^+} + \mathcal{F}_{\pi^-} + \mathcal{F}_q \ , \tag{11}$$

where

$$\mathcal{F}_\sigma = \frac{1}{2}\sum\!\!\!\!\!\!\int_P \log[P^2 + m_\sigma^2] \ , \tag{12}$$

$$\mathcal{F}_{\pi^0} = \frac{1}{2}\sum\!\!\!\!\!\!\int_P \log[P^2 + m_\pi^2] \ , \tag{13}$$

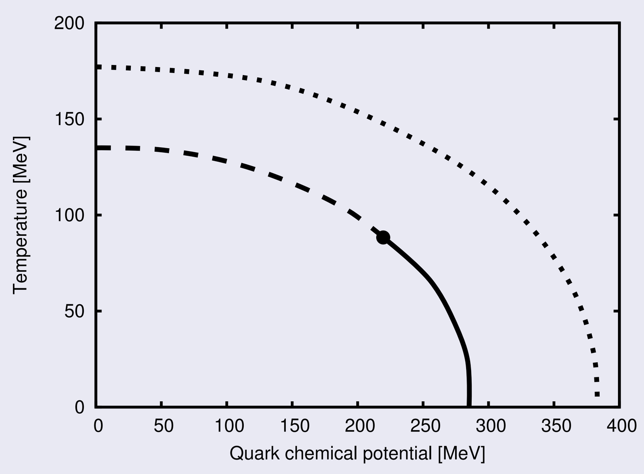
$$\mathcal{F}_{\pi^\pm} = \frac{1}{2}\frac{|qB|T}{2\pi}\sum_{P_0,n}\int_{p_z} \log[P_0^2 + p_z^2 + M_B^2] \ , \tag{14}$$

$$\mathcal{F}_q = -Tr\log[i\gamma_\mu(P_\mu + q_f A_\mu) + m_q - \mu\gamma_4] \ . \tag{15}$$

In the QM model, a common approximation is to neglect the quantum and thermal fluctuations of the mesons, hoping that the important effects come from the quarks [3, 4].
We consider the strong magnetic field $|q_f B| \gg m_q^2$, so only the term with $n = 0$ and $s = 1$ is not exponentially suppressed.
The effective potential of the QM model

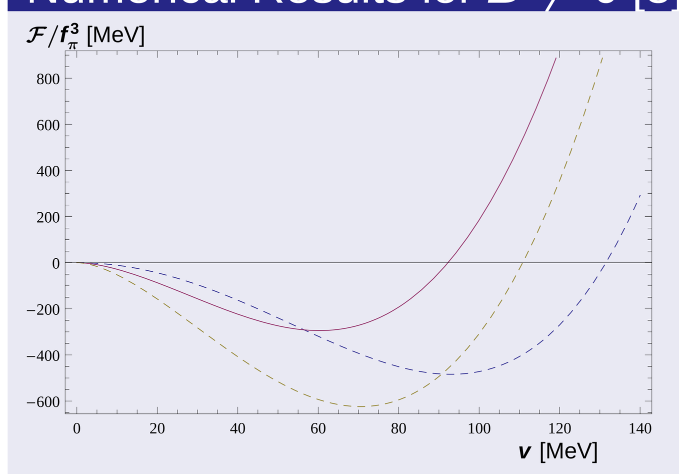
$$\begin{aligned}\mathcal{F}_{0+1} = & \frac{1}{2}m^2v^2 + \frac{\lambda}{24}v^4 - Hv + \frac{N_c m_q^4}{16\pi^2}\sum_f\left[\log\frac{\Lambda^2}{2|q_f B|} + 1\right] \\ & - \frac{N_c}{2\pi^2}\sum_f (q_f B)^2 \zeta^{(1,0)}(-1, \tfrac{m_q^2}{2|q_f B|}) \\ & - \frac{N_c m_q^2}{8\pi^2}\sum_f |q_f B| \log\frac{m_q^2}{2|q_f B|} \\ & - \frac{N_c|q_f B|T}{2\pi^2}\int_0^\infty dp \log\left[1 + e^{-\beta(\sqrt{p^2+m_q^2}\pm\mu)}\right] \ . \end{aligned} \tag{16}$$

Numerical Results for $B = 0$ [5]

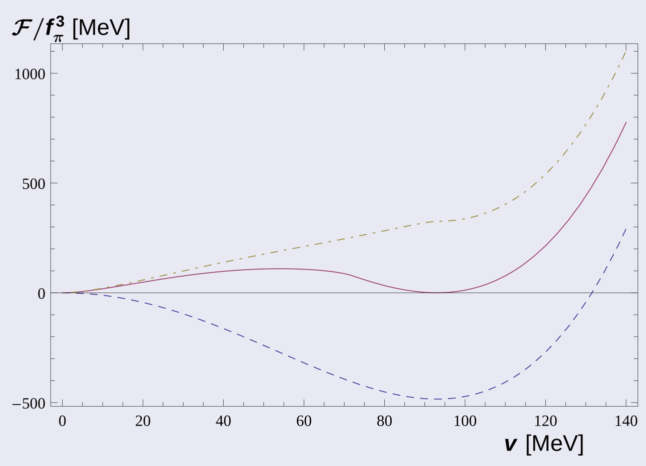


Phase diagram at the physical point. The First-order transition starts (lower curve) at $\mu_c = 287$ MeV and $T = 0$ and ends at a critical point $\mu = 220$ and $T = 90$ MeV [5]. Comparing our results with Scavenius [6] *et al.* $\mu = 207$ and $T = 99$ MeV, and Bowman and Kapusta [7] $\mu = 283$ and $T = 75$ MeV. If we include all contributions in the model, i.e. bosonic and fermionic vacuum contributions, the critical end point disappears, and the transition is crossover (upper curve).

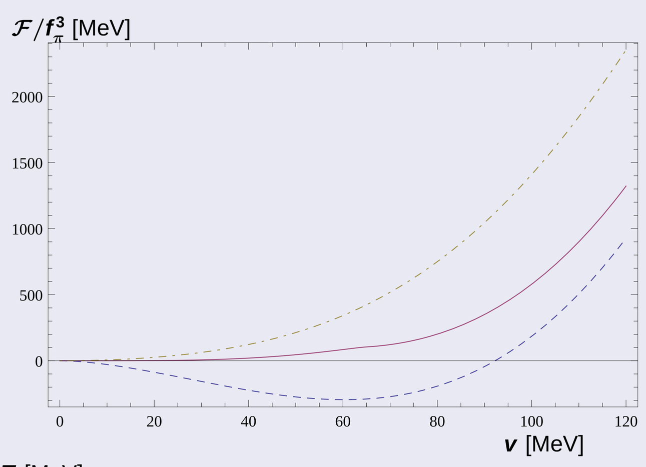
Numerical Results for $B \neq 0$ [8]



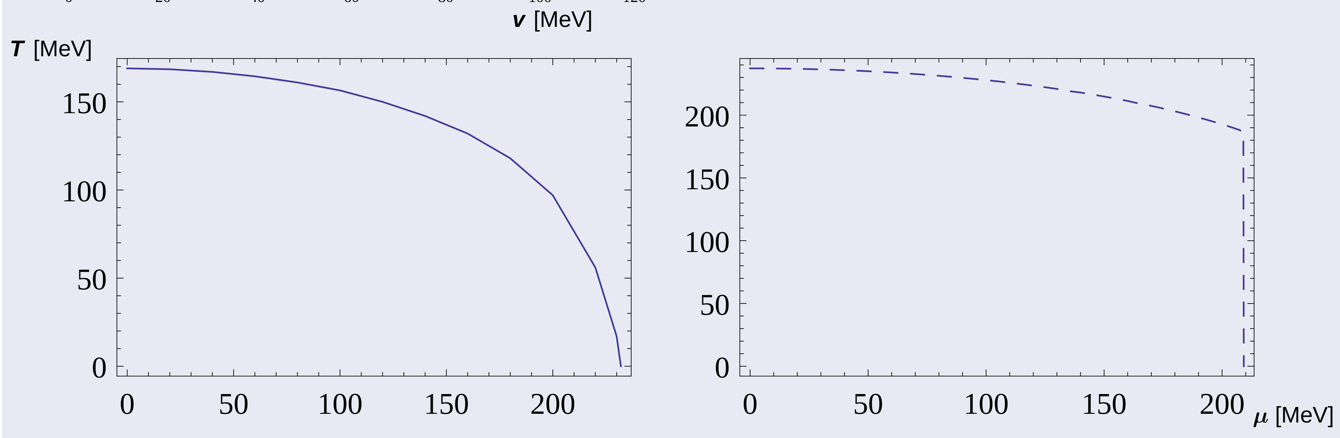
Tree-level potential (dotted-curve) as well as the one loop effective potential in the vacuum, i .e. for $T = \mu = 0$ for two different values of B: solid curve for$|qB| = 5m_\pi^2$ and the dash-dotted for $|qB| = 10m_\pi^2$. We can see that the potential becomes deeper with increasing magnetic field and minimum moves to larger value of v.



Effective potential without fermionic vacuum fluctuations for $T = 0$, $|qB| = 5m_\pi^2$ and for different values of μ: $\mu = 0$, $\mu = 232$, $\mu = 300$ MeV. Clearly, the transition is first order.



The effective potential including fermionic vacuum fluctuations at $T = 0$ and $|qB| = 5m_\pi^2$ for different values ofμ: $\mu = 0$, $\mu = 209.2$, $\mu = 400$ MeV. The transition is second order.



Numerical Results for $B \neq 0$ [8]

The phase diagram in the μ-T plane for $|qB| = 5m_\pi^2$. Left panel shows first- order phase transition and right panel second-order.
In the case where we include quantum fluctuation, the critical temperature for $\mu = 0$ is $T_c = 237$ MeV and the critical chemical potential for $T = 0$ is $\mu_c = 209.2$ MeV.
In comparison, the critical temperature for vanishing magnetic field is $T_c = 190$ MeV, the critical chemical potential is $\mu_c = 345$ and phase transition is second-order.

In case where we do not include quantum fluctuation, the critical temperature for $\mu = 0$ is $T_c = 169$ MeV and the critical chemical potential for $T = 0$ is $\mu_c = 232$ MeV.
In comparison, the critical temperature for vanishing magnetic field is $T_c = 140$ MeV, the critical chemical potential is $\mu_c = 296$ and phase transition is first-order [5].
The critical temperature for $\mu = 0$ increases with the strength of the magnetic field, and the critical chemical potential μ_c decreases at $T = 0$.

Conclusions and Outlook

We calculated the one-loop free energy for the QM model at finite temperature and baryon density in external magnetic field. Ignoring the vacuum and thermal contributions from the mesons to the effective potential, we find a second-order phase transition for $\mu = 0$ in accordance with universality arguments.

Moreover, the order of the phase transition also depends on whether we include or neglect the vacuum contribution from fermions. The transition is first order if they are omitted. This was also the case for vanishing magnetic field [5, 9]. At the physical point, if the quantum fluctuations are included, we get a crossover for all μ.

The next step is to use more sophisticated techniques to include the quantum and thermal effects of the mesons. This requires some type of resummation [10, 11].

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