Quark Recombination
and Heavy Quark
Diffusion

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Overview

- The case for a microscopic hadronization model
- Instantaneous and resonance recombination
- Recombination in equilibrium
- Heavy quarks in a strongly interacting medium: resonance scattering and resonance recombination

Summary

[He, RJF & Rapp, PRC 82, 034907 (2010)]
[He, RJF & Rapp, to appear (2011)]
[He. RJF & Rapp, arxiv:1103.6279]
Recombination: A Microscopic Hadronization Model
Hadronization

- Bulk QGP: equation of state in hydrodynamics
  - Local equilibrium assumption, no microscopic information.
- At high $P_T$: in-vacuum fragmentation for single partons.
  - Based on QCD factorization, little microscopic information.
- Recombination of quarks: a simple *microscopic* hadronization model.
  - Useful for partons in a medium.
- Experimental evidence:
  - Large baryon $R_{AA}$ at intermediate $P_T$.
  - Large baryon/meson ratios
  - Elliptic flow scaling in $KE_T$ and quark number $n_q$. 

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Resonance Recombination

- Early recombination models:
  - Instantaneous projection of states
  - $2 \rightarrow 1$ & $3 \rightarrow 1$ processes: no energy conservation

- Resonance recombination:
  - Mesons appear as resonances of quark-antiquark scattering
  - Described by Boltzmann equation, start with ensemble of quarks/antiquarks
    $$\frac{\partial}{\partial t} f_M(t, \vec{p}) = -\frac{\Gamma}{\gamma_p} f_M(t, \vec{p}) + g(\vec{p})$$
    - Breit-Wigner resonance cross sections: $\sigma(s) = C_M \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$
    - Long-time limit: $E \frac{dN_M}{d^3P} = \frac{E\gamma}{8(2\pi)^3\Gamma} \int \frac{d^3x d^3p_{rel} f_a(x, p_1) f_a(x, p_2) \sigma(s)v_{rel}(P, p_{rel})}{(2\pi)^3}$

  - Conserves energy and momentum, should be able to attain equilibrium.
  - Compatible with the picture of a strongly interacting medium.

[Ravagli & Rapp PLB 655 (2007)]
[Ravagli, van Hees & Rapp, PRC 79 (2009)]
Resonance Recombination in Equilibrium
RRM in Equilibrium

- Energy conservation + detailed balance + equilibrated quark input → equilibrated hadrons!
- Numerical tests: compare blast wave hadrons at $T_c - \varepsilon$ to hadrons coalesced from quarks of the same blast wave at $T_c + \varepsilon$:

  - Excellent agreement of spectra and $v_2$.
  - Here: hadronization hypersurface at const. time

[He, RJF & Rapp, PRC 82 (2010)]
Equilibrium: Arbitrary Hypersurface

- Realistic hadronization hypersurface $\Sigma$:
  - Extract equal-time quark phase space distributions $f_q$ along $\Sigma$ from hydro or kinetic model.
  - Apply RRM locally (cell-by-cell) $\rightarrow$ meson phase space distr. $f_M$ along $\Sigma$.
  - Compute meson current across $\Sigma$ a la Cooper-Frye:
    \[ \frac{dN}{p_T dp_T d\phi dy} = \int_{\Sigma} \frac{p_\mu d\sigma^\mu(\tau, x, y)}{(2\pi)^3} f_M(\tau, x, y; p) \]

- Result for charm-light system using AZHYDRO output at $T_c$:

[He, RJF & Rapp, to appear (2011)]
Lessons from RRM in Equilibrium

- Resonance recombination is compatible with equilibration and hydro.
  - Will work with any hydrodynamic flow field and hadronization hypersurface.
  - Important consistency check.
  - Can extract quark spectra at $T_c$ from spectra of hadrons freezing out just below $T_c$ (multi-strange hadrons).

- Microscopic information lost in kinetic equilibrium: how can we understand simultaneous $KE_T$- and $n_q$-scaling at low $P_T$?
  - Not a manifestation of recombination (at low $P_T$!)
  - Approximate $KE_T$-scaling provided by hydrodynamic flow.
  - Further improved by sequential freeze-out from heavier to lighter hadrons.
  - Additional $n_q$-scaling accidental?
Application of Recombination in the Kinetic Regime: Heavy Quarks
Setup of the HQ Formalism

- Our goal: a formalism for consistent description of heavy quarks in a strongly interacting medium via resonance interactions.

- Three important ingredients:
  - Langevin simulation of heavy quarks/mesons with realistic non-perturbative transport coefficients; model approach to equilibrium
  - Relativistic Hydrodynamic background for the medium.
  - Resonance recombination appropriate for the medium, consistently augmented by fragmentation.

[See also: Moore and Teaney, van Hees and Rapp, … ]
Langevin Dynamics

- Langevin Equations
  \[ dx = \frac{p}{E} dt, \]
  \[ dp = -\Gamma(p)p dt + \sqrt{2D(p + dp)} dt \rho \]

- Transport coefficients:
  - HQ relaxation rates in QGP: (T-matrix approach: resonant correlations around \( T_c \). [Riek and Rapp, Phys. Rev. C 82, 035201 (2010)]
  - Hadronic phase: coming soon.

- Initial distribution: binary collision density \( \otimes p\text{QCD} \) spectrum.

- Running Langevin in AZHYDRO background.
  - Test particles in lab frame \( \rightarrow \) boost to fluid rest frame for Langevin step.
  - Stop at hydro hadronization hypersurface, extract phase space distributions.
Heavy Quarks: Results

- Equilibrium check (artificially large relaxation rates)

- Realistic Riek-Rapp coefficients
Hadronization of Heavy Quarks

- Calculate recombination probability $P_{\text{coal}}(p)$ of heavy quarks for given heavy quark momentum using RRM in local fluid rest frame.
  - Assume RRM probability $\rightarrow 1$ for perfectly co-moving partners.

- We see a consistent picture:
  - Low/thermal HQ momenta (in co-moving frame) = RRM dominated.
  - High momenta (not co-moving) = fragmentation dominated.

- $P(p)$ decides whether to recombine or fragment a given heavy quark.
  - Apply fragmentation with probability $1 - P(p)$.
Heavy Mesons: Preliminary Results

- Charm (D)

Recombination: large effect on elliptic flow.

- Bottom (B)
Semi-Leptonic Decays and Data

- $R_{AA}$

- Electrons

[He, RJF & Rapp, to appear (2011)]
Semi-Leptonic Decays and Data

- What’s missing?
- RRM not yet completely implemented.
- Flow!
  - Have to tune hydro; currently gravely underestimate flow at $T_c$.
- Diffusion in the hadronic phase.
  - Just published transport coefficient for hadronic charm.

[See also: Laine (2011), Ghosh et al. (2011), Abreu et al. (2011)]

[He, RJF & Rapp, arXiv:1103.6279]
Summary

- Resonance Recombination is a microscopic hadronization model compatible with kinetic equilibrium.

- $KE_T$ and quark number scaling compatible with recombination in the equilibrium regime (but not a consequence of it).

- Formalism for heavy quarks in strongly interacting medium based on resonance interactions:
  - Setup complete with Langevin dynamics, hydro background.
  - Consistent blend of recombination and fragmentation.
  - Preliminary results available
  - To do: tuning of (equilibrium) flow, hadronic diffusion.

- To do: resonance recombination for baryons: check $n_q$ scaling at intermediate $P_T$. 
Backup Slides
Exp. Evidence and Challenges

- Leading Particle Effect (forward rapidities)
  - Charm quark recombination with beam remnants.
- Large baryon/meson ratios and large baryo $R_{AA}$, $R_{CP}$ at intermediate $P_T$.
  - Failure of "hydro+jet" models.
- Elliptic flow scaling in $KE_T$ and quark number $n_q$.
  - Signs of quark recombination?
KE_T^2-Scaling (Low P_T)

- Blastwave + two-stage freeze-out works within exp. uncertainties.
  - Group I: multi-strange particles freeze-out at T_c.
  - Group II: all others including pions freeze-out at ~ 110 MeV.
  - Can extract quark distributions at T_c from data on multi-strange hadrons.

- \nu_2(KE_T) tends to be straight line for the parameters at RHIC → additional quark number scaling becomes trivial.

- Expectations:
  - KE_T scaling fairly robust (at low P_T!)
  - KE_T+n_q scaling maybe accidental
**$KE_T$-Scaling (Low $P_T$)**

- Easy to find reasonable parameters in the Retiere and Lisa blast wave to achieve $KE_T$- and $n_q$-scaling within 10% accuracy.

- Contours in space of flow asymmetry and fireball eccentricity.

- Pions prefer slightly lower freeze-out temperatures but are compatible with group II within the 10% accuracy (not changed by resonance decays!).

- This behavior should qualitatively also hold for a full hydro simulation.

[He, Fries & Rapp, PRC (2010)]
Hadronic Diffusion

- Charm relaxation in hadronic medium.
- Use parameterization of existing D+h amplitudes from the literature.
- \( h = \pi, K, \rho, \eta, \omega, N, \Delta \)
- Lots of channels, partial waves missing: lower bound!
- Relaxation rates at \( T_c \) comparable to non-perturbative QGP estimates.
- Minimum of Ds at \( T_c \)?

[He, RJF & Rapp, arXiv:1103.6279]

[See also: Laine (2011), Ghosh et al. (2011), Abreu et al. (2011)]