

# Identified particles from viscous hydrodynamics

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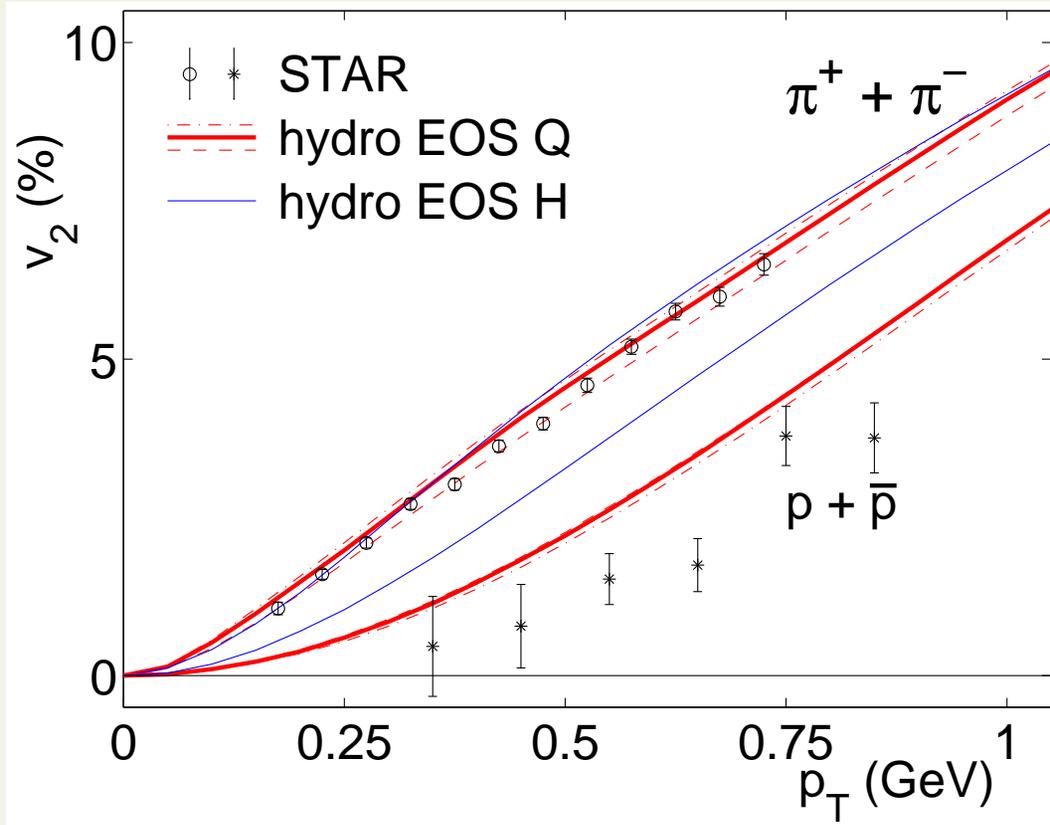
in collaboration with Zack Wolff, Cyrus Vandrevala (students)

# Motivation

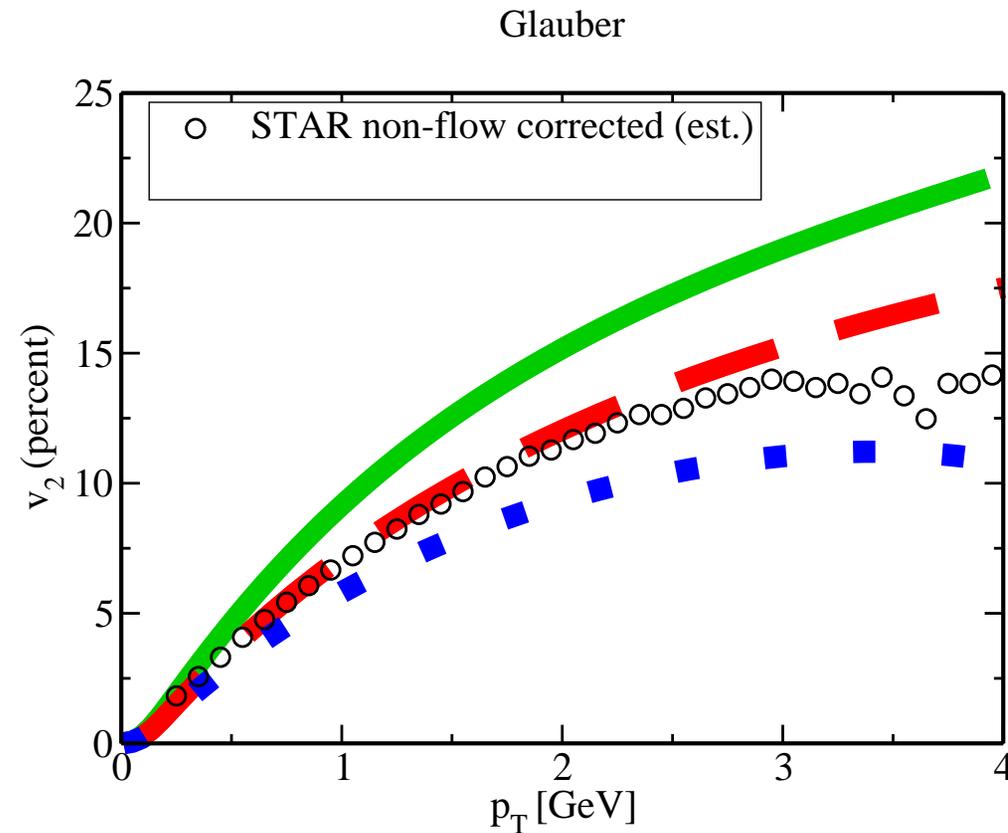
~ 2000 - Kolb et al, nucl-th/0305084:

2008 - Romatschke & Luzum, PRC78 ('08):

equation of state



shear viscosity



need identified particles to extract both EOS and transport properties

# Hydro $\rightarrow$ particles

In hydro and hydro+transport studies one must convert fluid to particles.

**two effects:** - dissipative corrections to hydro fields  $u^\mu, T, n$   
- dissipative corrections to thermal distributions  $f \rightarrow f_0 + \delta f$

- in local equilibrium (ideal hydro) - “one to one”

$$T_{LR}^{\mu\nu} = \text{diag}(e, p, p, p) \quad \Leftrightarrow \quad f_{eq,i} = \frac{g_i}{(2\pi)^3} e^{-p^\mu u_\mu / T}$$

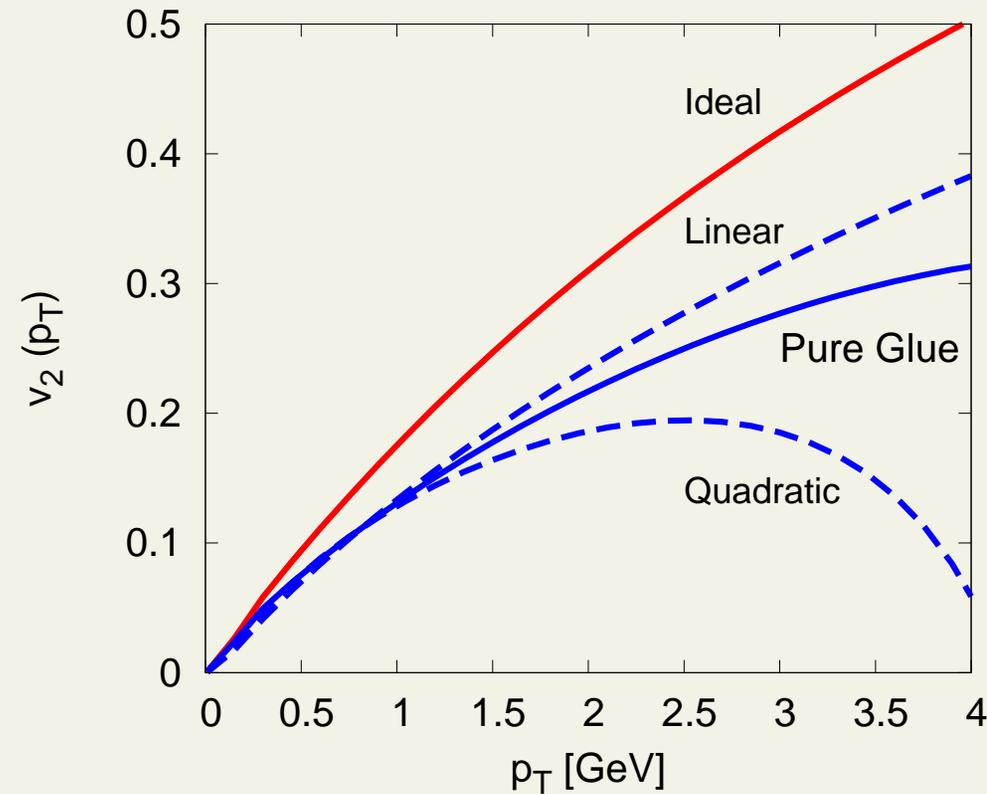
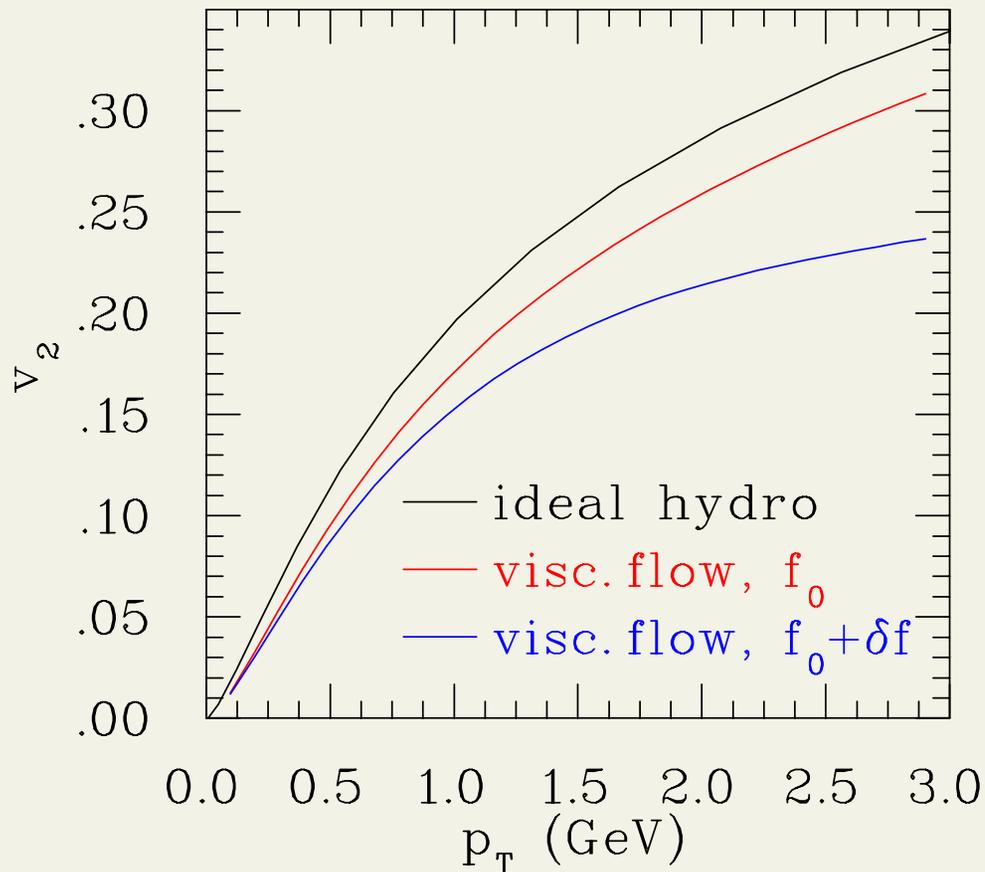
- near local equilibrium (viscous hydro) - “few to many”

$$T^{\mu\nu} = T_{ideal}^{\mu\nu} + \pi^{\mu\nu} \quad \Leftarrow \quad f = f_{eq,i} + \delta f_i$$

common choice - “democratic” **Grad ansatz:**  $\delta f_i \equiv f_i^{eq} \times \frac{\pi^{\mu\nu}}{2(e+p)} \frac{p_{\mu,i} p_{\nu,i}}{T^2}$

Huovinen & DM ('08) **Grad**  $\delta f \propto p^2$

Dusling et al, ('09) - **linear response**  $\delta f_{q,g} \sim p^{1.5}$



large effects at higher momenta ( $\delta f$  blows up, leads to  $f < 0$ )

**$\Rightarrow$  investigate region of applicability with nonlinear transport**

**Setup - 1D Bjorken**  $\rightarrow f_i = f_i(p_T, \xi, \tau)$ , where  $\xi \equiv \eta - y$

- i) compute  $f_i$  from full nonequilibrium transport  $p\partial f_i = \sum_j C_{ij}^{2\rightarrow 2}[f_i, f_j]$
- ii) from  $f_i$ , determine  $T^{\mu\nu}$  and  $\delta f_i$
- iii) **estimate  $\delta f_i$  from  $T^{\mu\nu}$  alone**, using some ansatz (e.g, Grad's  $\delta f \propto p^2$ )
- iv) compare  $\delta f_i^{estimated}$  with  $\delta f_i^{real}$

**Compare spectra  $dN_i(\tau)/dp_T^2 dy|_{y=0}$  and partial shear stresses  $\pi_{L,i}(\tau)/p(\tau)$ .**

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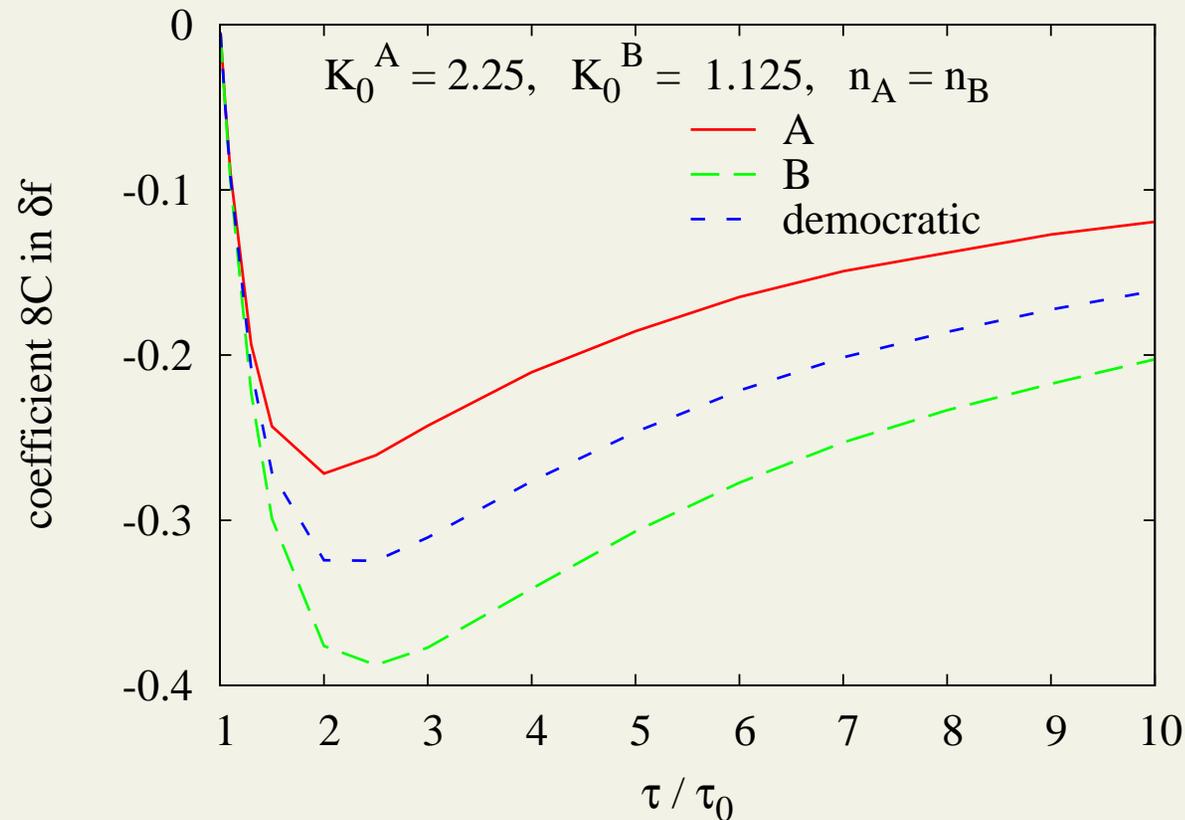
**expect dynamics to be governed by inverse Knudsen numbers:**

$$K_i \equiv \frac{\tau}{\lambda_i} = \tau \sum_j n_j \sigma_{ij} = \sum_j K_{i(j)}$$

## Two-component, massless system. $A$ set to equilibrate faster than $B$ .

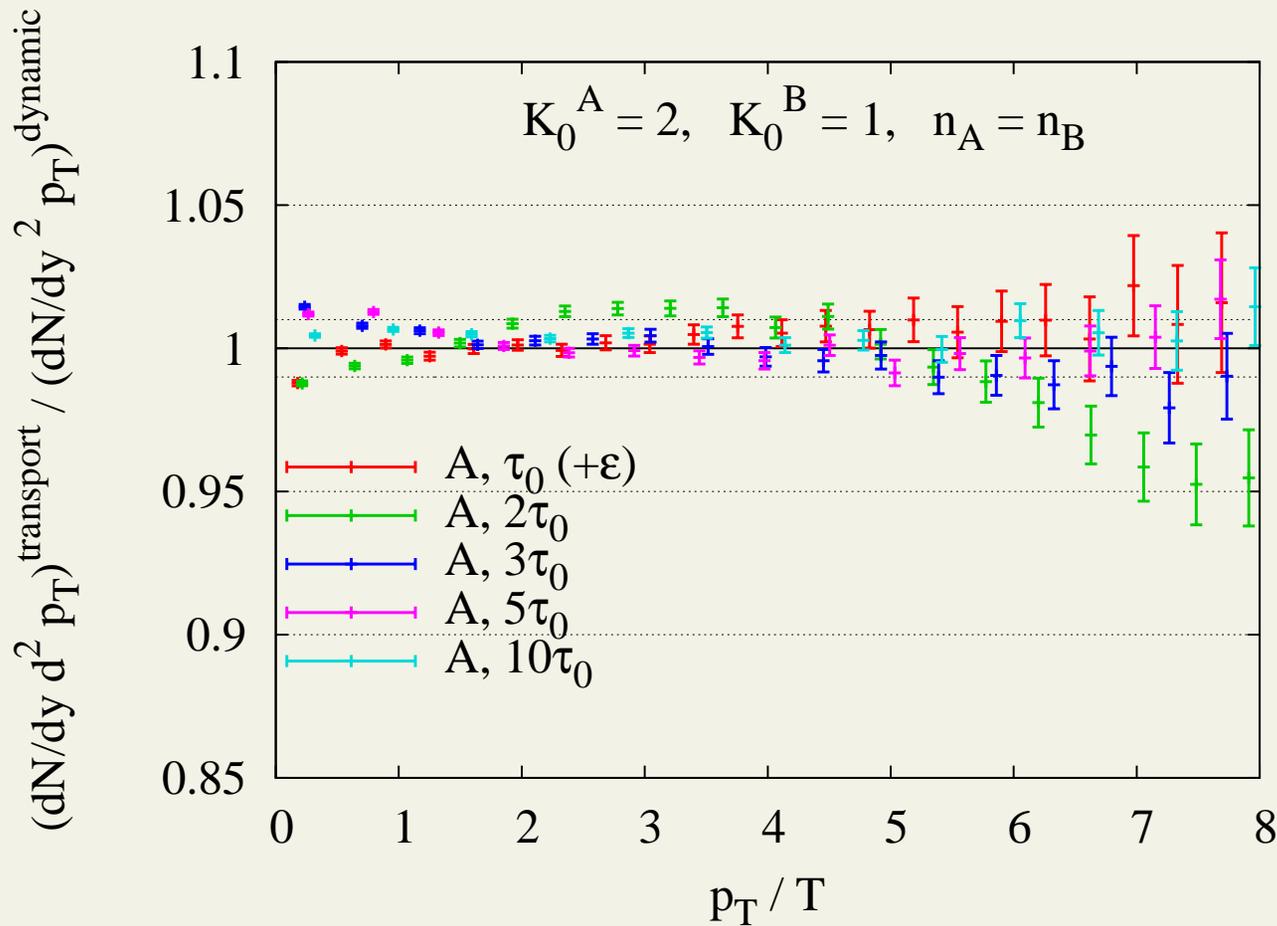
$$\delta f_i^{Grad} = C_i (p_T/T)^2 (\text{sh}^2 y - 1/2) f_i^{eq} \quad \pi_{L,i}/p_i = 8C_i$$

DM ('10)



**“democratic”**  $C_i = \text{const}$  ansatz misses viscous effects by  $\sim 20 - 30\%$

ratio - transport / dynamical Grad spectra DM ('10)  $\eta/s \approx 0.2$

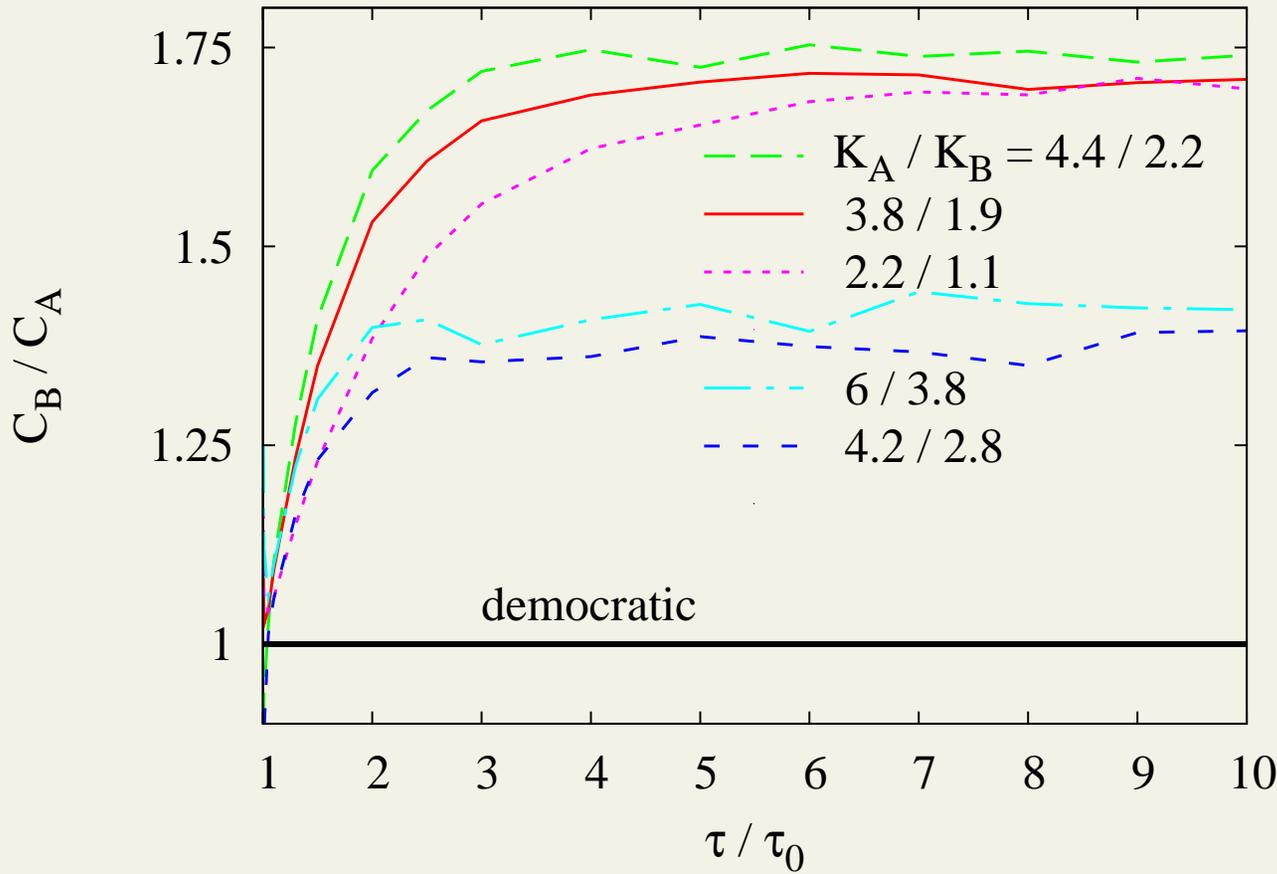


$$\delta f_i^{\text{Grad}} \propto \pi_{L,i} \cdot p^2$$

with dynamical values of partial shear stresses,  $\pi_{L,i}$ , Grad's quadratic ansatz is highly accurate

$$\delta f_i = C_i (p_T/T)^2 (\text{sh}^2 y - 1/2) f_i^{eq}$$

$$\pi_{L,i}/p_i = 8C_i$$



$n_A : n_B$	$\sigma_{AA} : \sigma_{AB} : \sigma_{BB}$
<b>3 : 1</b>	<b>20 : 10 : 5</b>
<b>2 : 2</b>	<b>20 : 10 : 5</b>
<b>2 : 2</b>	<b>12 : 6 : 3</b>
<b>3 : 1</b>	<b>24 : 24 : 12</b>
<b>2 : 2</b>	<b>20 : 13.3 : 8.89</b>

though viscous corrections are not proportional to  $K_i$ , shear stress sharing seems universal at late times

# $\delta f$ from linear response

standard linear response to flow shear  $\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial u)$ , same as computation of shear viscosity de Groot, et al ('70s)... Arnold, Moore, Jaffe, JHEP 0011...

$$p\partial f_i = \sum_j C_{ij}^{2\rightarrow 2}[f_i, f_j]$$

small deviations from local equilibrium  $f_i = f_{0i} + \delta f_i$ , 2-component case:

$$\begin{aligned} p\partial f_{0A} &= C_{AA}[f_{0A}, \delta f_A] + C_{AA}[\delta f_A, f_{0A}] + C_{AB}[\delta f_A, f_{0B}] + C_{AB}[f_{0A}, \delta f_B] \\ p\partial f_{0B} &= C_{BB}[f_{0B}, \delta f_B] + C_{BB}[\delta f_B, f_{0B}] + C_{BA}[\delta f_A, f_{0B}] + C_{BA}[f_{0A}, \delta f_B] \end{aligned}$$

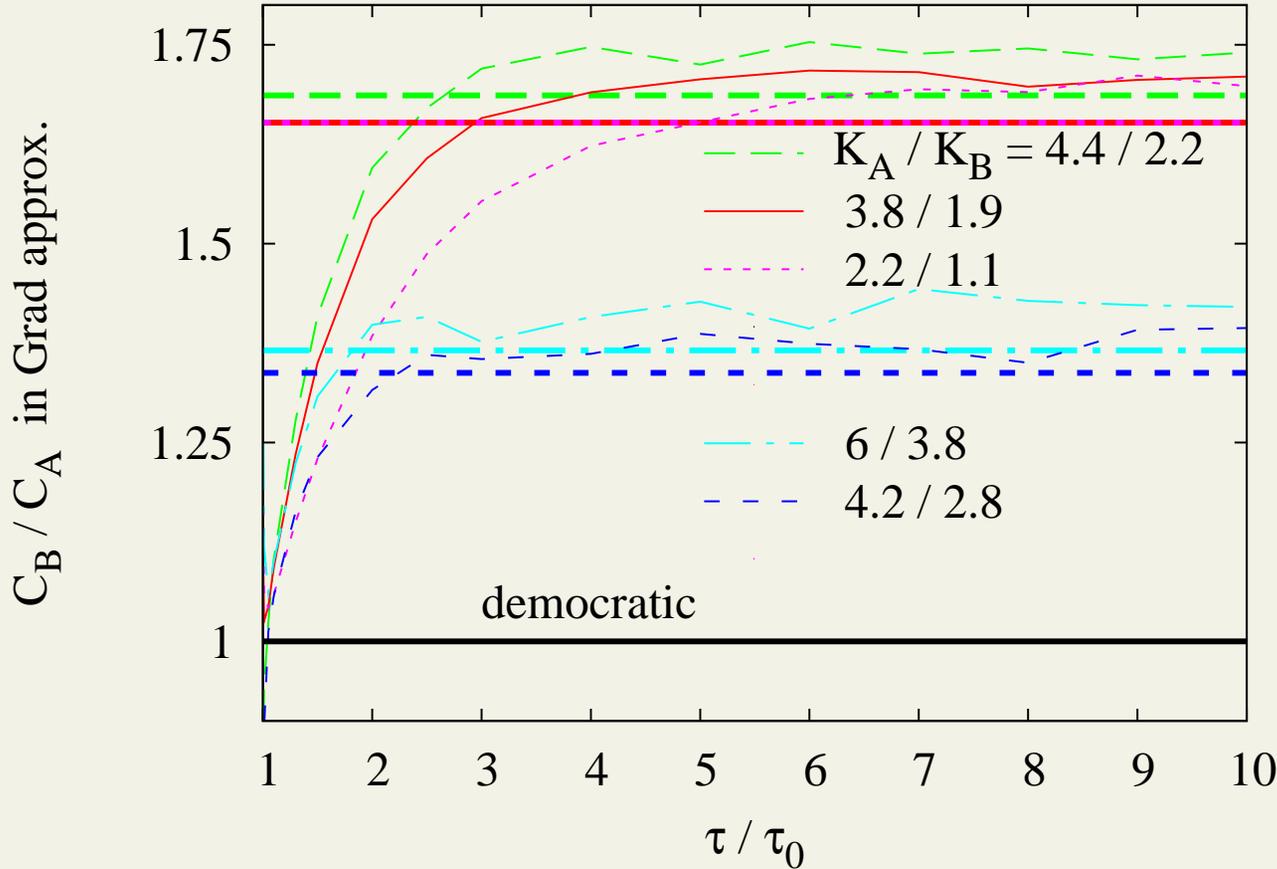
No  $\delta f$  on LHS - relaxation implicitly assumed.

Can be recast as a variational problem:

$$\delta Q[\delta f_A, \delta f_B] = 0$$

where  $Q_{max}$  is proportional to the shear viscosity.

$$\Rightarrow \left( \frac{C_B}{C_A} \right)_{lin.response}^{Grad} = \frac{5K_A + 2(K_{A(B)} + K_{B(A)})}{5K_B + 2(K_{A(B)} + K_{B(A)})}$$



$$\delta f_i = C_i (p_T/T)^2 (\text{sh}^2 y - 1/2) f_i^{eq}$$

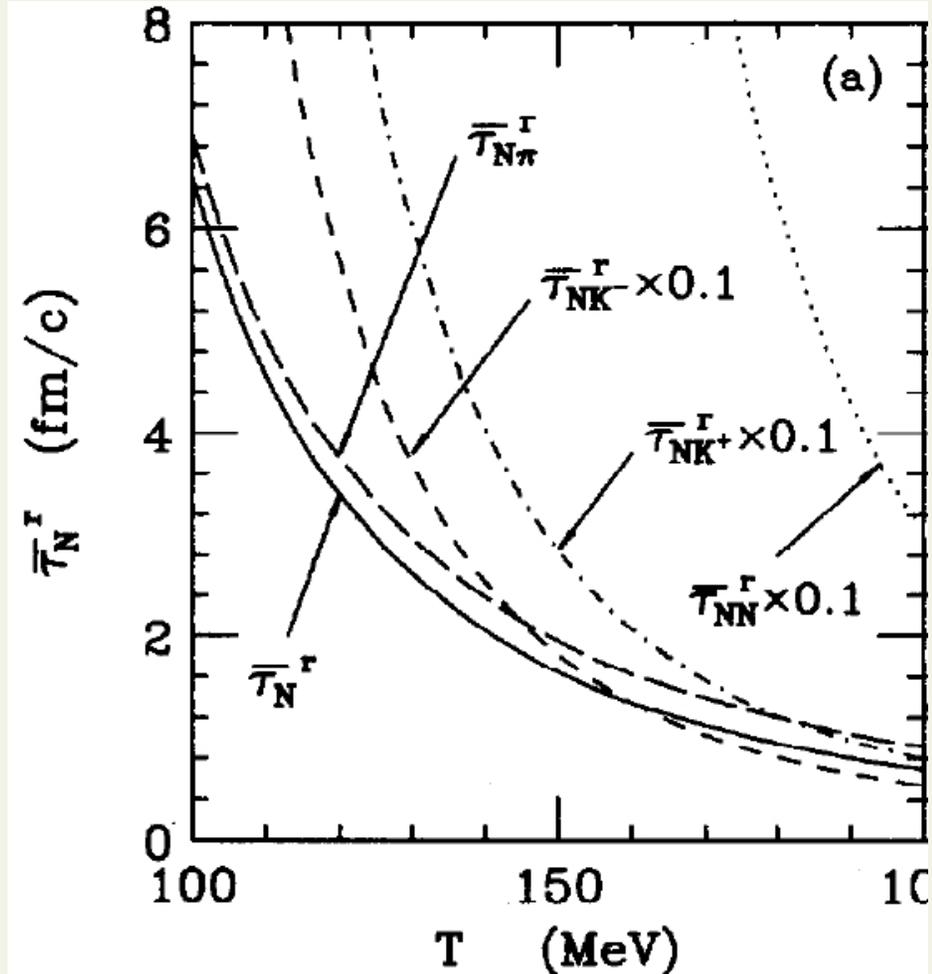
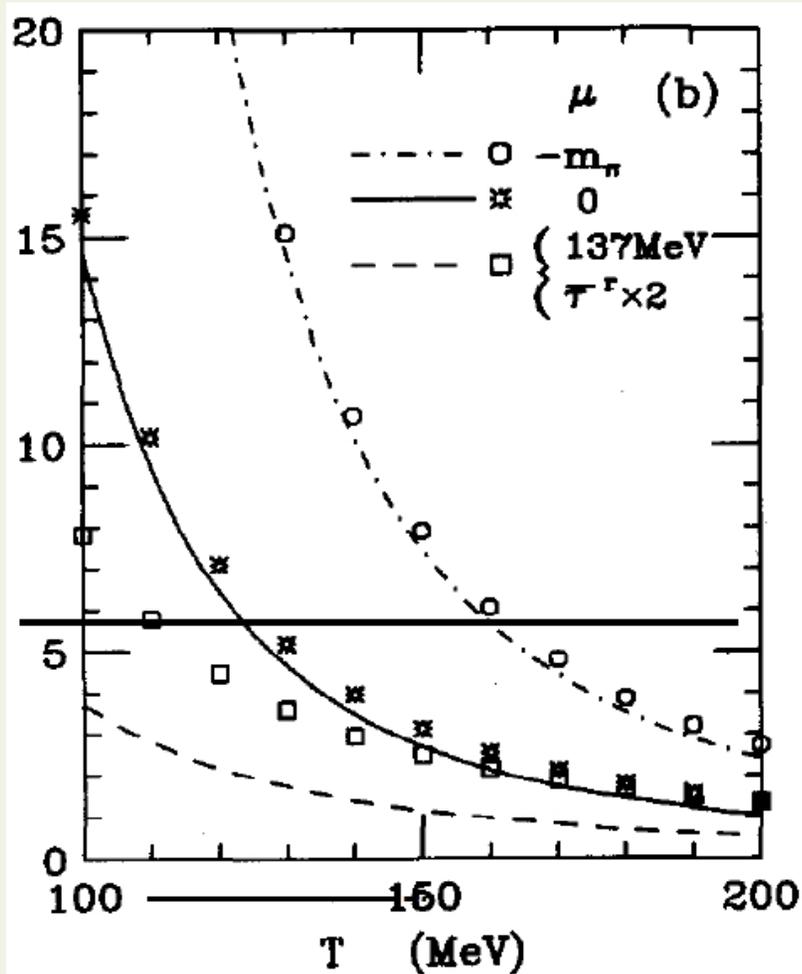
$$\pi_{L,i} / p_i = 8C_i$$

**linear response with  $\delta f \propto p^2$  “gets” late-time  $\pi^{\mu\nu}$  sharing within 10%**

# Apply to hadron gas. Approximation: $\pi - N$ system.

Prakash et al, Phys.Rep. 227 ('93):  $\bar{\tau}_{\pi\pi} \approx 1/\bar{\nu}_{\pi\pi}$

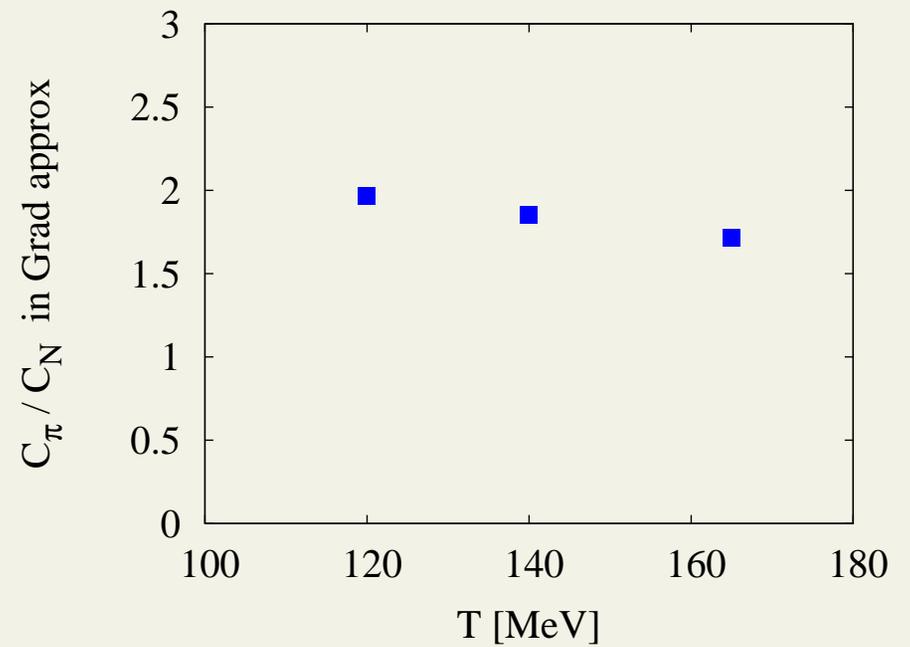
$\bar{\tau}_{\pi N} \approx 1/\bar{\nu}_{\pi N} \sim \tau_{\pi\pi}/2$



quite well captured with isotropic  $\sigma_{eff}^{\pi\pi} = 30 \text{ mb}$ ,  $\sigma_{eff}^{\pi N} = 50 \text{ mb}$

⇒ protons are closer to equilibrium

$$C_\pi \sim 2C_N$$

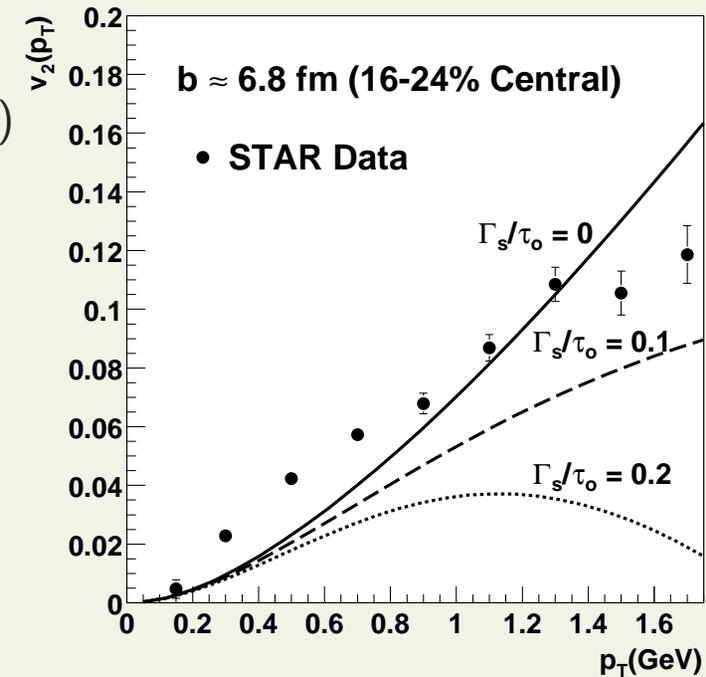


illustrate effect on  $v_2(p_T)$  - 'a la' Teaney, PRC68 ('03)

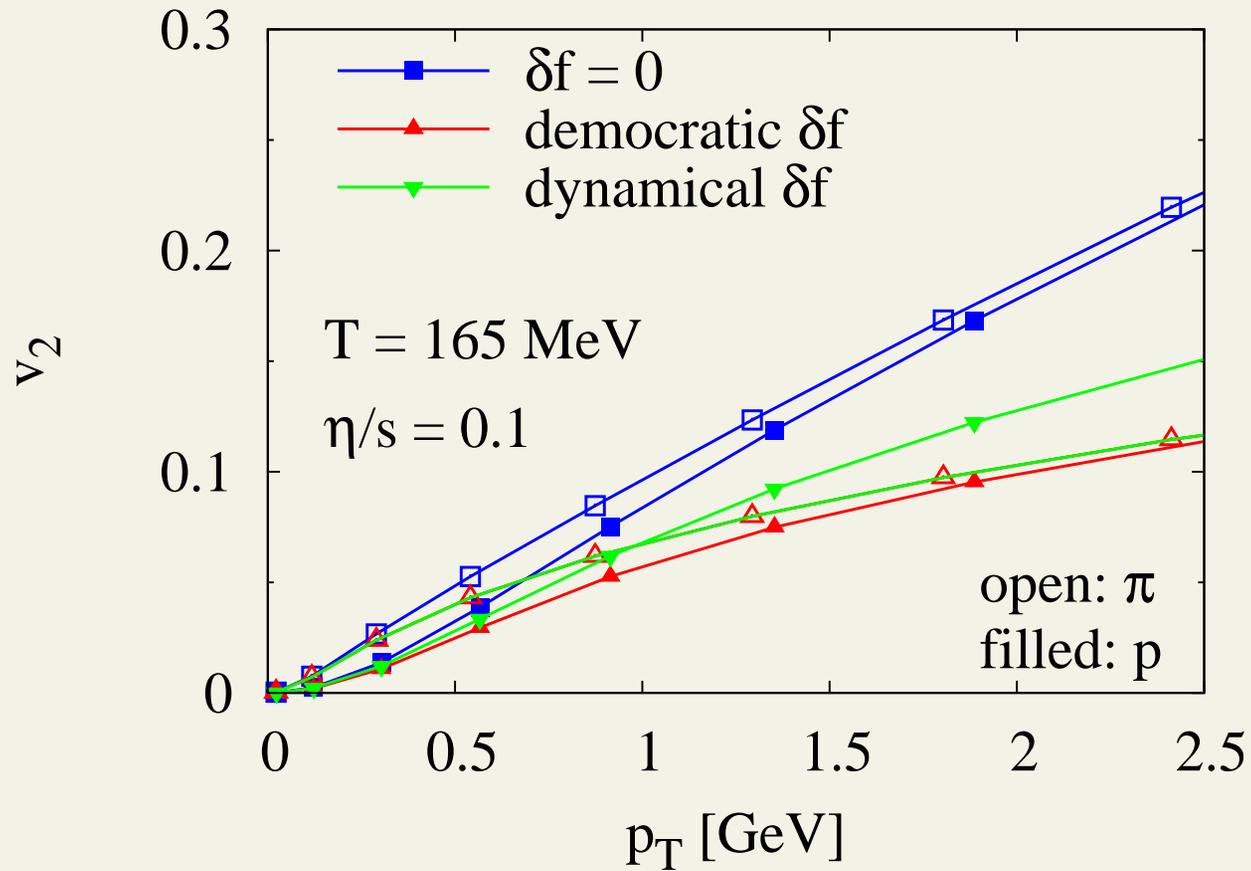
$$\pi_{NS}^{\mu\nu} = \eta[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial u)]$$

but with real hydro AZHYDRO-0.2p2

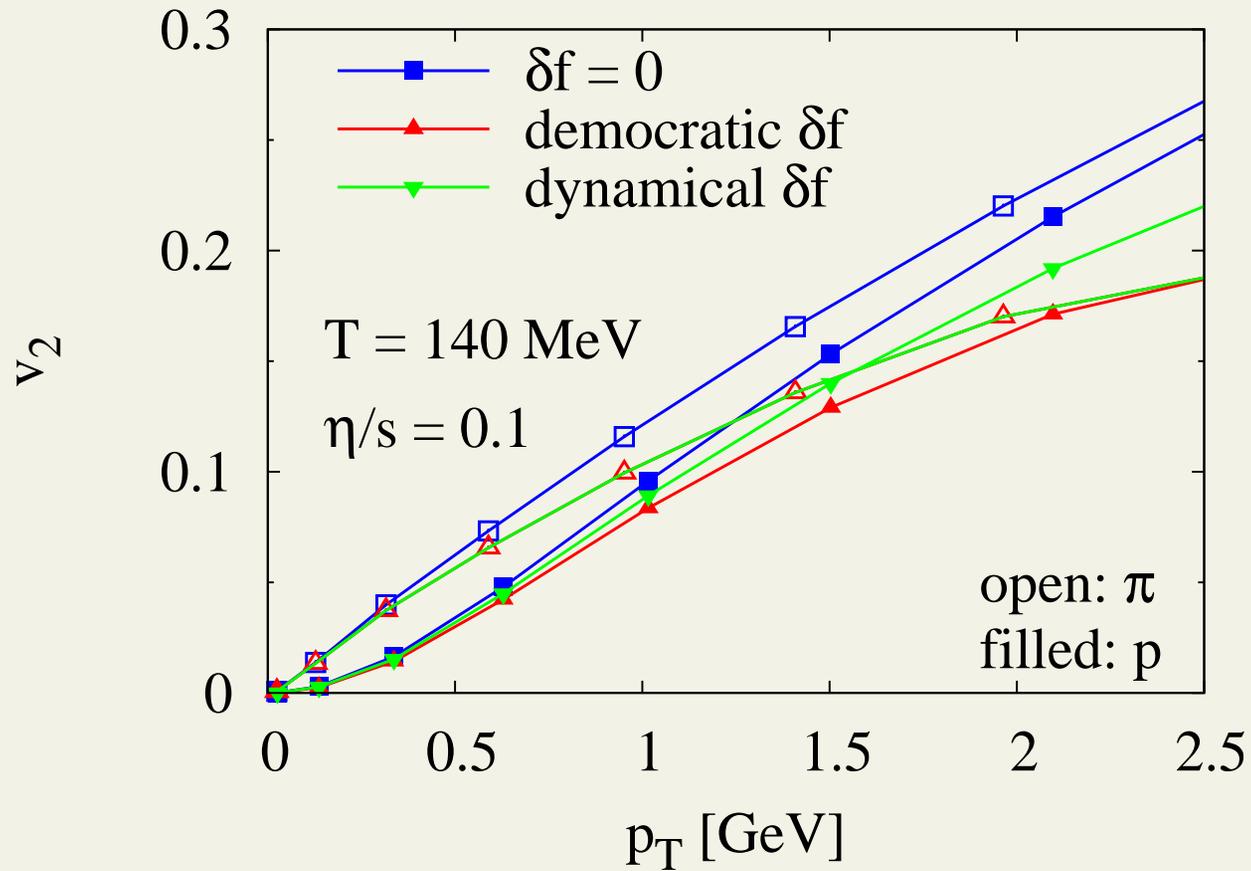
(EOS s95p-v1, Glauber profile,  $\tau_0 = 0.5$  fm)



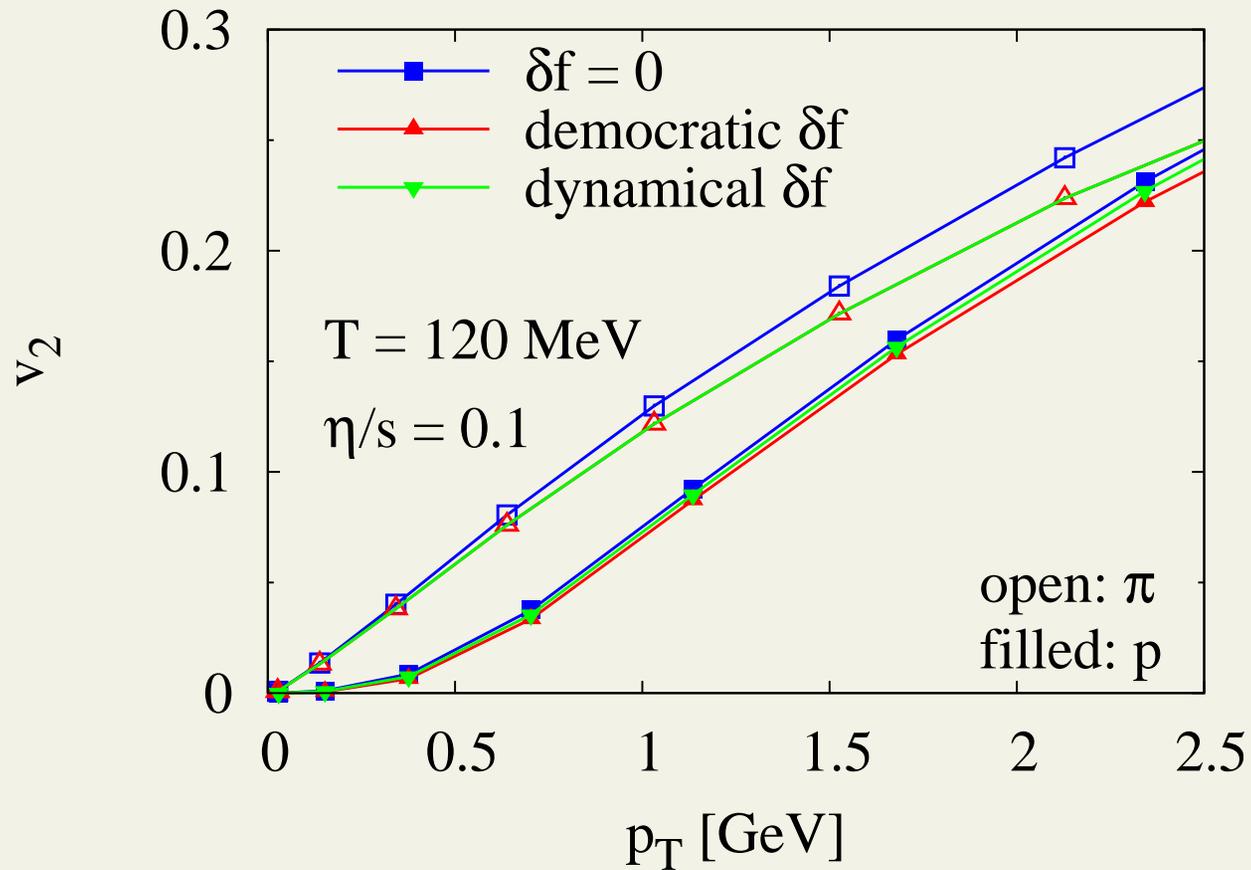
# Au+Au, b=7 fm - without resonance decays



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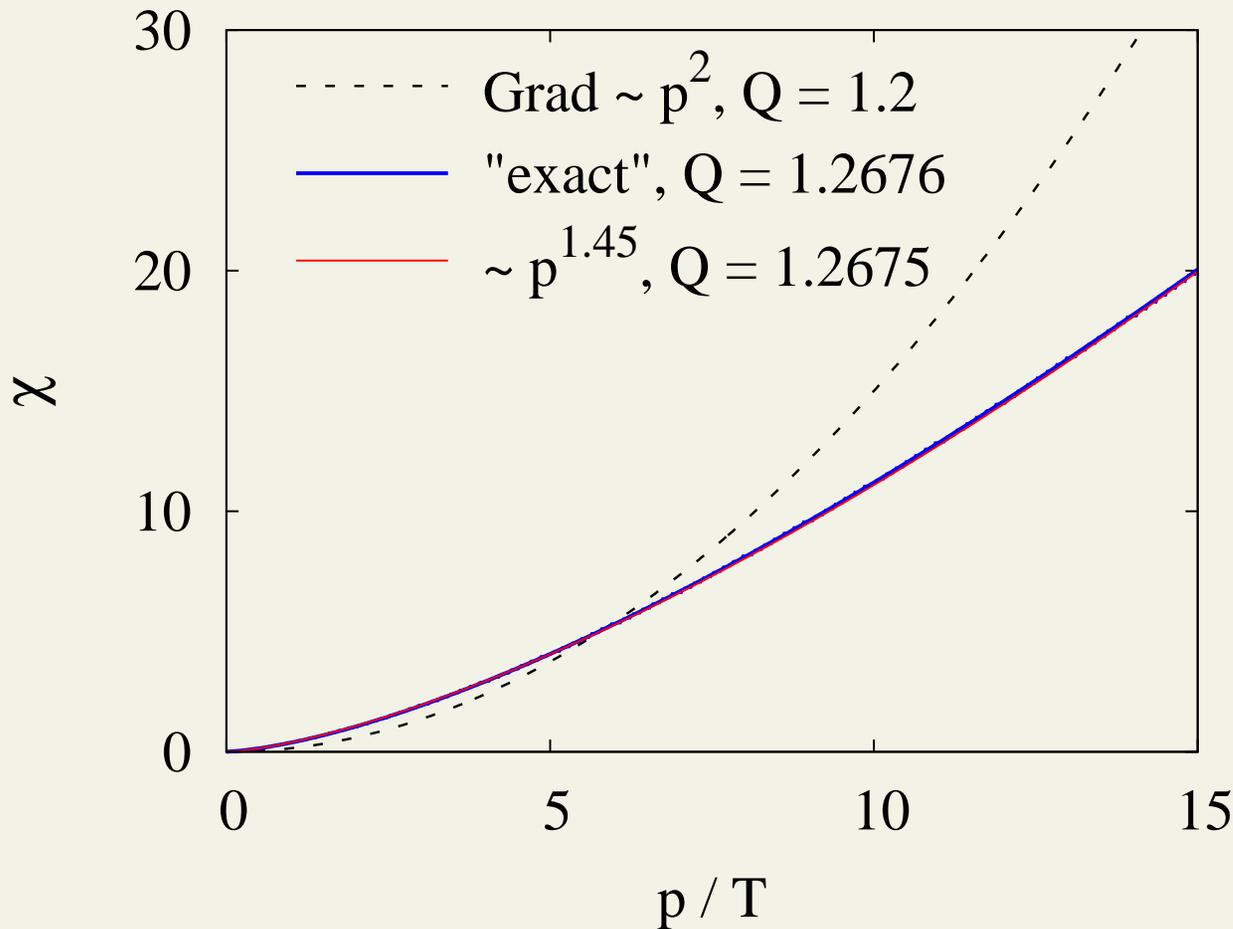
## Au+Au, b=7 fm - without resonance decays



significant reduction of  $\pi - p$  splitting at  $T_{switch} = 140$  and  $165 \text{ MeV}$

# Momentum dependence - Grad **inconsistent** with linear response!

e.g., one-component massless gas with isotropic  $\sigma = \text{const}$ :

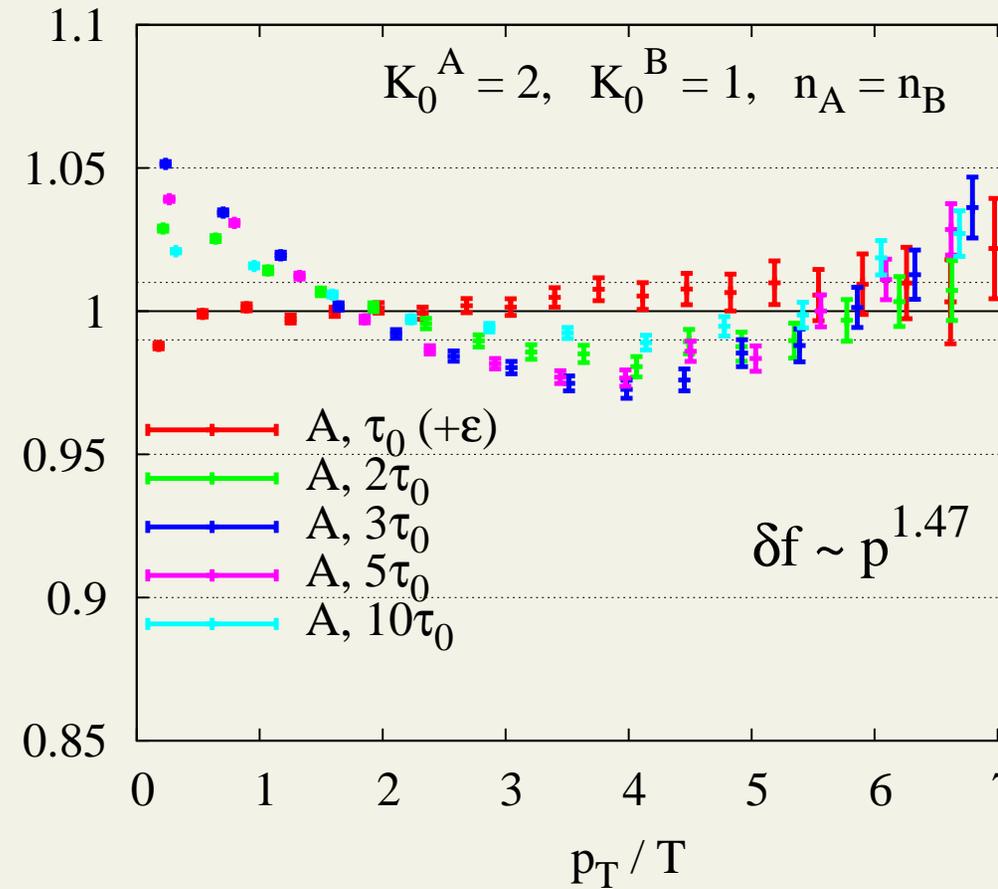
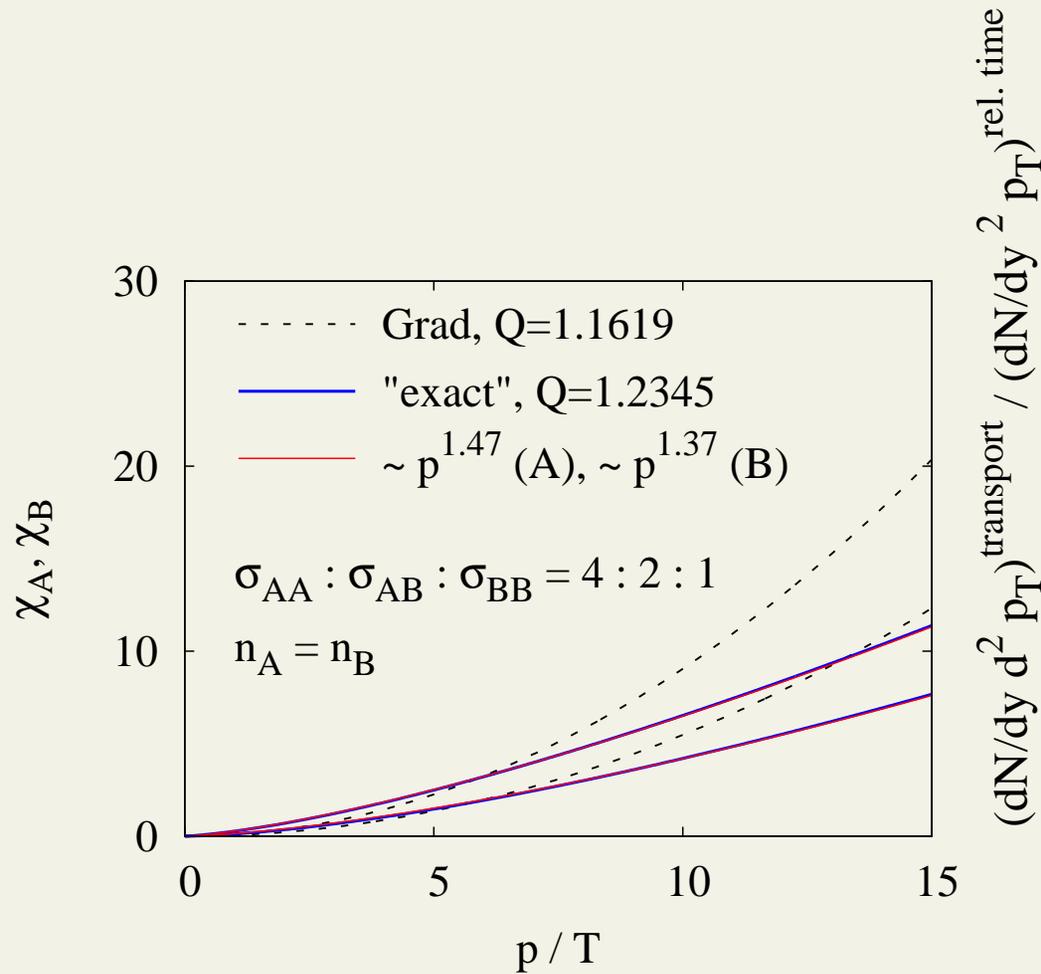


$$\delta f = \chi(p/T) \frac{T}{n\sigma} \frac{\pi_{\mu\nu} \hat{p}^\mu \hat{p}^\nu}{\eta T}$$

$$\eta \simeq \frac{1.2676T}{\sigma}$$

Duslin et al get similar  $\sim p^{1.5}$  but with forward-peaked  $2 \rightarrow 2$  and  $1 \leftrightarrow 2$

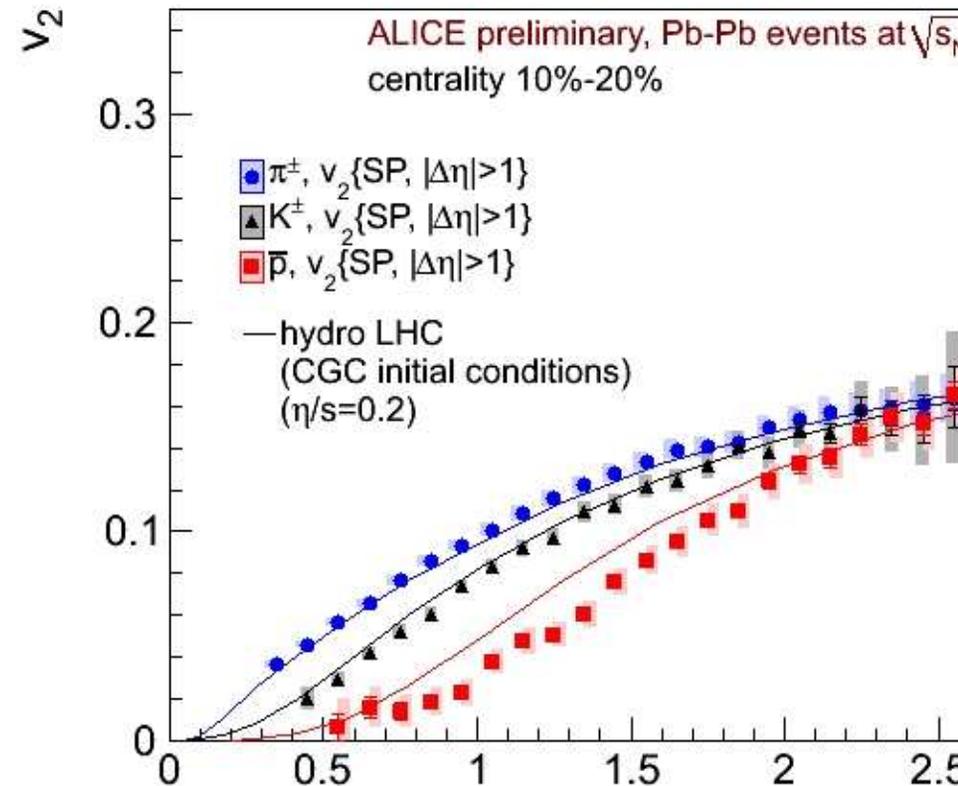
two-component, massless - **not** quadratic either  $\rightarrow$  fails 1D Bjorken test



# Summary

Instead of the “democratic” ansatz, viscous hydro and hydro+transport studies should **use per-species  $\delta f_i$ , based on microscopic physics.**

Until then, comparison with data has significant uncertainties.



Linear response describes shear stress sharing in a multicomponent system well, but the momentum dependence  $\delta f \sim p^{1.4-1.5}$  does not agree with nonlinear  $2 \rightarrow 2$  transport  $\sim p^2$ . Looks like for  $\eta/s \sim 0.2$ , relaxation phenomena cannot be ignored (too rapid expansion).

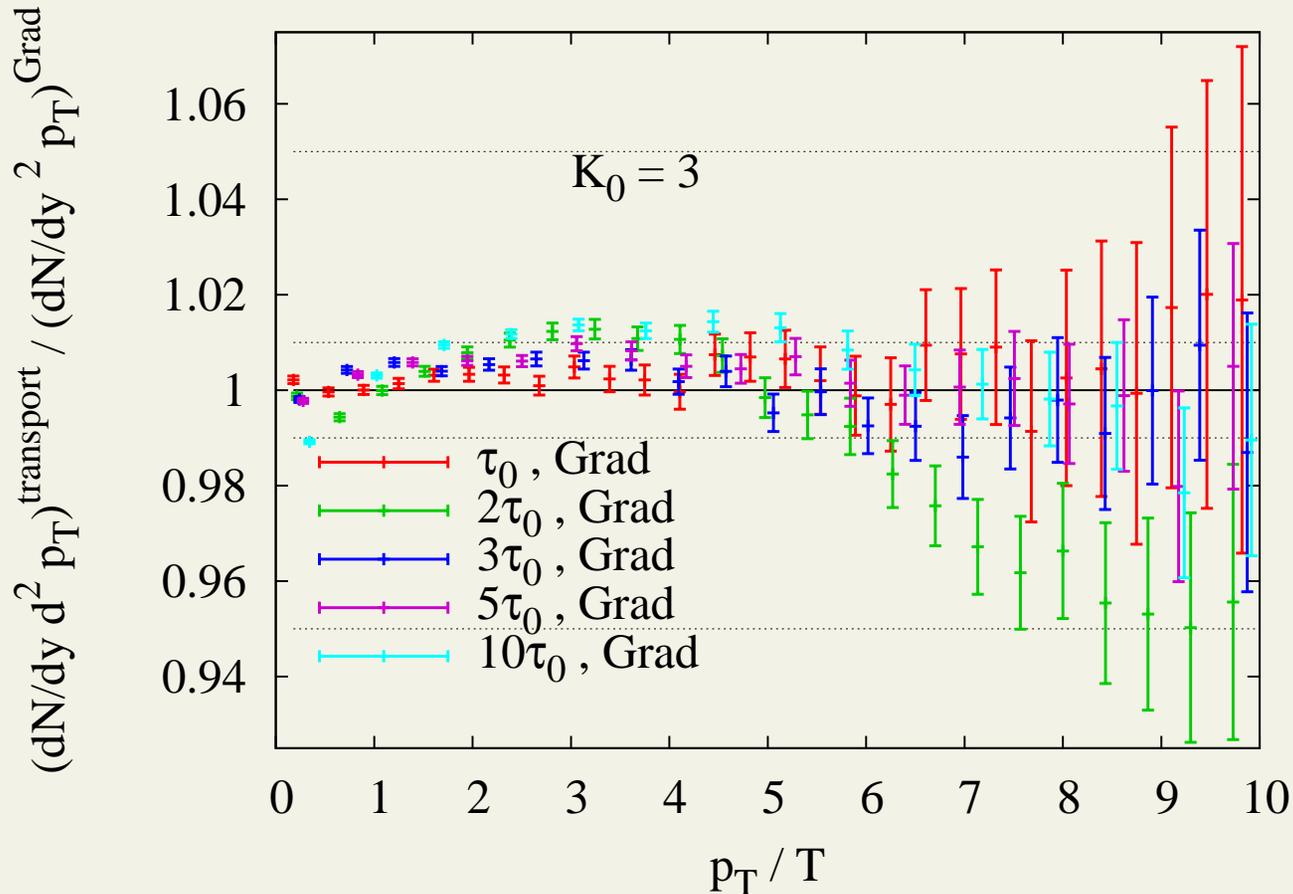
→ higher-order linear response? Multicomponent fluid dynamics?

# Backup slides

# One-component, massless system.

ratio - transport spectra / quadratic approx,  $\eta/s \sim 0.1$

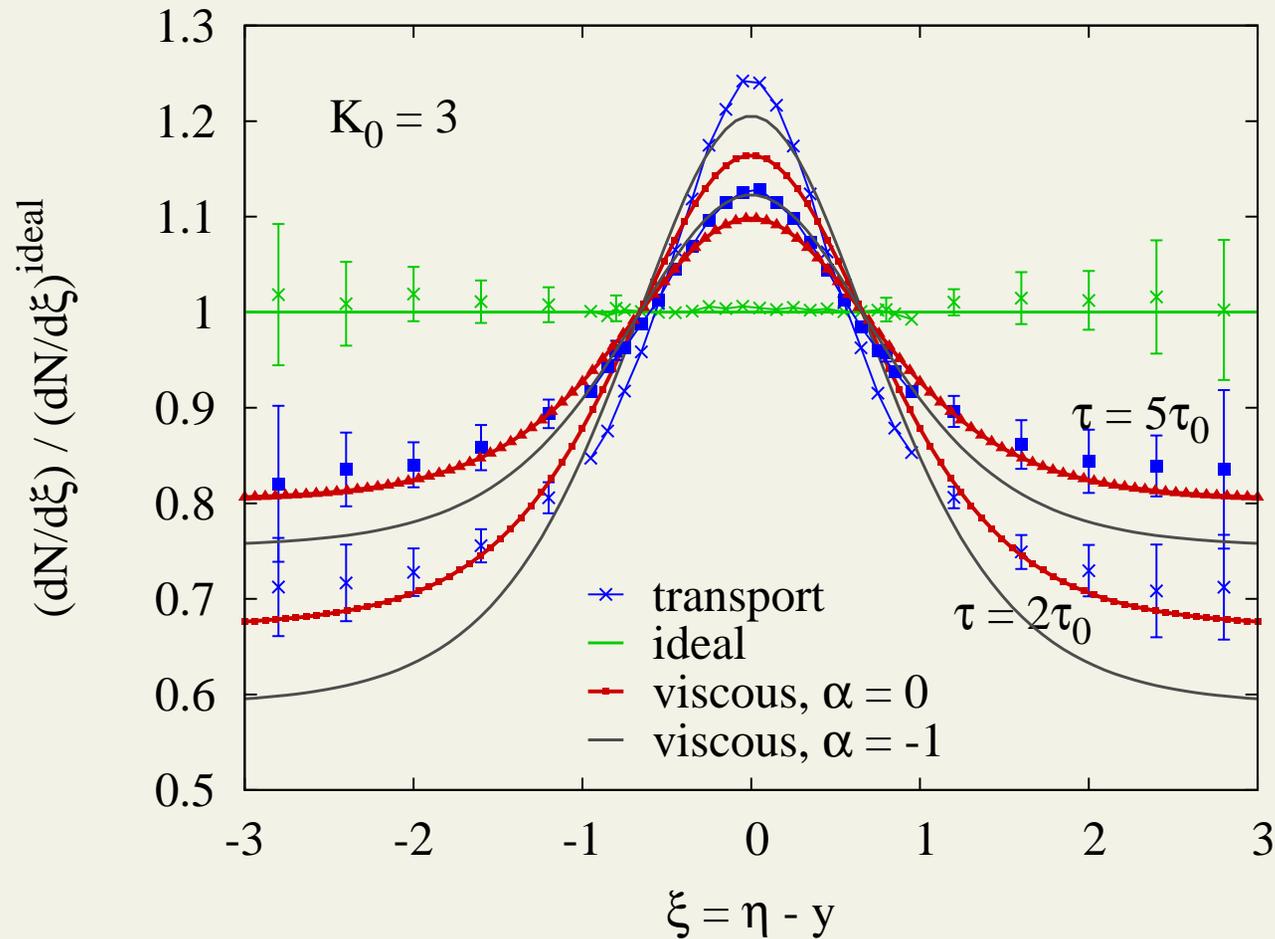
DM ('09):



quadratic  $\delta f \propto p^2$  is  $\approx 1\%$  accurate even at  $p_T/T = 6$

although Grad ansatz not as good for rapidity  $\xi \equiv \eta - y$  correlation

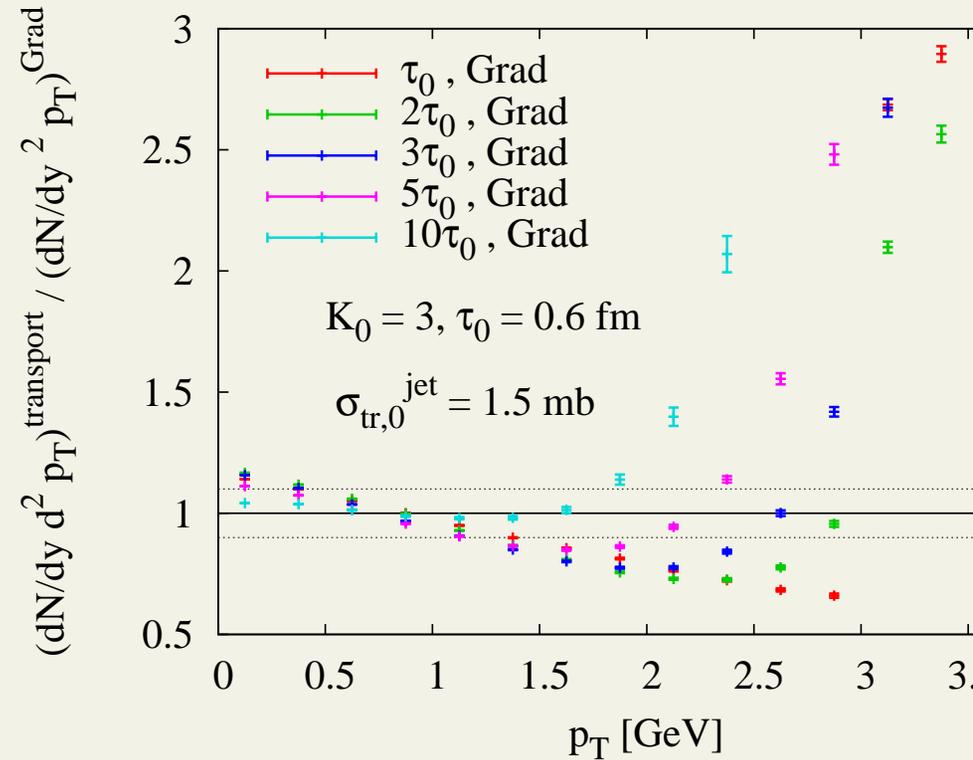
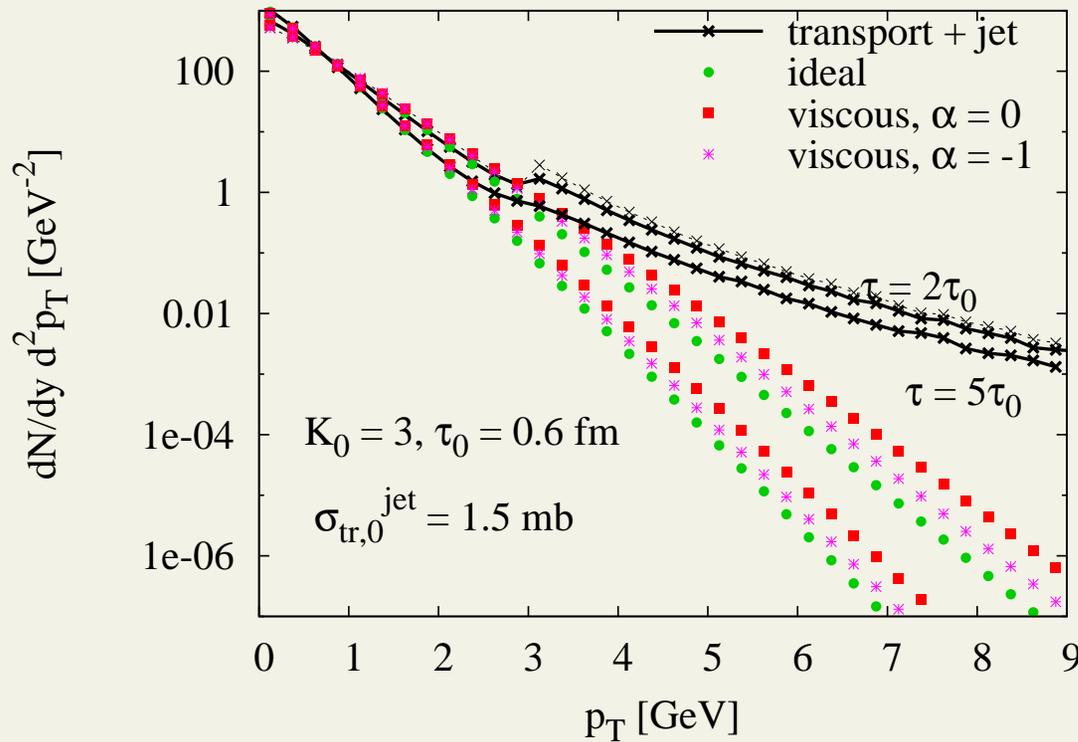
$\frac{dN/d\xi}{dN^{ideal}/d\xi}$  distributions relative to IDEAL hydro,  $\eta/s \sim 0.1$  DM ('09-'10)



in practice, though, accuracy is limited by pQCD power-law tails DM ('09)-('10)

spectra  
approximation

transport spectra / quadratic



[bulk:  $\eta/s \sim 0.1$ , jets:  $\sigma_{tr} = T_0^2/T^2 \cdot 1.5 \text{ mb}$ ,  $T_0 = 385 \text{ GeV}$ ]

with Grad, even  $\approx 10\%$  accuracy extends only up to  $p_T \sim 1.5 \text{ GeV}$

## viscous correction to spectra of species A in a 2-component system

points: full transport, lines: “democratic” Grad ansatz

