Holographic Thermalization

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Thermalization

Characteristic participant parton momentum scale: $Q_s$

How long does it take?

Characteristic parton momentum scale: $T \ll Q_s$
(at weak coupling)

How thermal is it?

QGP

How does the thermalization process work at strong coupling?

If not “bottom up”, what else?
AdS/CFT dictionary

- Want to study strongly coupled phenomena in QCD
- Toy model: $\mathcal{N} = 4 \, SU(N_c) \, SYM$

**AdS/CFT**

- $\mathcal{N} = 4 \, SYM$
- Strong coupling
- Vacuum
- Thermal state (equilibrated plasma)
- Thermalization
- HI collision

**holo. dual**

- AdS/CFT
- Weak coupling
- Empty AdS$_5$
- AdS BH
- BH formation

**String theory in AdS$_5$**

- Energy injection

$$\left( \frac{R_{AdS}}{l_s} \right)^4 = g_{YM}^2 N_c \equiv \lambda$$

$R_{AdS}$, $l_s$, $g_{YM}$, $N_c$, $\lambda$
Questions to answer

- What is the measure of thermalization on the boundary?
  - Local operators are not sufficient
    \[ \langle T_{\mu\nu} \rangle \text{ etc.} \]
  - Nonlocal operators are more sensitive
    \[ \langle O(x)O(x') \rangle \text{ etc.} \]

- What is the thermalization time?
  - When observables reach their thermal values
Thermality probes

- **Local operators** like $\langle T_{\mu\nu} \rangle$ measure moments of the momentum distribution of field excitations
  - e.g. $\langle k_x^2 \rangle$ vs. $\langle k_z^2 \rangle$

- **Nonlocal operators**, like the equal-time Green’s function, are sensitive to the momentum distribution and to the spectral density of excitations:
  - $G(\vec{x}) = \int d\vec{k} dk^0 \sigma(k^0, \vec{k}) \left[ n(\vec{k}) + 1 \right] \exp(i\vec{k} \cdot \vec{x})$

- **Entropy** is the “gold standard” of thermalization:
  - $S = - \text{Tr}[\rho \ln(\rho)]$ probes all degrees of freedom.
  - Coarse graining mechanism: *Entanglement entropy*. 
Probes we consider

- 2-point function
  - $\langle \mathcal{O}(x)\mathcal{O}(x) \rangle$
  - Bulk: geodesic (1D)

- Wilson line
  - $W = P\{\exp[\int_C A_\mu(x)dx^\mu]\}$
  - Bulk: minimal surface (2D)

- Entanglement entropy
  - $S_A = -\text{Tr}_A[\rho_A \log \rho_A]$, $\rho_A = \text{Tr}_B[\rho_{\text{tot}}]$
  - Bulk: codim-2 hypersurface

For details: V. Balasubramanian, et al., PRL 106, 191601 (2011); arXiv:1103.2683

See also: S. Caron-Huot, P.M. Chesler & D. Teaney, arXiv:1102.1073

Use semiclassical approximation

(same dimension as boundary space)
Entanglement entropy

Modes with momentum $k$ “leak” into surrounding by $\Delta x \sim 1/k \Rightarrow \text{entanglement with environment}$

Entanglement entropy of localized vacuum domain is proportional to surface area (Srednicki 1994).

$$\gamma(V) \sim |\partial V|$$

$$\gamma(V) \sim |V|$$

$T \neq 0$: $S$ proportional to volume $\Leftrightarrow$ area of horizon of dual BH (Ryu & Takayanagi 2006)

$$S = \frac{\text{Area}[\gamma(V)]}{4G_N}$$
Vaidya-AdS geometry

- Light-like (null) infalling energy shell in AdS (shock wave in bulk)
  - Vaidya-AdS space-time (analytical)
    \[ ds^2 = \frac{1}{z^2} \left[ -(1 - m(v)z^d) dv^2 - 2dz dv + d\vec{x}^2 \right] \]
  - \( z = 0 \): UV \( z = \infty \): IR
  - Homogeneous, sudden injection of entropy-free energy in the UV
  - Thin-shell limit can be studied semi-analytically
  - We studied AdS\(_{d+1}\) for \( d = 2,3,4 \)
  - \( \leftrightarrow \) Field theory in \( d \) dimensions

Injection moment
Examples

Geodesic line “punching” through the falling shell

Wilson surface “punching” through the falling shell
Probing thermalization

Equal-time geodesics for fixed $t_0 = 2$ and $\ell = 3.0, 4.6, 68.2$

Geodesics staying outside the falling shell only probe “thermal” part of bulk space

$\langle O(x)O(x') \rangle \sim \exp[-\delta \mathcal{L}] \quad$ with $\quad \delta \mathcal{L} = \mathcal{L} - \mathcal{L}_{\text{AdS}}$

$\tau_{\text{max}} \quad \tau_{\text{crit}}$

steepest slope

$\tau_{1/2}$

thermal limit

Monday, May 23, 2011
2-point functions

\[ \delta L - \delta \tilde{L}_{\text{thermal}} \]

\[ t_0 \]

\[ \Delta \]

\[ \text{thermal} \]

\[ \tau_{\text{AdS}_3} = \ell/2 \]

\[ \tau_{\text{crit}} \]

\[ \tau_{\text{max}} \]

\[ \tau_{1/2} \]

\[ \ell \]

\[ \text{AdS}_3 \]

\[ \text{AdS}_4 \]

\[ \text{AdS}_5 \]
Higher dim. observables

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<th>Rectangular Wilson loop</th>
<th>Wilson “sphere”</th>
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Entropy thermalizes slowest

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$\tau_{\text{crit}} = \frac{\ell}{2}$

Entanglement entropy of spherical volume in $d = 2, 3, 4$

**Thermalization time for entanglement entropy**

= time for light to escape from the center of the volume to the surface

**Other observables thermalize faster.**
Entropy density growth rate is nearly volume independent for small volumes, but slowly decreases for large volumes (numerically difficult to study in $d > 2$).

(Very crude) phenomenology:

$$\tau_{\text{crit}} \sim 0.5 \frac{\hbar}{T} \approx 0.3 \text{ fm/c for } T = 300 - 400 \text{ MeV}$$
Conclusions

- Long-distance observables sensitive to IR modes take longer to thermalize
  - Top-down rather than bottom-up thermalization
- Entropy is the last observable to reach thermal value
- Thermalization proceeds as fast as constrained by causality i.e. at the speed of light
  - True for homogeneous energy injection
  - Speed of sound is expected to govern equilibration of spatial inhomogeneities
- Future research opportunities: Many.
  - See next page....
Outlook

- Compute other observables in the Vaidya model
  - Unequal time correlators; light-cone Wilson loops
- Extend methods to different geometries
  - Colliding shock waves; boost invariant geometries
  - Expanding longitudinal flux tubes
- Extraction of QFT state as function of time
- Entanglement entropy of non-spherical domains
- Beyond the semi-classical approximation
- Non-AdS backgrounds
  - Confining geometries, improved holographic QCD models
- Whatever else you can think of!
Je vous remercie

de votre attention