Charge fluctuations in chiral models and the QCD phase transition

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Polyakov-loop extended quark-meson model

- Model based on symmetries of QCD
  $SU(2)_L \otimes SU(2)_R$, scalar condensate $\sigma \propto \langle \bar{q}q \rangle$
  $Z(3)$, Polyakov loop $\ell = \frac{1}{N_c} \langle \mathrm{Tr}_c \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\boldsymbol{x}, \tau) \right] \rangle$
- Lagrangian of PQM model:

$$\mathcal{L} = \bar{q} \left[i\gamma^\mu D_\mu - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi}) \right] q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - U(\sigma, \boldsymbol{\pi}) - \mathcal{U}(\ell, \ell^*)$$

$\mathcal{U}(\ell, \ell^*)$ – Z(3)-invariant Polyakov loop potential
Gluons are coupled to quarks q by covariant derivative $D_\mu = \partial_\mu - iA_\mu$

$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - c\sigma$ is meson potential

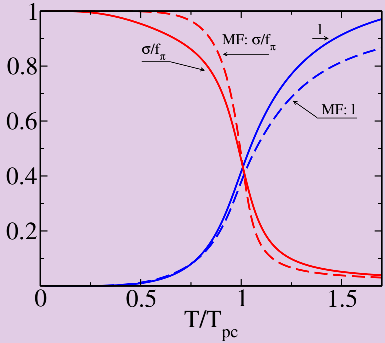
Functional renormalization group

- **accounts for the universal critical behaviour near chiral transition**
- reproduces scaling properties and critical exponents (Berges ’00, B. Stokic et. al. ’10)
- respects symmetries (the Goldstone theorem fulfilled, a second-order phase transition in the chiral $O(N)$ model)
- easily extended to finite T and μ

FRG review: J. Berges, N. Tetradis & C. Wetterich, Phys.Rept.363:223-386, ’02
FRG formulation of the PQM model: V. S., B. Stokic, B. Friman & K. Redlich, PRC, ’10

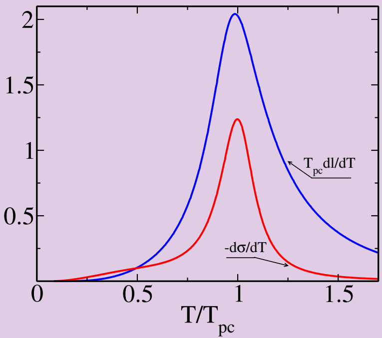
Order parameters

Order parameters in FRG and mean-field approximation:



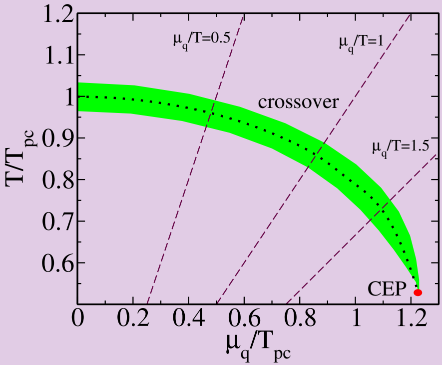
Mean-field: dashed lines
FRG: solid lines
Mesonic fluctuations: smoothen chiral transition

Temperature derivatives of the order parameters:



T_{pc} is pseudocritical temperature at $\mu_q = 0$

Phase diagram



Crossover: $|\partial\sigma/\partial T| > 0.95 \cdot \max(|\partial\sigma/\partial T|)$

Fluctuations

- Fluctuations of net-quark number χ_n^q and net-baryon charge χ_n^B

$$\chi_n^q = \frac{\partial^n (P/T^4)}{\partial(\mu_q/T)^n} \quad | \quad \chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = \left(\frac{1}{3}\right)^n \chi_n^q$$

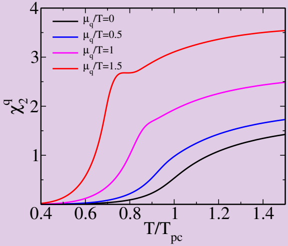
- Fluctuations of electric charge χ_n^Q

$$\chi_n^Q = \frac{\partial^n (P/T^4)}{\partial(\mu_Q/T)^n}$$

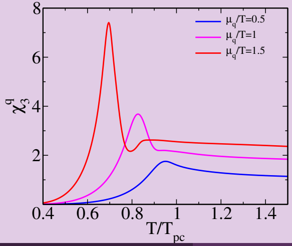
Properties:

- At CEP: for $n \geq 2$, $\chi_n \propto \xi^{n\beta\delta/\nu-3} \approx \xi^{5n/2-3}$, e.g. $\chi_4 \sim \xi^7$
  (M. Stephanov '09)
- $m_\pi = 0$ (critical line):
  at $\mu_B = 0$, $\chi_n \propto \xi^{(n+2\alpha-4)/(2\nu)}$ for even $n \geq 6$, e.g. $\chi_6 \sim \xi^{1.1}$
  at $\mu_B \neq 0$, $\chi_n \propto \xi^{(n+\alpha-2)/\nu}$ for $n \geq 3$, e.g. $\chi_3^B \sim \xi^{1.1}$
- $m_\pi \neq 0$: rapid change in crossover region
  (B. Friman, F. Karsch, K. Redlich and V.S. '11)

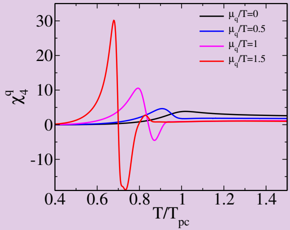
$$\chi_2^q = \frac{1}{VT^3}\langle(\delta N_q)^2\rangle \rightarrow \frac{N_c N_f}{3}\left[1 + \frac{3}{\pi^2}\left(\mu_q/T\right)^2\right]$$



$$\chi_3^q = \frac{1}{VT^3}\langle(\delta N_q)^3\rangle \rightarrow \frac{2N_c N_f}{\pi^2}\left(\mu_q/T\right):$$



$$\chi_4^q = \frac{1}{VT^3}\left(\langle(\delta N_q)^4\rangle - 3\langle(\delta N_q)^2\rangle^2\right) \rightarrow \frac{2N_c N_f}{\pi^2}:$$



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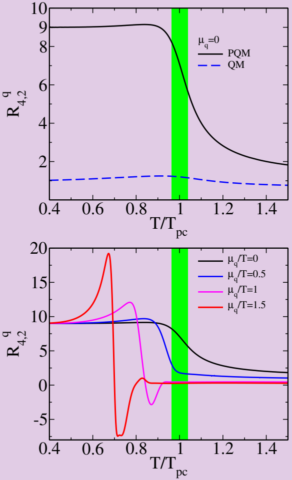
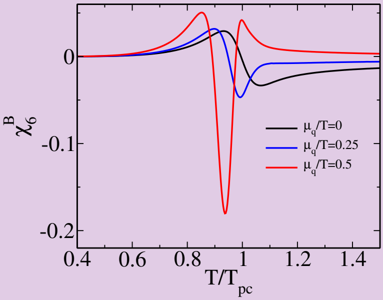
- χ_2^q: non-monotonic structure (diverges at CEP)
- χ_3^q: non-negative in range of μ_q/T considered
  (c.f. MF calculations, M. Asakawa et. al. '09)
- χ_4^q: negative for nonzero μ_q

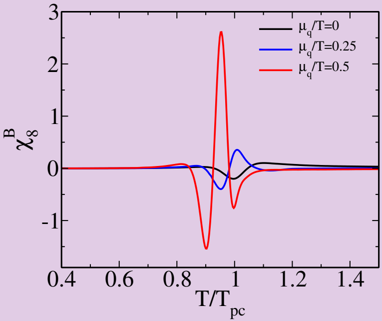
Kurtosis of net-quark number density

Kurtosis $R^q_{4,2} = \frac{\chi^q_4}{\chi^q_2} = \frac{\langle(\delta N_q)^4\rangle}{\langle(\delta N_q)^2\rangle} - 3\langle(\delta N_q)^2\rangle$ (S. Ejiri, F. Karsch, and K. Redlich '05):
quark content of effective degrees of freedom that carry baryon number

- **Low temperature phase**: dominance of effective three-quark states:
  $P_{\text{baryons}}/T^4 \approx \sum_i F(m_i/T)\cosh(3\mu_q/T)$
  $\leadsto R^q_{4,2} = 9$
- **High-temperature phase**:
  $P_{q\bar{q}}/T^4 \approx N_f N_c \left[\frac{1}{12\pi^2}\left(\frac{\mu_q}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu_q}{T}\right)^2 + \frac{7\pi^2}{180}\right]$
  $\leadsto R^q_{4,2} = (6/\pi^2) \approx 1$
- *PQM: statistical confinement*
- $m_\pi = 0$:
  - $\mu_q = 0$: kurtosis is finite
  - $\mu_q \neq 0$: kurtosis diverges
    $R^q_{4,2} \sim \left(\frac{\mu_q}{T}\right)^4 / t^{2+\alpha}$ ($t \propto$ distance to chiral critical line)
- $m_\pi \neq 0$: extrema scale as $\propto 1/m_\pi^{(2+\alpha)/\beta\delta} \approx 1/m_\pi$.



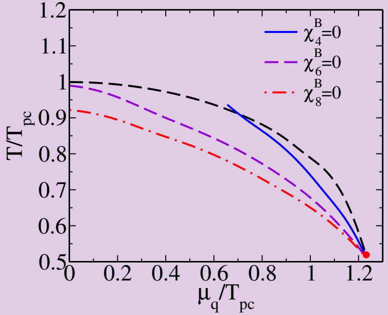




- Negative also at $\mu_q = 0$
- Temperature range of negative cumulants correlates with crossover temperature

Zeros of baryon cumulants

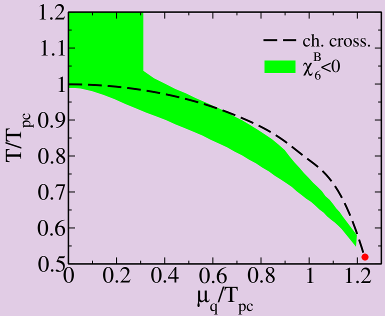
First zero (closest to hadronic phase) of χ_n^B:

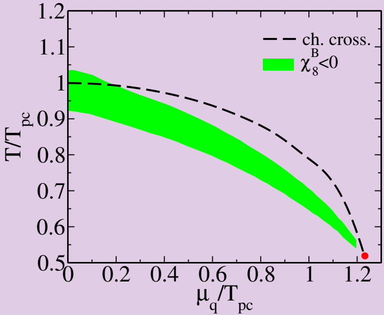


- HRG: $\chi_n^B > 0$
- PQM: $\chi_4^B > 0$ for small μ_q/T
- Zeros of χ_4^B close to CEP: 3d Ising model scaling constraints on singular part (M. Stephanov '11)
- Zeros of high-order cumulants at $\mu_q \approx 0$: close to crossover temperature

High-order baryon cumulant

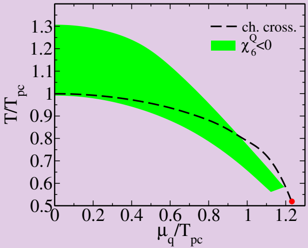
Temperature interval of negative cumulants closest to hadronic phase:



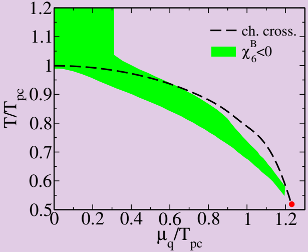


- Negative values (in broken phase!) of high-order cumulants: indicates proximity of freeze-out to crossover
- Accessible experimentally

Electric charge:



Baryon charge:



Electric charge fluctuations follow similar pattern as baryon fluctuations

Conclusions

- χ_6^B and χ_8^B are promising probes of chiral crossover in HIC at LHC and RHIC
- χ_6^B and χ_8^B cumulants are negative close to chiral crossover
- χ_4^B (and kurtosis) is negative close to chiral crossover at finite chemical potential
- Chiral crossover $\rightsquigarrow$ qualitative differences to HRG

Thank you for attention

Backup slides

Functional renormalization group

The general flow equation for the effective action

$$\partial_k \Gamma_k[\Phi,\psi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_{kB}\left(\Gamma_k^{(2,0)}[\Phi,\psi] + R_{kB}\right)^{-1}\right\} - \mathrm{Tr}\left\{\partial_k R_{kF}\left(\Gamma_k^{(0,2)}[\Phi,\psi] + R_{kF}\right)^{-1}\right\}$$

The flow equation for the PQM model

$$\partial_k \Omega(k, \rho \equiv \frac{1}{2}[\sigma^2 + \boldsymbol{\pi}^2]) = \frac{k^4}{12\pi^2}\left\{\frac{3}{E_\pi}\left[1 + 2n_B(E_\pi; T)\right] + \frac{1}{E_\sigma}\left[1 + 2n_B(E_\sigma; T)\right] - \frac{4N_f N_c}{E_q}\left[1 - N(\ell,\ell^*; T,\mu_q) - \bar{N}(\ell,\ell^*; T,\mu_q)\right]\right\}$$

$n_B(E;T)$ is the boson distribution functions
$N(\ell,\ell^*;T,\mu_q)$ are fermion distribution function modified owing to coupling to gluons
E_σ and E_π are the functions of k, $\partial\Omega/\partial\rho$ and $\rho\partial^2\Omega/\partial\rho^2$
$E_q = \sqrt{k^2 + 2g\rho}$

FRG defines $\Omega(k, \rho; T, \mu_Q, \mu_B)$.
Physically relevant quantity is the thermodynamical potential
$\bar{\Omega}(T, \mu_Q, \mu_B) \equiv \Omega(k \to 0, \rho \to \rho_0; T, \mu_Q, \mu_B)$, where ρ_0 is the minimum of Ω.

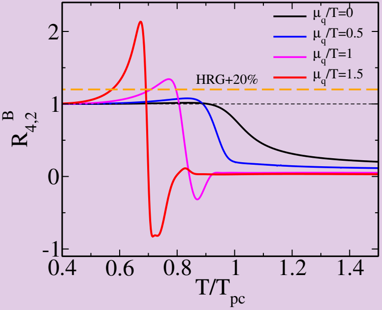
- thermodynamical limit
- assumption of full equilibrium
- survival of fluctuations after chemical freeze-out (mainly affects p_T-fluctuations)
- at low collisional energies:
  baryon and electric charge fluctuations constrained by conservation laws

Deviation from HRG

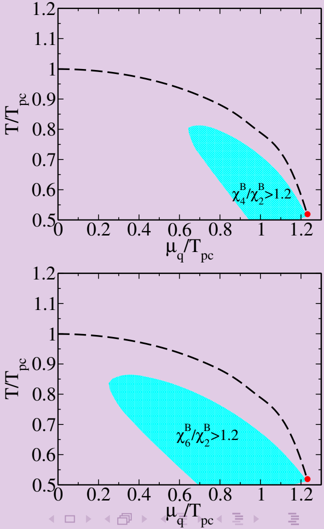
- HRG:

$$R^B_{4,2} \equiv \chi^B_4/\chi^B_2 = 1$$
$$R^B_{6,2} \equiv \chi^B_6/\chi^B_2 = 1$$

- The FRG PQM model:

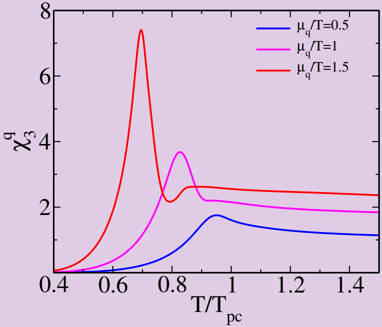


20% increase of cumulant ratios over HRG

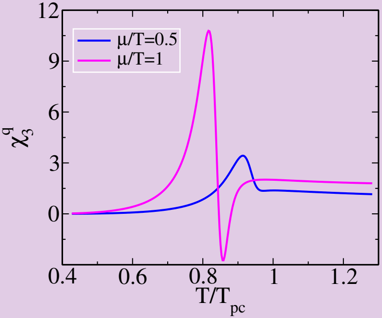


3D ORDER CUMULANT

FRG:



MF:



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