Universität Bielefeld

Electrical Conductivity and Thermal Dilepton Rate from Quenched Lattice QCD

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Quark Matter 2011
Annecy
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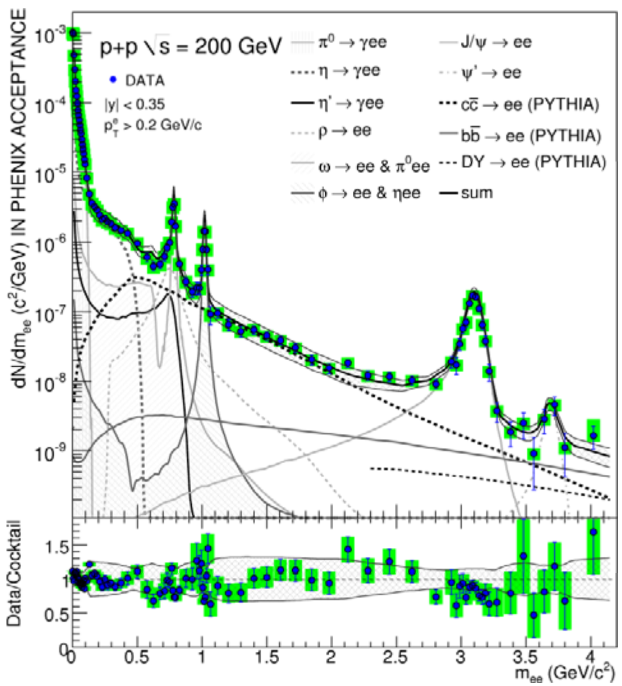
Motivation – PHENIX results for the dilepton rates

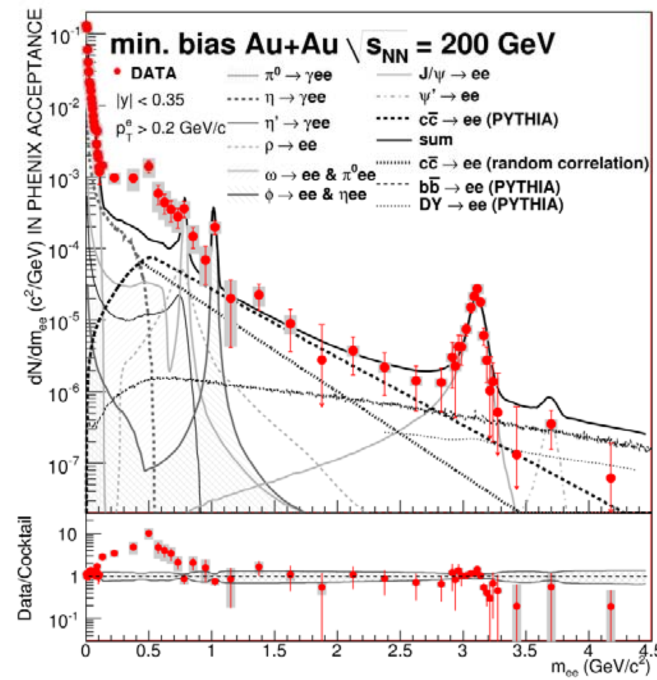
pp-data well understood by hadronic cocktail

large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP$\rightarrow$ spectral functions from lattice QCD





[PHENIX PRC81, 034911 (2010)]

Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\;\rho_{\mathbf{V}}(\omega,\vec{\mathbf{p}},\mathbf{T})$$

Outline

Introduction

Vector correlation function on the lattice

- **Temporal correlators vs. free correlators**
- **Volume and cut-off dependence**
- **Thermal moments of the spectral function**
- **Continuum extrapolation**

Electrical conductivity and dilepton rates at $T \approx 1.5\, T_c$

- **Using an Ansatz for the spectral function**
  - **in the continuum**

Conclusions & Outlook

Vector correlation functions at high temperature

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

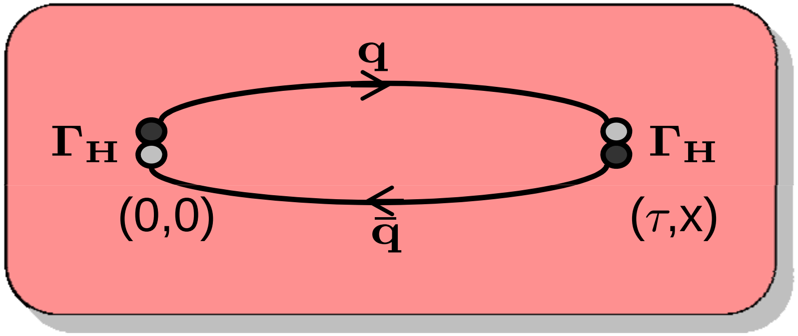
$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x})$$

← local, non-conserved current, needs to be renormalized

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x}) e^{i\vec{p}\vec{x}}$$

← only $\vec{p} = 0$ used here



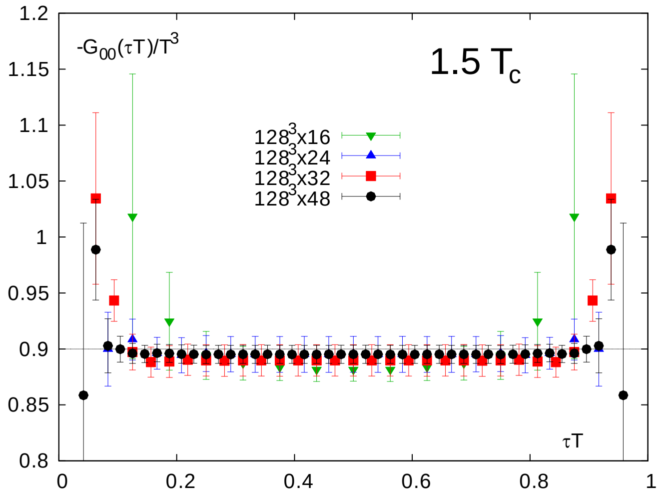
Vector correlation functions at high temperature

$$G_V(\tau,\vec{p}) \quad = \quad -G_{00}(\tau,\vec{p}) + G_{ii}(\tau,\vec{p})$$

J_0 is a conserved current

→ G_{00} is τ independent at p=0 $\sim$ quark number susceptibility χ_q

→ $$G_{00}(\tau,\vec{p}=0) \quad \equiv \quad \chi_q T + \mathcal{O}(a^2)$$



ratios of correlators free of renormalization ambiguities:

$$R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)} \quad ; \quad R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau) G_V^{free}(\tau)}$$

Spectral functions at high temperature

Free theory:

free non-interacting vector spectral function (infinite temperature):

$$\begin{aligned}\rho_{00}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) \\ \rho_{ii}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\, \omega^2 \tanh(\omega/4T)\end{aligned}$$

δ-functions exactly cancel in $\rho_V(\omega)=-\rho_{oo}(\omega)+\rho_{ii}(\omega)$

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δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{oo}(\omega)$+$\rho_{ii}(\omega)$

With interactions:

while ρ_{oo} is protected, the δ-funtion in ρ_{ii} gets smeared:

$\kappa = \frac{\alpha_s}{\pi}$ at leading order

Ansatz:

$$\begin{aligned} \rho_{00}(\omega) &= 2\pi \chi_q \omega \delta(\omega) \\ \rho_{ii}(\omega) &= 2\chi_q c_{BW} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T) \end{aligned}$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

Electrical Conductivity ⟷ slope of spectral function at ω=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

$$C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2 \;=\; \begin{array}{lll} 5/9\ e^2 & \text{for} & n_f = 2 \\ 6/9\ e^2 & \text{for} & n_f = 3 \end{array}$$

Using our Ansatz for $\rho_{ii}(\omega)$:

$$\frac{\sigma}{T} = \frac{2}{3}\,\frac{\chi_q}{T^2}\,\frac{T}{\Gamma}\,c_{BW}\;C_{em}$$

previous studies using staggered fermions (need to distinguish ρ_{even} and ρ_{odd}):

S.Gupta, PLB 597 (2004) 57: N_τ=8-14, $N_\sigma \leq$ 44
G.Aarts et al., PRL 99 (2007) 022002: N_τ=16,24, N_σ=64

Vector correlation function on large & fine lattices

Quenched SU(3) gauge configurations at T/T_c=1.5 (separated by 500 updates)

Lattice size $N_\sigma^3\, N_\tau$ with N_σ = 32 – 128
N_τ = 16, 24, 32, 48

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to the chiral limit, $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

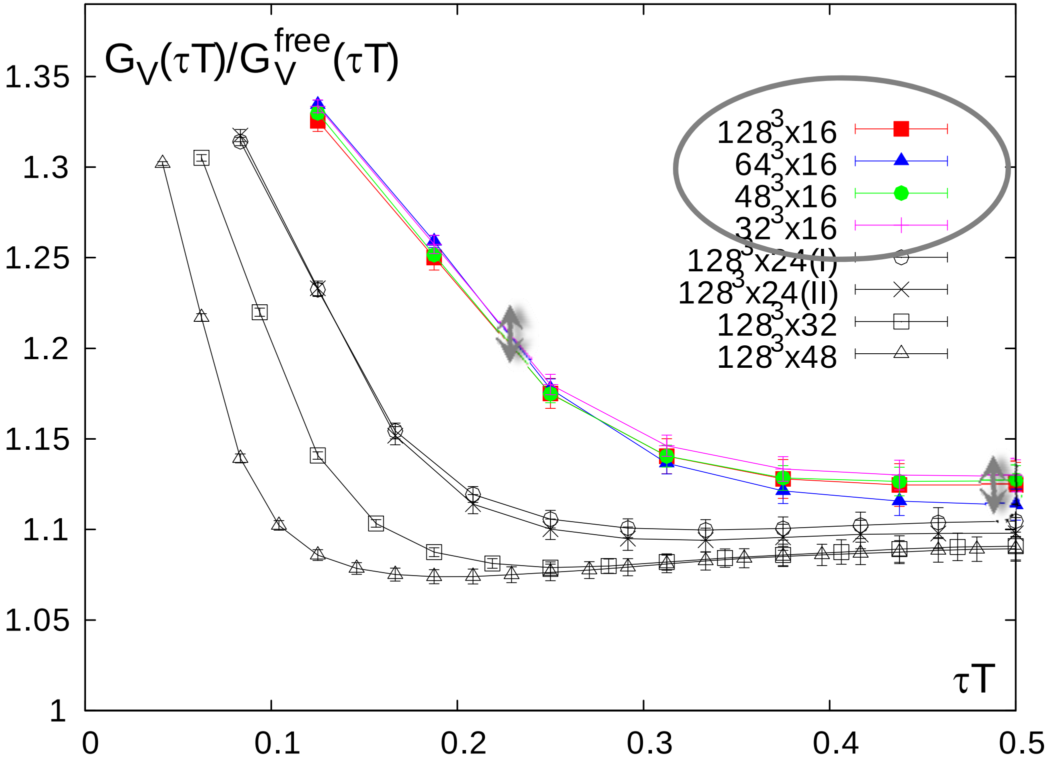
Volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	$1/a$[GeV]	a[fm]	#conf
16	32	6.872	1.4124	0.13495	0.829	6.43	0.031	60
16	48	6.872	1.4124	0.13495	0.829	6.43	0.031	62
16	64	6.872	1.4124	0.13495	0.829	6.43	0.031	77
16	128	6.872	1.4124	0.13495	0.829	6.43	0.031	129
24	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	19.30	0.010	431

cut-off dependence & continuum extrapolation

close to continuum

Vector Correlation Function – volume & cut-off dependence



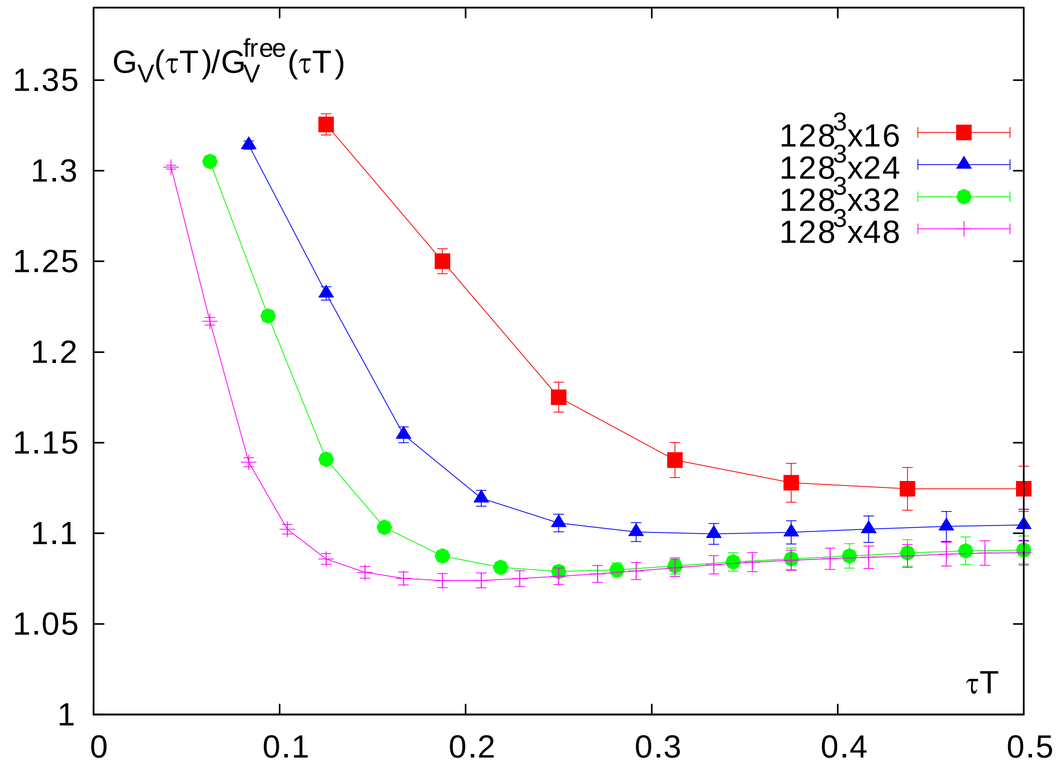
only small volume dependence

Vector correlation function normalized by free (non-interacting) correlator

analytic in the continuum (for massless quarks):

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

Vector Correlation Function – volume & cut-off dependence



only small volume dependence

fix lattice volume $V=128^3$

vary N_t at fixed $T=1.5T_c$

cut-off effects are more severe

large N_t needed

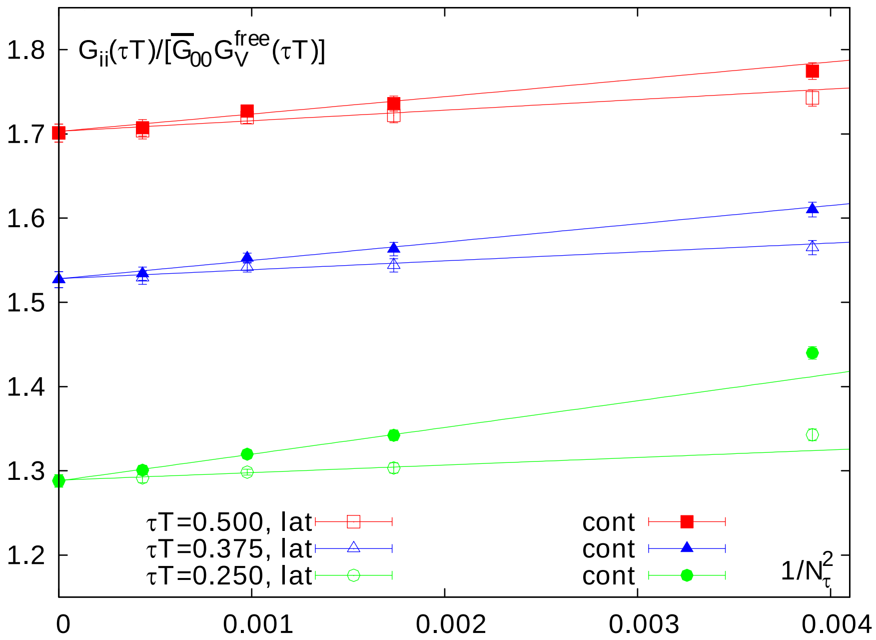
continuum extrapolation

$$G_V(\tau,\vec{p}) \quad = \quad -G_{00}(\tau,\vec{p}) + G_{ii}(\tau,\vec{p})$$

incomplete cancelation between $G_{00}(\tau T)$ and BW-contribution to $G_{ii}(\tau T)$?

→ **information on transport properties?**

Vector Correlation Function – continuum extrapolation



τT	$\dfrac{G_{ii}(\tau,T)}{\bar{G}_{00}G_V^{free}(\tau,T)}$
0.5	1.701(11)
0.375	1.527(9)
0.25	1.288(7)

normalization with $\overline{G}_{00} \equiv \chi_q/T^2$ defines renormalization invariant quantity

extrapolation in $(aT)^2=(1/N_\tau)^2$ under control for $\tau T \geq 0.2$

extrapolation is independent of the chosen normalization

extrapolation at other values use spline interpolation at fixed cut-off

→ **well defined continuum extrapolated values for this ratio**

Vector Correlation Function – curvature at $\tau T = 1/2$

Taylor expansion around the mid-point $\tau T = 1/2$:

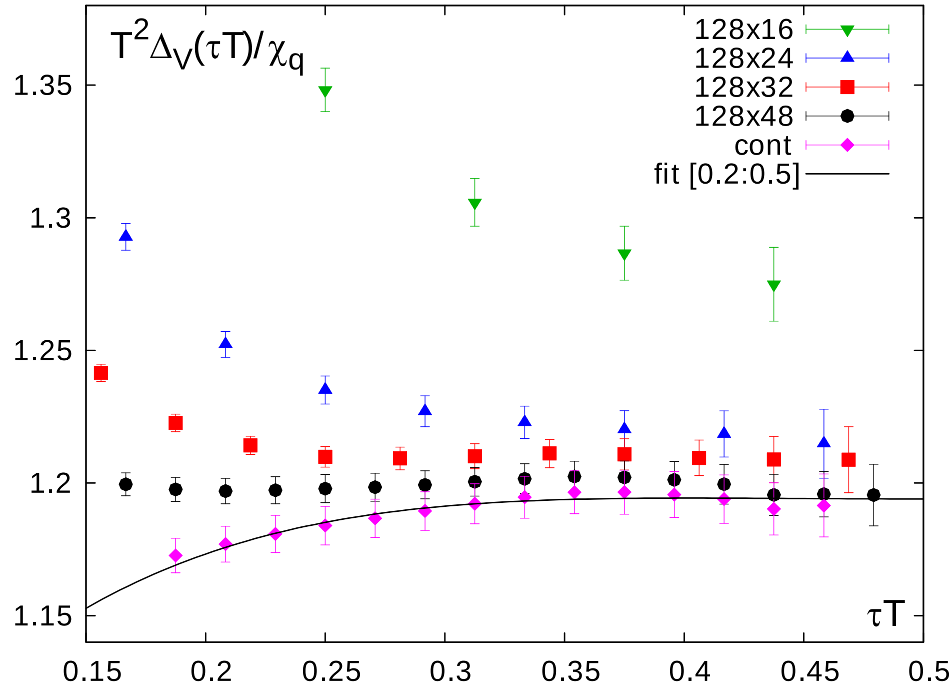
Thermal moments of the spectral function

$$G_V(\tau T) = G_V^{(0)} \sum_{n=0}^{\infty} \frac{G_V^{(2n)}}{G_V^{(0)}} \left(\frac{1}{2} - \tau T\right)^{(2n)}$$

$$G_V^{(n)} = \frac{1}{n!} \frac{\mathrm{d}G_V(\tau T)}{\mathrm{d}(\tau T)^n}\bigg|_{\tau T=1/2} = \frac{1}{n!}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left(\frac{\omega}{T}\right)^n \frac{\rho_V(\omega)}{\sinh(\omega/2T)}$$

Ratio of midpoint subtracted correlator:

$$\Delta(\tau T) = \frac{G_V(\tau T) - G_V(1/2)}{G_V^{free}(\tau T) - G_V^{free}(1/2)} = \frac{G_V^{(2)}}{G_V^{(2),free}} \times \left(1 + (R_V^{(4,2)} - R_{V,free}^{(4,2)})\left(\frac{1}{2} - \tau T\right)^2 + \ldots\right)$$



Vector Correlation Function – curvature at $\tau T = 1/2$

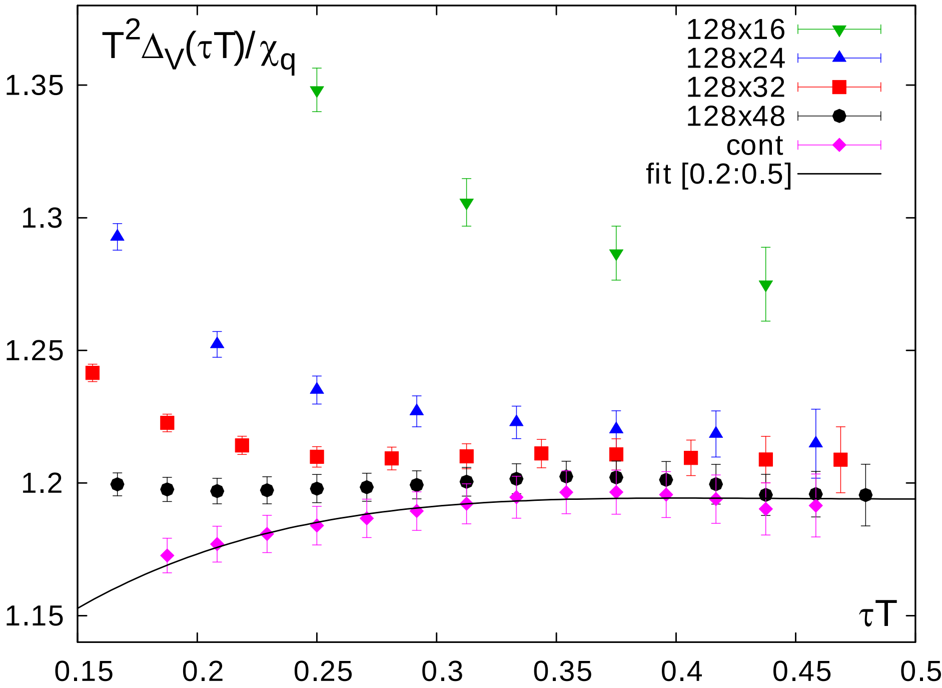
Quadratic fit to $\mathbf{\Delta(\tau T)}$ with varying lower fit range $[(\tau\mathbf{T})_{\mathbf{min}}, \mathbf{1/2}]$

$$G_V^{(0),free} = 2T^3$$
$$G_V^{(2),free} = \frac{28\pi^2}{5}T^3$$
$$G_V^{(4),free} = \frac{124\pi^4}{21}T^3$$

$$\frac{G_V^{(0)}}{G_V^{(0),free}} = 1.086 \pm 0.008$$
$$\frac{G_V^{(2)}}{G_V^{(2),free}} = 1.067 \pm 0.012$$

Ratio of midpoint subtracted correlator:

$$\Delta(\tau T) = \frac{G_V(\tau T) - G_V(1/2)}{G_V^{free}(\tau T) - G_V^{free}(1/2)} = \frac{G_V^{(2)}}{G_V^{(2),free}} \times \left(1 + (R_V^{(4,2)} - R_{V,free}^{(4,2)})\left(\frac{1}{2} - \tau T\right)^2 + \ldots\right)$$



normalized curvature close to that of the free vector correlator

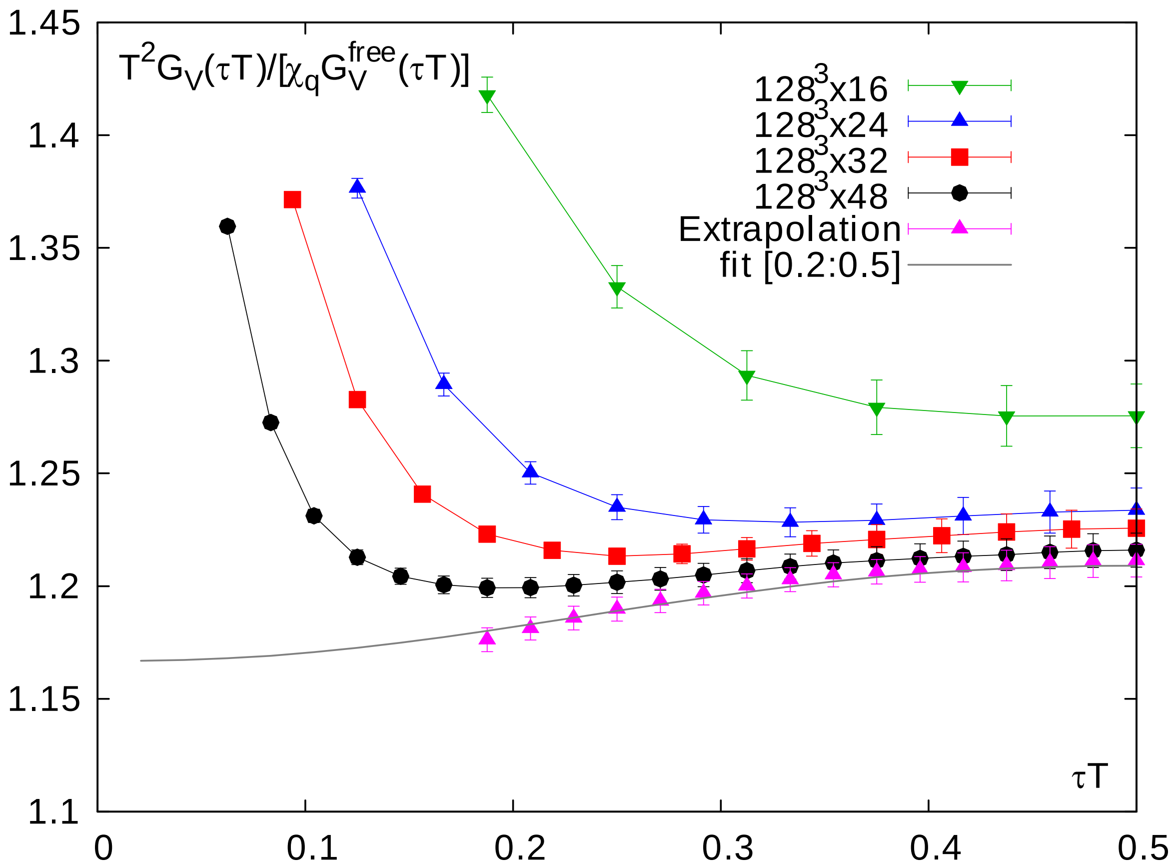
Vector Correlation Function – continuum extrapolation

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

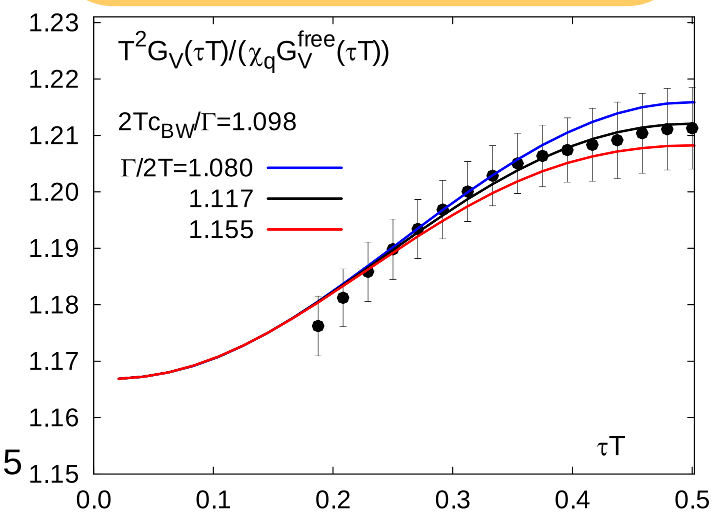
and fit to the continuum extrapolated values $\frac{\mathbf{G_V(\tau,T)}}{\mathbf{\bar{G}_{00}G_V^{free}(\tau,T)}}$ & $\mathbf{G_V^{(2)}}$



$$\frac{2C_{BW}\chi_q}{\Gamma} = 1.098(27)$$

$$\frac{\Gamma}{T} = 2.235(75)$$

$$\kappa = 0.0465(27)$$



Spectral function and electrical conductivity

Use our Ansatz for the spectral function

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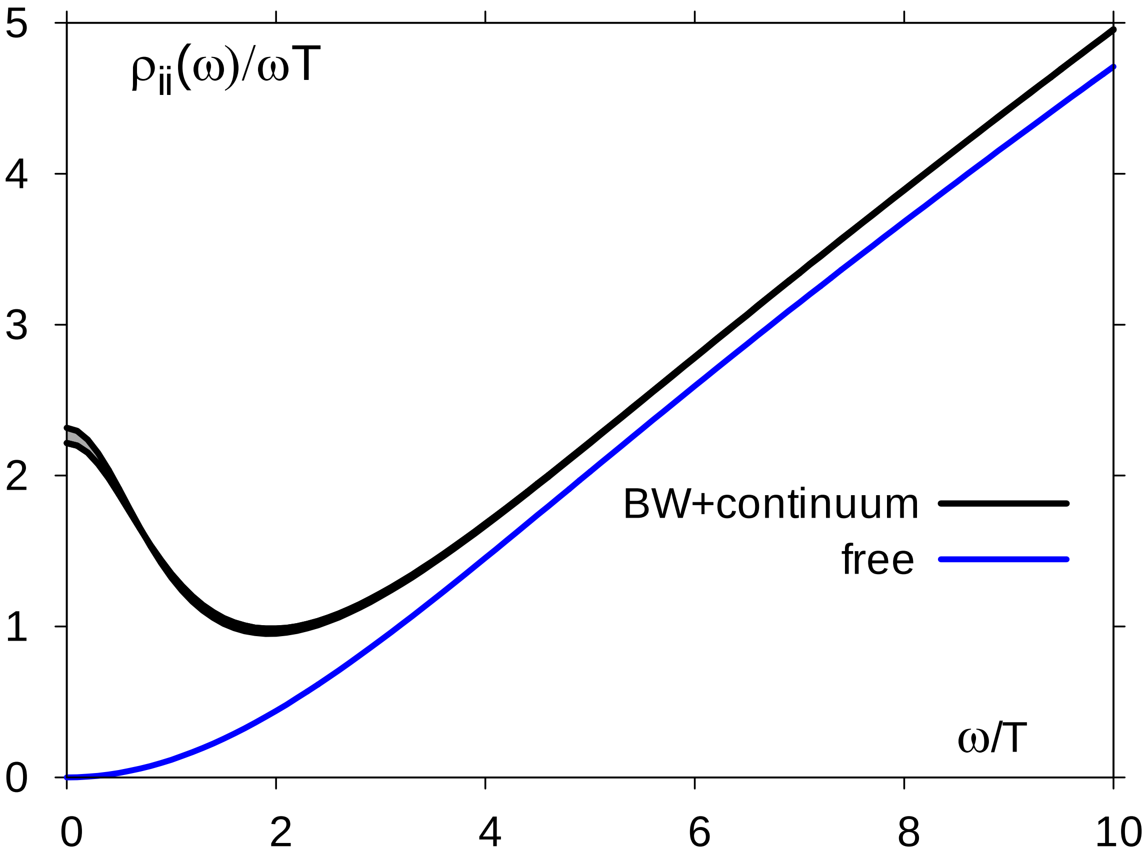
$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

shaded area by varying Γ/T within its errorband



$$\frac{2C_{BW}\chi_q}{\Gamma} = 1.098(27)$$
$$\frac{\Gamma}{T} = 2.235(75)$$
$$\kappa = 0.0465(27)$$

electrical conductivity

$$\frac{\sigma}{T} = (0.37 \pm 0.01)C_{em}$$

Spectral function and electrical conductivity

Use our Ansatz for the spectral function

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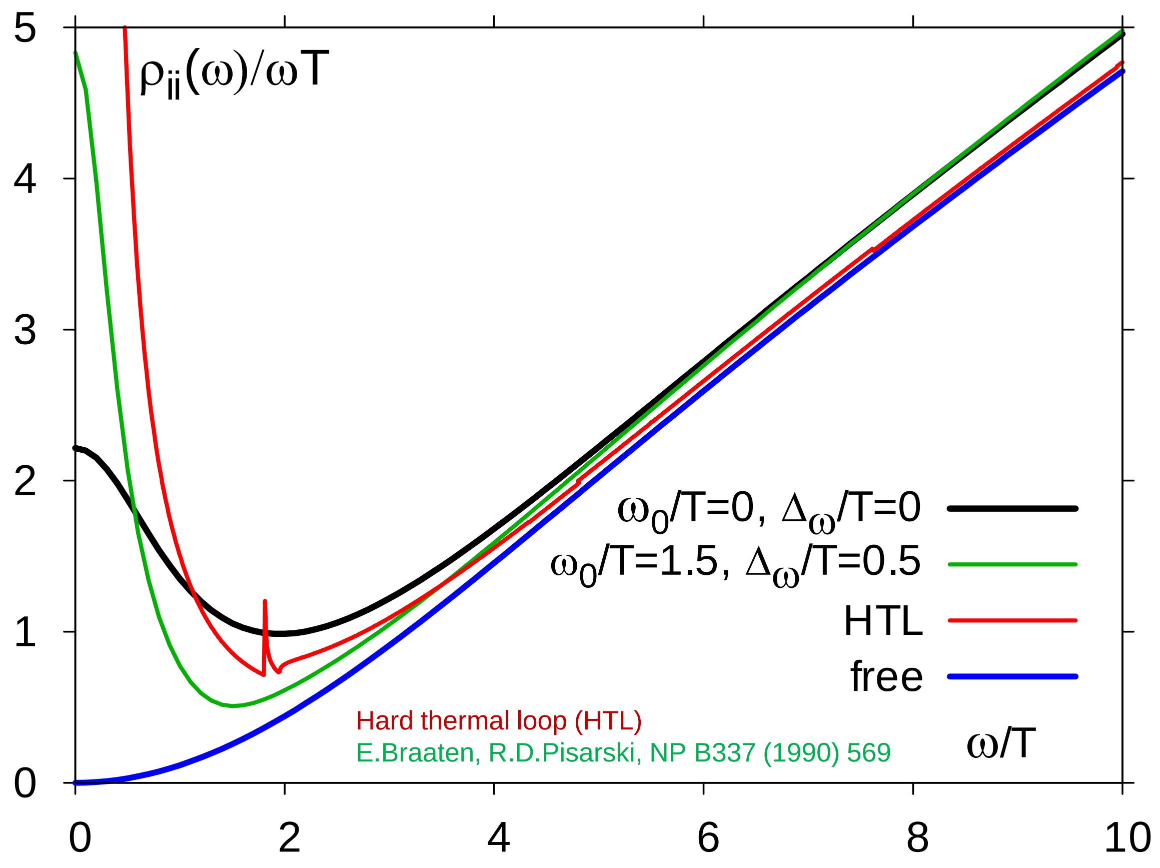
$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\;\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

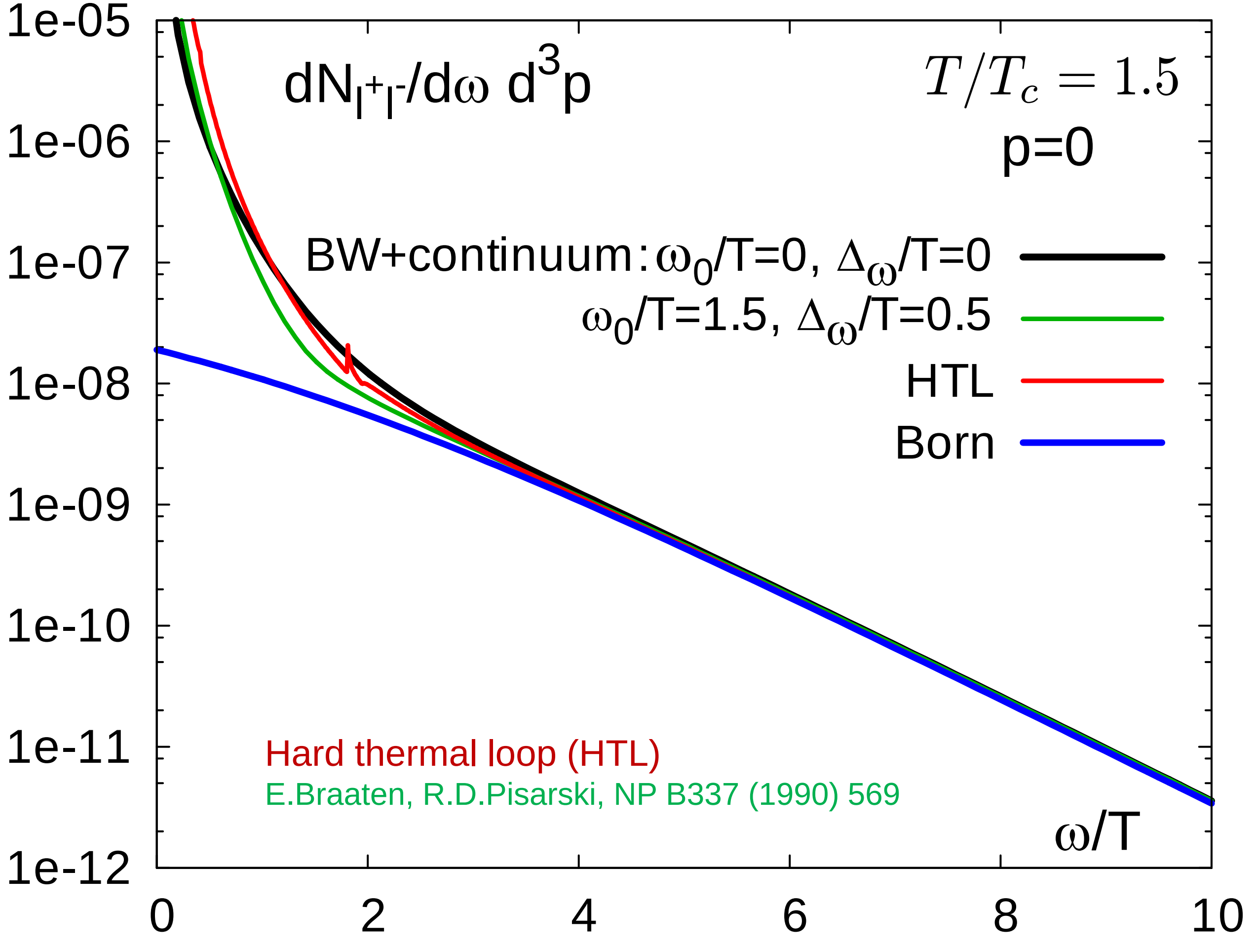
$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

electrical conductivity

$$1/3 \le \frac{1}{C_{em}}\frac{\sigma}{T} \le 1$$

Dileptonrate directly related to vector spectral function:

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\rho_V(\omega,\vec{p},T)$$



Conclusions and Outlook

Conclusions:

Detailed knowledge of the **vector correlation function** at $T=1.5T_c$ in quenched QCD

→ **continuum extrapolation** of correlation function and thermal moments

$G_V(\tau T)$ well reproduced by **Breit-Wigner plus continuum** Ansatz for $\sigma_V(\omega)$

→ **Electrical conductivity:** $$1/3 \leq \frac{1}{C_{em}} \frac{\sigma}{T} \leq 1$$

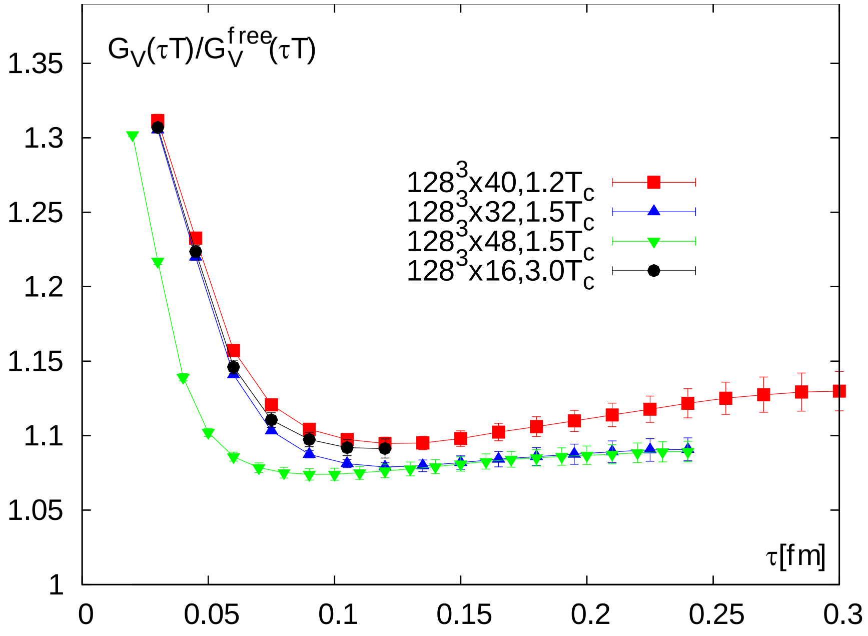
→ **Dilepton rate** approaches leading order Born rate for $\omega/T \geq 4$
enhancement at small ω/T

Outlook:

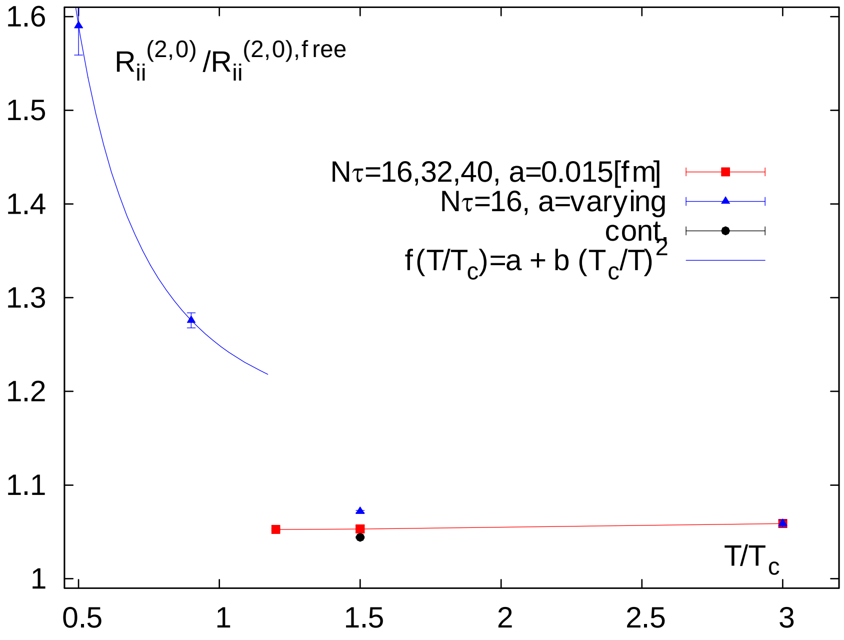
include HTL result for $\sigma_V(\omega)$ at large ω/T in the Ansatz

vector correlation function at **other temperatures** and **non-zero momentum**

Outlook



$G_V(\tau T)/G_V^{free}(\tau T)$
128^3x40,1.2T_c
128^3x32,1.5T_c
128^3x48,1.5T_c
128^3x16,3.0T_c
τ[fm]
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.05
0.1
0.15
0.2
0.25
0.3



$R_{ii}^{(2,0)}/R_{ii}^{(2,0),free}$
Nτ=16,32,40, a=0.015[fm]
Nτ=16, a=varying
cont.
$f(T/T_c)=a + b\,(T_c/T)^2$
T/T_c
1.6
1.5
1.4
1.3
1.2
1.1
1
0.5
1
1.5
2
2.5
3