Flow - theory perspective

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Flow in heavy-ion collisions

Large elliptic flow is one of the most striking observations at RHIC and now at LHC.

\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum_n (2v_n \cos(n\phi))\right)
\]

Pressure gradients evolve coordinate space anisotropy into momentum space anisotropy. Well described by hydrodynamics.
Flow in heavy-ion collisions

Focus lay on elliptic flow for long: Symmetry: odd coefficients vanish.

Interesting part of the expansion then

\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left( 1 + 2v_2 \cos(2\phi) + 2v_4 \cos(4\phi) \right)
\]

Single event: no symmetry

Prepare for a lot more \( v_n \) this year.

Interesting part of the expansion now

\[
\frac{N}{2\pi} \left( 1 + 2v_1 \cos(2(\phi - \psi_1)) + 2v_2 \cos(2(\phi - \psi_2)) \\
+ 2v_3 \cos(3(\phi - \psi_3)) + 2v_4 \cos(4(\phi - \psi_4)) \\
+ 2v_5 \cos(5(\phi - \psi_5)) + \ldots \right)
\]
Hydrodynamics: conservation laws for long wavelength modes

\[ \partial_\mu T^{\mu\nu} = 0 \]

Generally:

\[ T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu}. \]

First order Navier Stokes theory:

\[ \pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} = \eta \left( \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u^\alpha \right). \]

\[ \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \]

Second order theory:

\[ \pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} + \text{second derivatives}. \]

Full Israel-Stewart theory for a conformal fluid, BRSSS:

\[ \pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} - \tau_{\pi} \left( \frac{4}{3} \pi^{\mu\nu} \partial_\alpha u^\alpha + \Delta^\mu_\alpha \Delta^\nu_\beta u^\sigma \partial_\sigma \pi^{\alpha\beta} \right) \]

in flat space and neglecting vorticity and all terms that seem numerically unimportant

Hydrodynamics

Using the set of equations

\[ \partial_\mu T^{\mu\nu} = 0 \]

and

\[ \pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} - \tau_\pi \left( \frac{4}{3} \pi^{\mu\nu} \partial_\alpha u^\alpha + \Delta^\mu_\alpha \Delta^\nu_\beta u^\sigma \partial_\sigma \pi^{\alpha\beta} \right) \]

is now standard.

When bulk viscosity is included (non-conformal fluid)

\[ T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu} - \Pi \Delta^{\mu\nu} \]


QCD enters through the equation of state: \( P = P(\epsilon) \)

(from e.g. lattice QCD / hadron gas model, more later)
Consensus about

\[
\langle \frac{\eta}{s} \rangle \leq 0.5 \approx \frac{6}{4\pi}
\]

three then existing 2+1D viscous hydro codes agree (TECHQM effort, Dusling, Song)
What happened since QM2009?

How to make progress from here

Identified uncertainties people work on:
- Non-conformal hydrodynamics (bulk viscosity!)
- Late hadronic gas stage (more realistic freeze-out)
- Pre-Hydro stage (thermalization)
- Equation of state, transport coefficients
- 3+1d viscous hydrodynamic simulations

Identified uncertainties people do not (?) work on:
- Measuring initial state eccentricity
- initial state fluctuations & viscosity
Equation of state

Comparison of different equations of state in hydrodynamic evolution:


Equation of state

Comparison of different equations of state in hydrodynamic evolution:


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**HotQCD:** HotQCD collaboration, Phys.Rev.D80:014504 (2009)


Equation of state

Comparison of different equations of state in hydrodynamic evolution:


Equation of state

Comparison of different equations of state in hydrodynamic evolution:


Conclusion:
“Differences in the lattice EoS parametrization in the literature are small and not observable in the $p_T$-differential elliptic flow.”

Another more recent lattice equation of state:
Bulk viscosity

Small effect compared to shear viscosity.
Especially in “critical slowing down” scenario, where relaxation time $\tau_\Pi$ peaks near $T_c$ like $\zeta/s$.


However, uncertainty in the value of $\zeta/s(T)$ and its initial value.
$\zeta/s$ could be larger than what stable hydrodynamic evolution allows.

also see
G. Denicol, T. Kodama, T. Koide, Ph. Mota, Phys.Rev.C80, 064901 (2009) find 15% reduction of $v_2$ at $p_T < 1$ GeV, no $\delta f$
Bulk viscosity, cavitation

But, cavitation can happen early:

\[ \tau \]

\[ \Phi = \text{shear stress} \]

\[ \Pi = \text{bulk stress} \]

\[ P_\xi = \text{longitudinal pressure} \]

1+1D hydrodynamics using \( \eta/s = 1/4\pi \) and bulk viscosity peaked at \( T_c \) as from lattice QCD without quarks.


Longitudinal pressure becomes negative, leading to cavitation (breakup into droplets).


Cavitation also by large (temperature dependent) \( \eta/s \) at LHC energies.

J.R. Bhatt, H. Mishra, V. Sreekanth, arXiv:1103.4333
Beyond 2\textsuperscript{nd} order viscous hydrodynamics

Rapid expansion $\rightarrow$ large momentum space anisotropies $\rightarrow$ shear $\geq$ isotropic pressure.
Breakdown of expansion in terms of shear corrections.

New developments: \textit{Hydrodynamics for highly anisotropic systems}.

Hydrodynamic expansion around anisotropic mom. dist. function.
$\rightarrow$ New evolution equations

- by requiring energy-momentum conservation
  and using an ansatz for an entropy source

  see poster by R. Ryblewski

- by taking moments of the Boltzmann equation

  see poster by M. Martinez

One formalism for anisotropic early time dynamics and late time near-equilibrium dynamics (right limits).
Expand entropy current to third order in the shear stress tensor.

Obtain new third order evolution equations.

Numerical solution here is for 1+1D boost invariant expansion:

\[
\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3} \frac{\pi}{\tau} + \frac{8}{27} \frac{\epsilon}{\tau} - \frac{3}{\epsilon \tau} \frac{\pi^2}{\tau}
\]

2nd order Israel Stewart

All orders approximation:

\[
\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3} \frac{\pi}{\tau} + \frac{8}{27} \frac{\epsilon}{\tau} - \frac{x \pi^2}{\epsilon \tau}
\]

Points are BAMPS parton cascade

see D. Molnar, P. Huovinen, Phys.Rev.C79:014906 (2009) for similar comparison of IS to transport
Derivation of 2\textsuperscript{nd} order relativistic viscous hydrodynamics

Derivation of 2\textsuperscript{nd} order dissipative relativistic hydro depends on method.

- IS: use 2\textsuperscript{nd} moment of Boltzmann equation
- New method: use definition of dissipative currents directly

→ new equations of the same form but with different coefficients.

New equations agree better with numerical solution of the Boltzmann equation.

Important when matching to kinetic theory at late times.


Points are BAMPS parton cascade
Viscous correction to particle distributions

Translating dissipative $T^{\mu\nu}$ to particles: corrections to the distribution function:

$$T^{\mu\nu}_{\text{hydro}} = \sum_{n=1}^{N} d_n \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{E_n} (f_{0n} + \delta f_n)$$

for an $N$ component system.

The ansatz

$$\delta f_n = \frac{C_n}{2T^3} f_0 (1 \pm f_0) \hat{p}^\alpha \hat{p}^\beta \chi(p) \frac{\pi_{\alpha\beta}}{\eta}$$

leaves uncertainties.

Usual procedure: Assume all $C_n$ to be equal, use $\chi(p) = p^2$:

$$\delta f_n = f_{0n} (1 \pm f_{0n}) p^\alpha p^\beta \pi_{\alpha\beta} \frac{1}{2(\epsilon + \mathcal{P})T^2} \quad \forall \ n$$

see talk by D. Molnar on problem with equal $C_n$
Viscous correction to particle distributions

Different underlying theories give different $\chi(p)$.


$\chi(p) \sim p^2$ is for a relaxation time approximation + Boltzmann approximation (very simple model).

Hadron gas, weakly coupled QCD: $\chi(p)$ between $\sim p$ and $\sim p^2$.

Uncertainty in $\delta f$ introduces large uncertainty in $v_2$ for $p_T > 1$ GeV.

Analysis of experimental data in
R. Lacey, A. Taranenko, R. Wei, N.N. Ajitanand, J.M. Alexander, J. Jia, R. Pak, D. Rischke, D. Teaney, K. Dusling

$\delta f$ large when $\eta/s(T)$ and freezing out late in dilute hadronic phase.
Combining hydrodynamic evolution with microscopic hadronic transport models.


Recent developments:

- **Ideal hydro + JAM:** T. Hirano, P. Huovinen, Y. Nara., Phys.Rev. C83 021902 (2011) and arXiv:1012.3955
Late hadronic gas stage: Afterburner

Why use an afterburner?
Can handle large dissipative corrections in dilute hadronic stage.

How is flow affected?

![Diagram showing $v_2$ vs $p_T$ for various models: pure hydro, viscous hydro + UrQMD, viscous hydro + resonant decay at $T_{sw}=165$ MeV, and viscous hydro + resonant decay at $T_{dec}=100$ MeV.]

Smaller $v_2$ than pure hydro with chem. freeze-out due to larger viscosity.

$p_T$-integrated $v_2$ gives most direct access to $\eta/s$.
Distribution of total generated mom. anisotropy among hadrons and $p_T$ depends on chem. composition and $p_T$-distributions.


find $0.08 < (\eta/s)_{QGP} \lesssim 0.2$ for $T_c < T < 2T_c$
with major uncertainty from initial conditions.
Beyond constant $\eta/s$

Determine dependence of $v_2$ on modeled $\eta/s(T)$.
L=low, H="high"
H=hadronic phase, Q=QGP

H. Niemi at al, arXiv:1101.2442
see talk by H. Niemi

Weak dependence on QGP $\eta/s(T)$ at RHIC. Dependent on minimum.
Different at LHC energies (longer QGP phase, smaller gradients in the hadronic phase)
Beyond constant $\eta/s$

**Strong dependence on the initial $\pi_0^{\mu\nu}$**

by influencing the initial effective pressure profile.

Need better understanding of the pre-thermal evolution and its matching to viscous hydrodynamics.


Navier Stokes:

$$\eta_0^{\mu\nu} = \eta_0 (\nabla^\mu u_0^\nu + \nabla^\nu u_0^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u_0^\alpha)$$

see talk by U. Heinz/C. Shen
Real event-by-event hydrodynamics

... as opposed to using an average over MC-Glauber or MC-KLN initial conditions (which includes only part of the effect of fluctuations)

Z. Qiu, U. Heinz, arXiv:1104.0650 2+1D viscous hydro

average, then evolve

evolve, then average

initial energy density

initial energy density
Real event-by-event hydrodynamics

... as opposed to using an average over MC-Glauber or MC-KLN initial conditions (which includes only part of the effect of fluctuations)

Z. Qiu, U. Heinz, arXiv:1104.0650 2+1D viscous hydro

also see talks by:
F. Grassi
H. Holopainen
S. Jeon
P. Mota
H. Petersen

...
Most central collisions: fluctuations **increase** elliptic flow.

Larger centralities: fluctuations **decrease** elliptic flow.

Flow results from e-b-e 3+1D viscous hydro

Why?
- Measure with respect to the event-plane:
  Maximizes flow.
  (also included if one averages and then evolves)
- Competing effect:
  Substructure of hotspots

Flow results from e-b-e 3+1D viscous hydro

Now add shear viscosity.
\[ \eta/s = 0.08 \text{ and } \eta/s = 0.16 \]

Flow results from e-b-e 3+1D viscous hydro

**Flow results from e-b-e 3+1D viscous hydro**


Another prediction using an average initial distribution with trianlarity and 2+1D viscous hydro.
Fluctuations and viscosity

For the first time we can see the effect of viscosity in event-by-event simulations! after 6 fm/c

initial \hspace{1cm} ideal \hspace{1cm} viscous

$\tau=0.4 \text{ fm/c}$ \hspace{1cm} $\tau=6.0 \text{ fm/c, ideal}$ \hspace{1cm} $\tau=6.0 \text{ fm/c, } \eta/s=0.16$

energy density in the transverse plane

A lot of progress has been made!

How to make progress from here

Identified uncertainties people work on:
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- Equation of state, transport coefficients
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Identified uncertainties people do not (?) work on:
- Measuring initial state eccentricity
- Initial state fluctuations & viscosity

Björn Schenke (BNL)
Flow at LHC

Many LHC pre- and postdictions using hydro simulations are available. Here they are (mostly) in order of appearance.

2+1D viscous hydro

comparing to ALICE data
2 upper curves are calculations with correct cuts

No significant change of $\eta/s$ from RHIC to LHC
Using MC-KLN initial conditions leaves room for viscous corrections in the QGP, MC-Glauber not so much. 

MC-KLN result hints at larger effective viscosity at LHC.
Flow at LHC

2+1D (shear+bulk) viscous hydro coupled to THERMINATOR


![Graph showing v2 vs p⊥ for Pb-Pb collisions at 2.76 TeV ALICE Data]

see talk by P. Bożek

Glauber initial conditions.
No significant increase of the effective $\eta/s$ at LHC.
2+1D (shear+bulk) viscous hydro coupled to THERMINATOR

Almost all models under-predict $v_2(p_T)$ for $p_T < 800$ MeV.

Possible explanation: contribution from non-thermalized particles from jet-fragmentation.
Assume constant jet $v_2$ and $v_2(p_T) = (1 - g(p_T))v_2^{\text{hydro}}(p_T) + g(p_T)v_2^{\text{jet}}(p_T)$, where $g(p_T)$ denotes the proportion of particles from jet fragmentation.

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see talk by P. Božek

see poster by B. Tomasik

for other model of hard parton contribution

B. Tomasik, P. Levai, arXiv:1104.3262
Flow at LHC

2+1D viscous hydro coupled to UrQMD hadron cascade (VISHNU)


Result indicates an increase of the effective $\eta/s$ at LHC.
However, $\eta/s(T)$ should be used.
Remember issue with initialization of $\pi^{\mu\nu}$.
Flow at LHC

2+1D viscous hydrodynamics

Glauber model initial conditions

No increase of the effective $\eta/s$ at LHC. Best fit: $\eta/s = 0.06 \pm 0.02$

see posters by A. Chaudhuri
3+1D event-by-event viscous hydrodynamics
B. Schenke, S. Jeon, C. Gale, arXiv:1102.0575

MC-Glauber initial conditions

more in S. Jeon’s talk

$\nu_3$-prediction

Alice collaboration, arXiv:1105.3865
Using higher harmonics $v_n$ in addition to $v_2$ will give a better handle on initial conditions and $\eta/s$
**Other recent developments**

 קיבל חדש ליניק

- **Dissipative hydro for multi-component systems.**

  Effective shear viscosity of mixture depends on partial shear pressures of components.

  see poster by A. El  

- **Thermal fluct. in sound and shear waves contribute to viscosity.**

  Important when microscopic $\eta/s = 0.08$ (breakdown of 2nd order visc. hydro)
  negligible when $0.16$  

- **Anisotropic flow far from equilibrium.**

  In the limit of a small number of collisions the solution of the Boltzmann equation leads to sizable elliptic flow.

  see talk by N. Borghini  

- **Shear viscosity from parton cascade with 2 ↔ 3 processes:**

  $\eta/s = 0.13 - 0.16$, also elliptic flow computation


  see talk by O.Fochler and poster by I. Bouras

- **Flow in multiphase transport model.**

A lot of progress has been made in the last 2 years including 3+1D viscous event-by-event hydro. Improvements (will) allow for more sophisticated extraction of medium properties and initial conditions. Hydrodynamics is a useful tool to study the bulk dynamics also at LHC. First results from many groups: medium at LHC similar to that at RHIC. More information from odd $v_n$. Different dependence on initial state/fluctuations than $v_2$. Real event-by-event viscous hydro could make all the difference for these higher harmonics. But uncertainties remain.
How to continue the progress

- Pre-equilibrium physics! Initial conditions for viscous hydro and transition from non-equilibrium to equilibrium
- Improve transition from hydrodynamics to particles ($\delta f$)
- Determine differences between 2+1D and 3+1D viscous hydro
- Better understand temperature dep. of transport coefficients
- Finite baryon number
- Include all the improvements in a single calculation to reduce the uncertainty on $\eta/s$.
  Currently, all results hint at $\langle \frac{\eta}{s} \rangle_{\text{QGP}} \lesssim \frac{3}{4\pi}$, but there are more uncertainties.

See Matt Luzum’s talk on Thursday for more on fluctuations, odd $v_n$ and correlations.
Initialization:

- **Sample Woods-Saxon distributions** to determine all nucleon positions
- Overlap those distributions using impact parameter $b$

$$b \text{ is sampled from } P(b)db = 2bdb/(b_{\text{max}}^2 - b_{\text{min}}^2)$$

- **Nucleon-nucleon collision occurs if distance is** $< \sqrt{\sigma_{NN}/\pi}$
- At position of collision **add** 2D-Gaussian energy density **distribution** with width $\sigma_0$.

For now we use $\sigma_0 = 0.4 \text{ fm}$. 
Freeze-out surface in a single event

The freeze-out finder works great in a single event:

Event plane

Event plane is defined by the angle:

\[ \psi_n = \frac{1}{n} \arctan \frac{\langle p_T \sin(n\phi) \rangle}{\langle p_T \cos(n\phi) \rangle} \]

using particle momenta.


\[ v_n = \langle \cos(n(\phi - \psi_n)) \rangle \]

... different angle for every flow coefficient.
Flow results from e-b-e viscous MUSIC

Event-by-event fluctuations and viscosity improve agreement with experimental data.

$v_2$ and $v_3$ as functions of pseudo-rapidity $\eta_p$


Flow results from e-b-e viscous MUSIC

Higher Fourier coefficients are suppressed more by finite viscosity.

![Graph showing the relationship between $(v_3/v_2)^2$ and $<N_{part}>$ for ideal and viscous conditions with η/s = 0.08 and η/s = 0.16.](image-url)
A little more detail

Dependence on the width $\sigma_0$, used to initialize the lumpy energy density distribution.

Qualitatively: Harder spectra and larger $v_2$ for smaller $\sigma_0$.
Quantitatively: Small effect - within statistical error bars for $p_T < 2$ GeV.
More results for $v_3$

Another prediction using an average initial distribution with triangularity and 2+1D viscous hydro.


More on fluctuations in Matt Luzum’s talk on Thursday.
Numerical viscosity

- In the Lax scheme, the time derivative is expressed by

\[
\frac{d}{dt} \rho = \frac{\rho_{j+1}^{n+1} - \rho_j^n}{\Delta t} - \frac{\rho_{j+1}^{n+1} - 2\rho_j^n + \rho_{j-1}^n}{2\Delta t}
\]

to deal with instability of the simplest discretization (FTCS).

In “continuum form” the last term reads \( \frac{(\Delta x)^2}{2\Delta t} \partial_x^2 \rho \), introducing numerical viscosity.

- In KT we have

\[
\frac{d}{dt} \bar{\rho}_j(t) = - \left[ \frac{1}{2\Delta x} \right] \left[ J \left( \bar{\rho}_{j+1/2}^+(t) + \bar{\rho}_{j+1/2}^-(t) \right) - J \left( \bar{\rho}_{j-1/2}^+(t) + \bar{\rho}_{j-1/2}^-(t) \right) \right]

+ \frac{1}{2\Delta x} \left\{ a_{j+1/2}(t) \left[ \bar{\rho}_{j+1/2}^+(t) - \bar{\rho}_{j+1/2}^-(t) \right] - a_{j-1/2}(t) \left[ \bar{\rho}_{j-1/2}^+(t) - \bar{\rho}_{j-1/2}^-(t) \right] \right\}
\]

Taylor expanding \( \bar{\rho}_{j\pm 1} \) in \( \bar{\rho}_{j\pm 1/2} \) around \( \bar{\rho}_j \) shows that numerical viscosity is of order \((\Delta x)^3\).

No \( 1/\Delta t \) term!
Entropy production

Ideal hydro: \( \partial_\mu S^\mu = 0 \) with \( S^\mu = s u^\mu \Rightarrow S = \int_V dx_\perp \tau d\eta s u^\tau = \text{const} \).

Viscous hydro: physical entropy production
Glauber and fKLN reproduce $v_2$ data but with very different $\eta/s$. So which describes the initial state better?


Divide measured $v_2(b)$ by modeled fKLN or Glauber initial eccentricity. If nature is like fKLN we get the top panels. If nature is like Glauber the bottom.

Experimental $v_2/\epsilon$ decreases with $b$, if we divide by Glauber or fKLN. Conclusion of this exercise: things look better for fKLN.

dependence on EOS, what if $\eta/s$ is not assumed constant?
There have been recent calculations of a forward peaked and increased $v_1$ at LHC due to tilted and moving initial state. The effect of the initial flow velocity distribution is superseding the pressure driven expansion.


a new type of $v_1$ due to fluctuations of the initial state


Hadrons with $|\eta| < 1$
20-60% centrality
Data points extracted from STAR correlation data arXiv:1010.0690
NeXSPeRIO ideal hydro
corrected: reduced net transverse momentum in the simulation