Large-N_c behavior of the critical temperature for the chiral phase transition

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The chiral phase transition

An important phase transition of QCD is the chiral phase transition [1, 2]. Two effective models have been widely used to describe it:

- NJL-model [3]: chiral phase transition is driven by quark loops
- Linear-σ model [4]: mechanism behind the phase transition are mesonic loops

A large-N_c [5] study of the critical temperature T_c is interesting: we expect that $T_c \sim N_c^0$ (Just as the deconfinement phase transition).

$\Rightarrow$ the behavior of the NJL and σ model do **not** coincide, linear-σ model fails

$\Rightarrow$ The behavior of the linear-σ model must be improved [6].

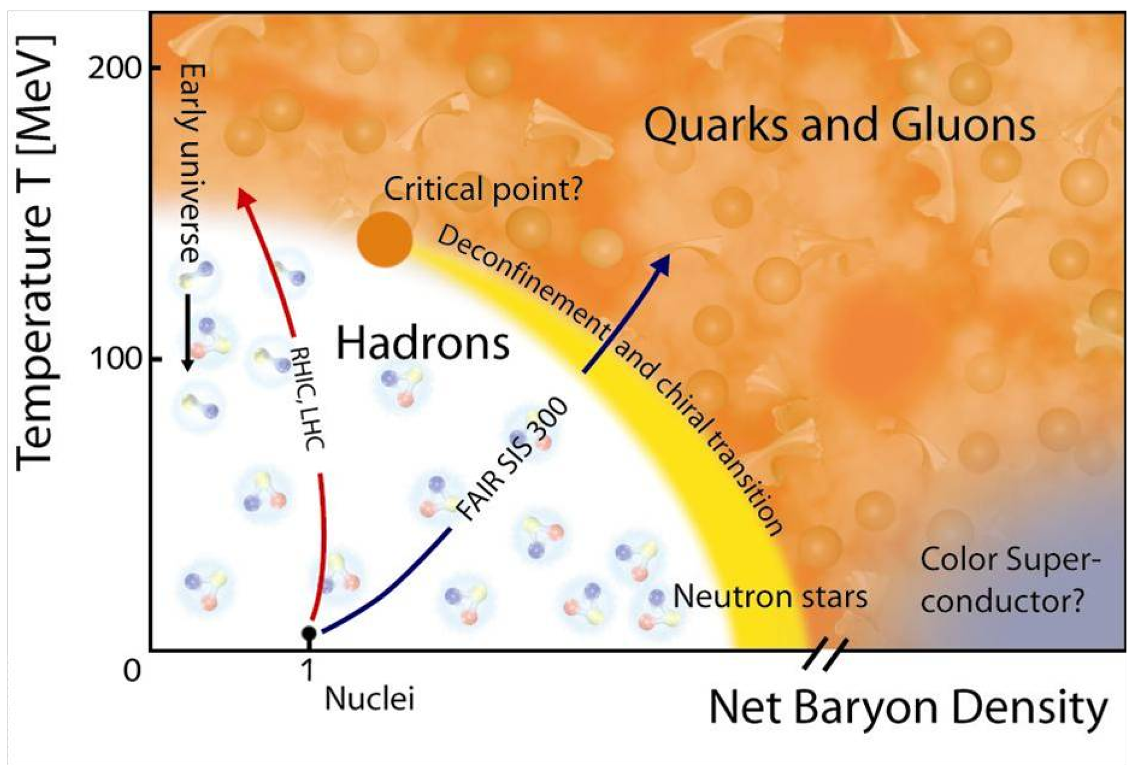


Figure 1: Schematic phase diagram of the QCD [7]

Nambu-Jona-Lasinio model

The Lagrangian of the NJL-model as function of N_c reads [3]:

$$\mathcal{L}_{NJL}(N_c) = \bar{\psi}(\imath\gamma^\mu\partial_\mu - m_q)\psi + \frac{3G}{N_c}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\psi)\right]^2, \qquad \psi^t = (u,d)\ . \quad (1)$$

The quark develops a constituent quark mass m^* which is proportional to $<\bar{q}q>$. As a result the scaling of the critical temperature $T_c(N_c)$ as a function of the number of colors reads:

$$T_c(N_c) \simeq \Lambda\sqrt{\frac{3}{\pi^2}}\sqrt{1-\frac{\pi^2}{6\Lambda^2 G}} \propto N_c^0\ . \quad (2)$$

To leading order the chiral transition is independent of N_c. The chiral phase transition is triggered via the N_c quark loops.

The result is in agreement with expectations [8].

Linear σ-model

The Lagrangian of the linear σ-model [4] with N_c scaling reads:

$$\mathcal{L}_\sigma(N_c) = \frac{1}{2}(\partial_\mu\Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4, \qquad \Phi^t = (\sigma, \vec{\pi})\ , \quad (3)$$

where the scalar field sigma σ and the pseudoscalar pion triplet $\vec{\pi}$ have been introduced. The chiral condensate reads: $\varphi_0 = \varphi(T=0) = \mu\sqrt{N_c/\lambda 3} = \sqrt{N_c/3}f_\pi$. The critical temperature $T_c(N_c)$ calculated with the CJT formalism [9] reads:

$$T_c(N_c) = \sqrt{2}f_\pi\sqrt{\frac{N_c}{3}} \propto N_c^{1/2}\ . \quad (4)$$

The critical temperature increases with the number of colors. Interactions vanish in the large-N_c limit the symmetry cannot be restored.

The result contradicts the result of the NJL-model [10].

Modified linear σ-model

At least one parameter is chosen to be temperature-dependent [6]:

$$\mu^2 \to \mu(T)^2 = \mu^2\left(1 - \frac{T^2}{T_0^2}\right)\ . \quad (5)$$

The quadratic behavior is in agreement with Ref. [11]. This change lead to the following correct scaling of T_c:

$$T_c(N_c) = T_0\left(1 + \frac{1}{2}\frac{T_0^2}{f_\pi^2}\frac{3}{N_c}\right)^{-1/2} \propto N_c^0\ . \quad (6)$$

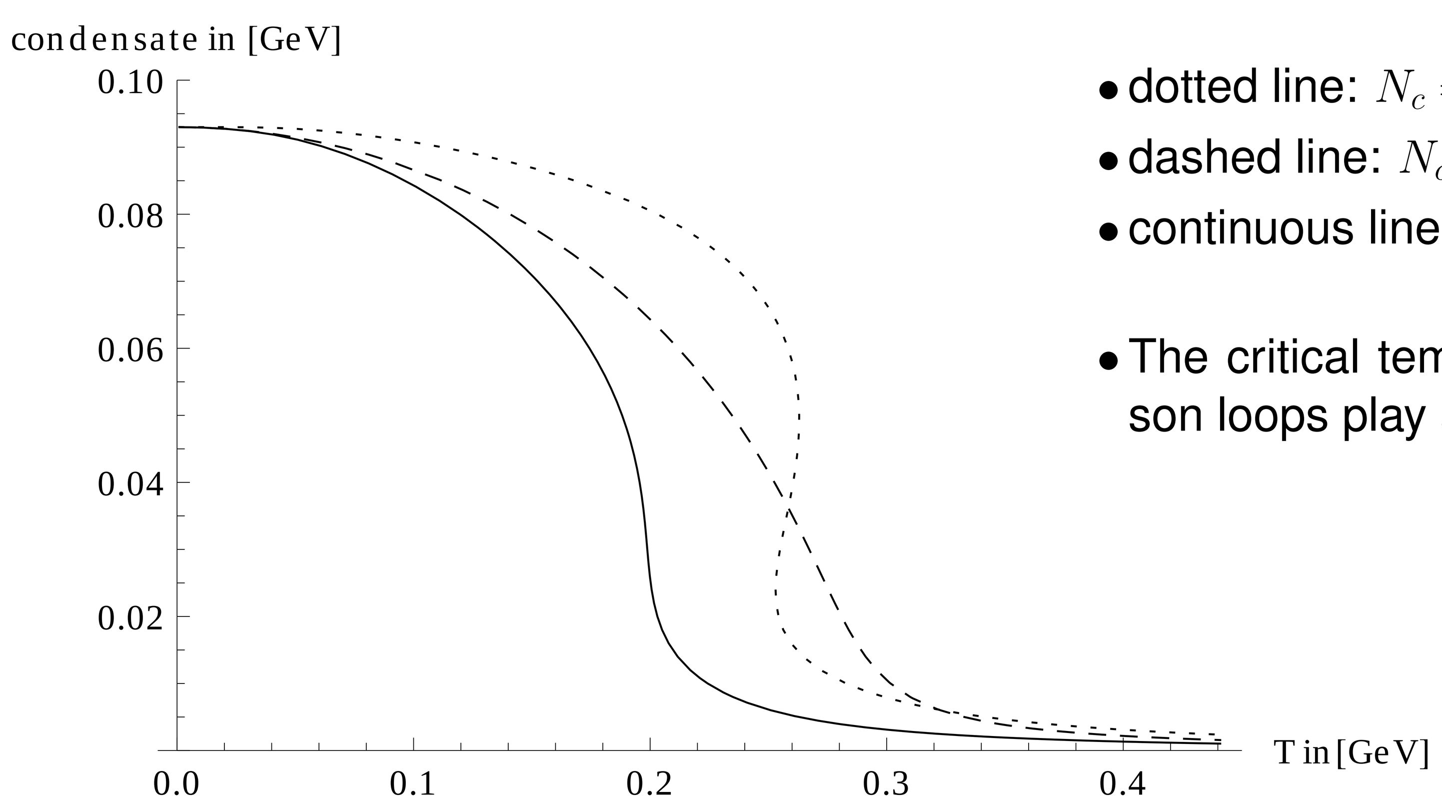
Phenomenological improved linear σ-model shows the correct large-N_c behavior. The mesonic loops pull the T_c to lower values. (Coupling to Polyakov loop leads to same scaling)

Finite temperature study

Linear-σ model including an additional scale T_0 in a large-N_c consistent version:

$$\mathcal{L}_\sigma(T_0) = \frac{1}{2}(\partial_\mu\Phi)^2 + \frac{1}{2}\mu^2\left(1 - \frac{T^2}{T_0^2}\right)\Phi^2 - \frac{\lambda}{4}\Phi^4 + \epsilon\sigma\ . \quad (7)$$

Finite temperature study within the CJT-formalism in a double-bubble approximation:



- dotted line: $N_c = 3$ and $T_0 \to \infty$
- dashed line: $N_c \to \infty$ and $T_0 = 270$ MeV
- continuous line: $N_c = 3$ and $T_0 = 270$ MeV
- The critical temperature T_c decreases when meson loops play a role.

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