AAMQS: a non-linear QCD description of new HERA data at small-x

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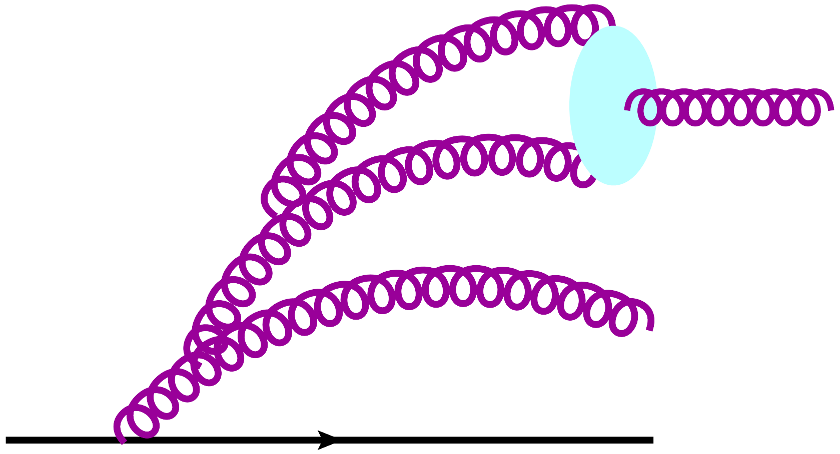
arXiv:1012.4408
arXiv:0902.1112

[Javier **A**lbacete, Nestor **A**rmesto, Guilherme **M**ilhano, P**Q**A and Carlos **S**algado]

Quark Matter 2011 - Annecy, France - 26 May 2011

Introduction

In the limit of small Bjorken-x [HE]:



deviations from standard collinear perturbation theory are expected on account of large gluon densities => non-linear processes become relevant

"BK-JIMWLK"

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k_t})}{\partial \ln(\mathbf{x_0}/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k_t}) - \phi(\mathbf{x}, \mathbf{k_t})^2$$

the **C**olor **G**lass **C**ondensate is a correct framework in which to address small-x physics

Unitarity sets upper limit on the growth rate of gluon densities: realized by inclusion of recombination processes

highly probable in high density enviroment

Interplay between radiation and recombination processes => dynamical transverse momentum scale: the saturation scale [onset of non-linear corrections]

- Phenomenology side: dipole model formulation of DIS [simple implementation of saturation effects]

- DGLAP linear evolution equations provide accurate description of data
  - relevant question: flexibility of i.c. hiding some interesting QCD dynamics [non-linear behavior]?
  - recent NNPDF [no i.c. bias] fits find <u>deviations</u> w.r.t. low x data excluded from fits

Outline

- AAMQS SETUP
  - dipole model formulation of DIS
  - initial conditions
  - inclusion of heavy quarks [additional fit parameters]
  - data samples
- Fit results:
  - light quarks, inclusion of heavy flavors, description of $F_{2,c}/\sigma_r$ and F_L
- Comparison to DGLAP [region where deviations appear]
  - delineating saturation boundaries

[With Guilherme Milhano and Juan Rojo]

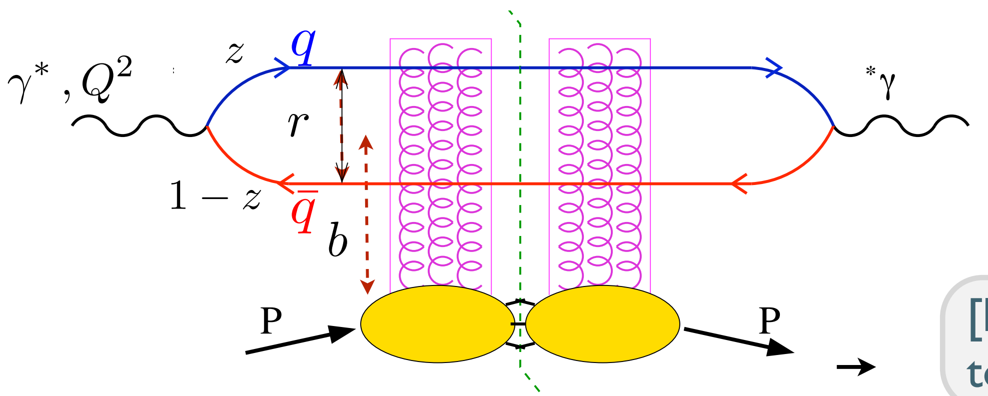
AAMQS setup. Dipole model formulation of e+p scatt. + rcBK eq.

- dipole model formulation of the e-p scattering process

$$\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$

x<<1

$$\begin{cases} F_2(x,Q^2) = \dfrac{Q^2}{4\pi^2\alpha_{em}}(\sigma_T + \sigma_L) \\ F_L(x,Q^2) = \dfrac{Q^2}{4\pi^2\alpha_{em}}\sigma_L \end{cases}$$



virtual photon-proton cross section [long. & trans. polarization of γ^*]

$$\sigma_{T,L}(x,Q^2) = 2\sum_f \int_0^1 dz \int d^2\mathbf{b}\, d^2\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}(\mathbf{b},\mathbf{r},x)$$

[light-cone wave function for γ^* to fluctuate into a q-qbar dipole]

Im. part of dipole-target scatt. amplitude [all strong interaction and x dependence]

- small-x dynamics of the dipole scattering amplitude described by rcBK equation

non-linear term

$$\frac{\partial N(r,x)}{\partial \ln(x_0/x)} = \int d^2r_1 \mathbf{K}^{run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2})[N(r_1,x) + N(r_2,x) - N(r,x) - N(r1,x)N(r_2,x)]$$

evolution kernel including rc corrections

Balitsky, Phys.Rev.D75:014001,2007

$$K^{\rm run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\,r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

Y.Kovchegov's talk

- Regularization of the coupling: phase space for all dipoles sizes explored [arbitrarily large] => need to regulate in the IR

$$\alpha_s(r^2 < r^2_{fr}) = \frac{12\pi}{(11N_c - 2n_f)\ln\left(\frac{4C^2}{r^2\Lambda^2_{QCD}}\right)}$$

$$\alpha_s(r^2 \geq r^2_{fr}) = \alpha_{fr}$$

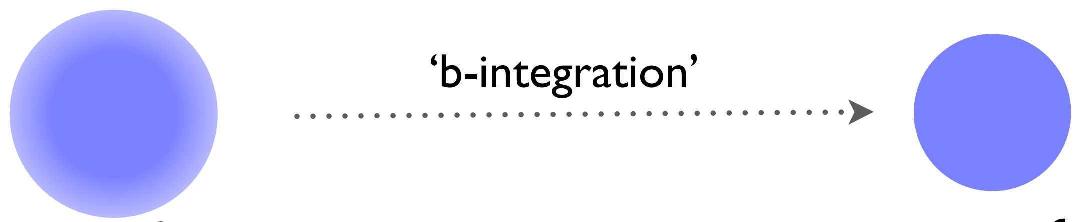
Fourier transform: momentum to coordinate space

AAMQS setup. Impact parameter. Initial condition [light]

- b-dependence of dipole amplitude $\mathcal{N}$(b,r,x): governed by long-distance non-perturbative phenomena [extra model input]

J.Albacete's talk

- AAMQS resorts to translational invariance approximation



$\mathcal{N}(\mathbf{b}, \mathbf{r}, x)$ $\qquad$ $\sigma_0 \mathcal{N}(\mathbf{r}, x)$

average over impact parameter

$$2 \int d\mathbf{b} \rightarrow \sigma_0$$

[average transv. area of quark distrib. in transv. plane]

- 2 families of initial conditions [for the rcBK evol. eq. $\frac{\partial \mathcal{N}(r,x)}{\partial \ln(x_0/x)}$] x_0<0.01: largest value of x (=0.00893)

- GBW $\quad \mathcal{N}^{GBW}(r, x_0) = 1 - e^{-\left(\frac{r^2 Q_{s,0}^2}{4}\right)^{\gamma}}$

- MV $\quad \mathcal{N}^{MV}(r, x_0) = 1 - e^{-\left(\frac{r^2 Q_{s,0}^2}{4}\right)^{\gamma} \ln\left(\frac{1}{r\Lambda_{QCD}}\right)}$

2 fit parameters:

initial saturation scale [at x_0]

anomalous dimension [steepness of dipole amplitude fall-off with decreasing r]

- Third family: 'scaling' i.c.: asymptotic solutions of rcBK are universal [independent of i.c.]

$\mathcal{N}(r, Y >> 1) \rightarrow \mathcal{N}^{scal}(\tau = rQ_s(Y))$ $\quad$ evolve rcBK to high rapidity. Then rescale back to i.c. [$\tau = rQ_s(Y) \rightarrow rQ_{s,0}$]

no good fits found [Pre-asymptotic effects slow down evol. for MV & GBW; scaling i.c. much faster evol.]

AAMQS setup. Fits including heavy quarks

- sum in the dipole model extended to heavy flavors
- light and heavy quarks may not have equal distribution

different normalization

$m_{u,d,s} = m_{light}$, fixed: m_{charm}=1.27 GeV, m_{beauty}=4.2 GeV

$$\sigma_{T,L}(x,Q^2) = \sigma_0^{light} \sum_{f=u,d,s} \int_0^1 dz d\mathbf{r} |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}^{light}(\mathbf{r}, x)$$
$$+\sigma_0^{heavy} \sum_{f=c,b} \int_0^1 dz d\mathbf{r} |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}^{heavy}(\mathbf{r}, x)$$

- allow for independent i.c.

$$\mathcal{N}^{GBW}_{light}(r,x_0) = 1 - exp\left[-\left(\frac{r^2 \bar{Q}^2_{0,light}}{4}\right)^{\gamma^{light}}\right] \qquad \mathcal{N}^{GBW}_{heavy}(r,x_0) = 1 - exp\left[-\left(\frac{r^2 \bar{Q}^2_{0,heavy}}{4}\right)^{\gamma^{heavy}}\right]$$

3 additional fit parameters when heavy quarks are included

- As a matter of consistency: variable flavor number scheme for the running of the coupling

$$\alpha_s(r^2 < r^2_{fr}) = \frac{4\pi}{\beta_{0,n_f} \ln\left(\frac{4C^2}{r^2 \Lambda_{n_f}}\right)}, \qquad \alpha_{s,n_{f-1}}(r^2_*) = \alpha_{s,n_f}(r^2_*), \qquad r^2_* = \frac{4C^2}{m^2_f}$$

Match the branches of the coupling with adjacent n_f at the scale corresponding to the quark masses

$$\alpha_s(r^2 \geq r^2_{fr}) = \alpha_{fr}$$

$$\Lambda_{n_{f-1}} = (m_f)^{1-\frac{\beta_{0,n_f}}{\beta_{0,n_{f-1}}}} (\Lambda_{n_f})^{\frac{\beta_{0,n_f}}{\beta_{0,n_{f-1}}}}$$

AAMQS setup. Summary and data sets

- calculate σ_r and F_2 according to the dipole model

$$\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$

$\propto \sigma_{T,L}$ $\propto \sigma_L$

$$\sigma_{T,L}(x,Q^2) = \sum_f \sigma_0^{l\,(heavy)} \int_0^1 dz d^2\mathbf{r} |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \mathcal{N}(\mathbf{r}, x)$$

normalization

mass of light quarks
[m_l=0.14 GeV or free]

- small-x dependence described by rcBK $\partial\mathcal{N}(r,x)/\partial\ln(x_0/x) \propto K^{run} \propto \alpha_s(r^2)$
  - uncertainty from F.T. $\alpha_s(r^2) \rightarrow C^2$
  - initial conditions $\mathcal{N}(r,x_0) \rightarrow Q^2_{s,0}\ ,\ \gamma\ ,\ Q^2_{s,0,c}\ ,\ \gamma^c$

initial saturation scale and anomalous dimension

AAMQS free parameters: 4(5) [only light], 7(8) [light+heavy]

Experimental data sets

kinematic shift $\tilde{x} = x(1+4m_f^2/Q^2)$

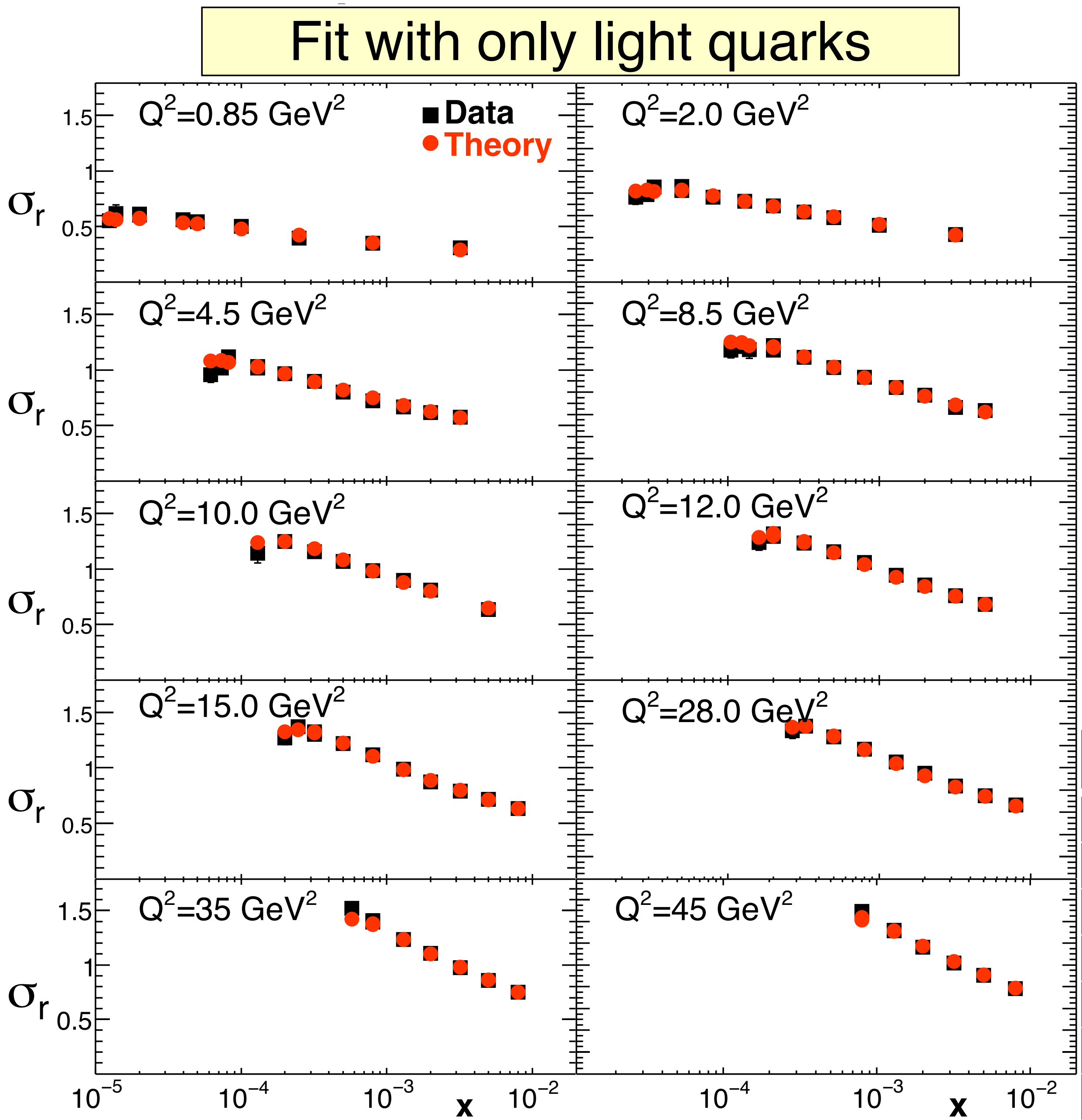
- reduced cross section σ_r from combined H1+ZEUS (HERA) analysis => reduces systematic uncertainties [new w.r.t. AAMS ('old' fits)]
- inclusive structure function F_2 from E665 (FNAL) & NMC (CERN-SPS) (σ_r not available)
- cuts $x \leq 10^{-2}$ $\quad 0.045 < Q^2 < 50\,\mathrm{GeV}^2$=> 325 data points [fits with only light quarks]
- all available data sets of F_2^c (within cuts) => 329 data points [fits with light+heavy quarks]

[b contribution to σ_r considered but F_2^b data not included]

excluded data not fulfilling $\tilde{x} < 0.01$ [more restrictive for m_c]

Fit results: AAMQS 1.0 [only light quarks]

AAMQS 1.0 combined H1 and ZEUS data [also non HERA]



- Reduced sigma data

  improved accuracy [more constraining conditions]
- Tiny error bars
- Fully consistent with AAMS (fits to 'old' F_2 data with large error bars)
  - mild changes in the parameters
  - but tension with high Q^2 data

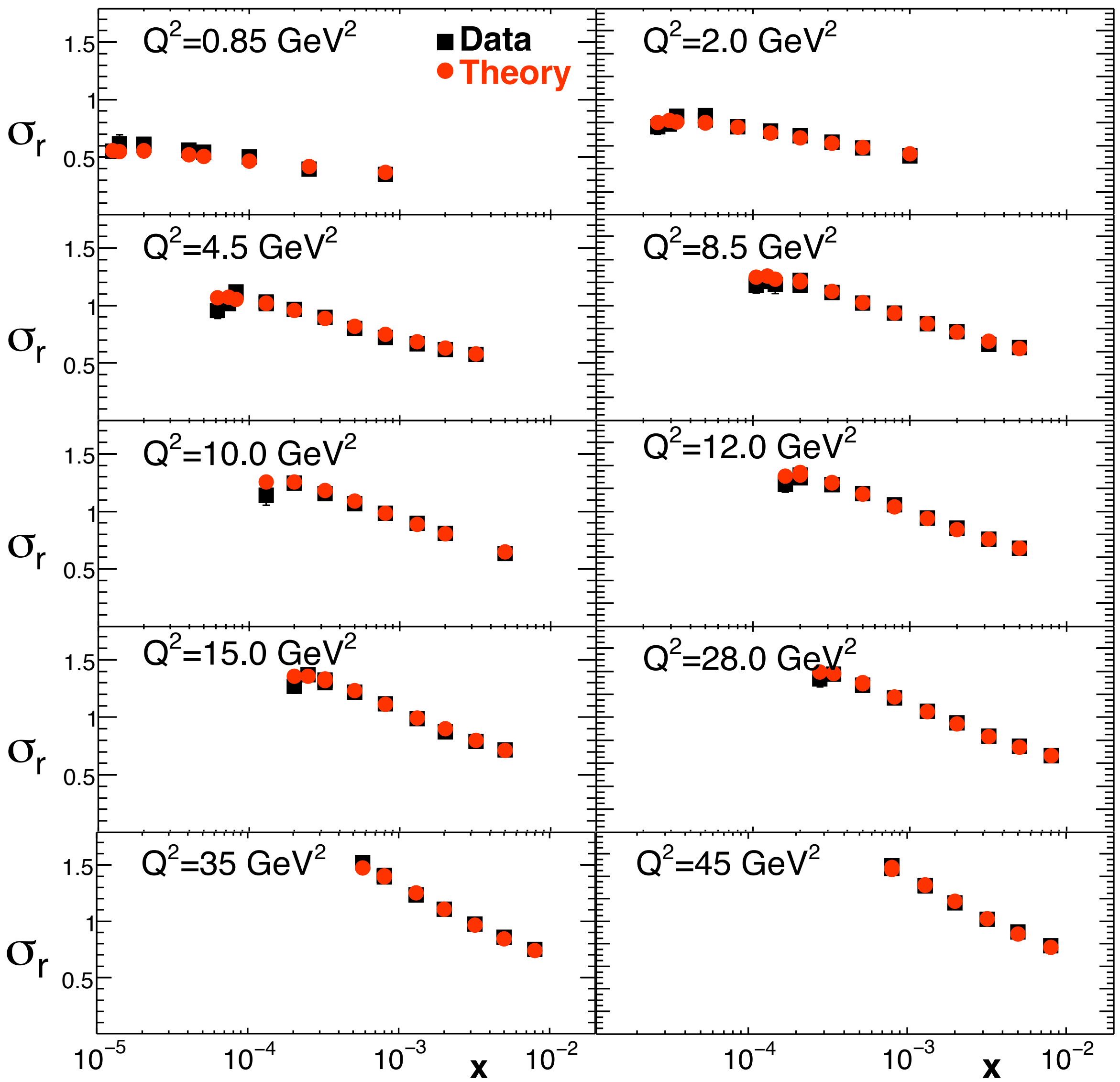
statistical and systematic errors added in quadrature

	fit	$\frac{\chi^2}{d.o.f}$	$Q^2_{S,0}$	σ_0	γ	C	m_l^2
	GBW						
a	$\alpha_f = 0.7$	1.226	0.241	32.357	0.971	2.46	fixed
a'	$\alpha_f = 0.7\ (\Lambda_{m_\tau})$	1.235	0.240	32.569	0.959	2.507	fixed
b	$\alpha_f = 0.7$	1.264	0.2633	30.325	0.968	2.246	1.74E-2
c	$\alpha_f = 1$	1.279	0.254	31.906	0.981	2.378	fixed
c'	$\alpha_f = 1\ (\Lambda_{m_\tau})$	1.244	0.2329	33.608	0.9612	2.451	fixed
d	$\alpha_f = 1$	1.248	0.239	33.761	0.980	2.656	2.212E-2
	MV						
e	$\alpha_f = 0.7$	1.171	0.165	32.895	1.135	2.52	fixed
f	$\alpha_f = 0.7$	1.161	0.164	32.324	1.123	2.48	1.823E-2
g	$\alpha_f = 1$	1.140	0.1557	33.696	1.113	2.56	fixed
h	$\alpha_f = 1$	1.117	0.1597	33.105	1.118	2.47	1.845E-2
h'	$\alpha_f = 1\ (\Lambda_{m_\tau})$	1.104	0.168	30.265	1.119	1.715	1.463E-2

Stability w.r.t. variations in:
- IR regularization of the coupling
- reference scale to determine Λ_{QCD}

Fit results: AAMQS 1.0 [light + heavy quarks]

Fit including heavy quarks



- Excellent global description
- χ^2/d.o.f. slightly larger than fits with only light

  [systematic deviation between different F_{2c} data sets]
- heavy contribution has smaller size

  $$\sigma_0^{light} > \sigma_0^{heavy}$$

  [average radius of heavy quark distrib.<light quarks]
- also gentler fall-off in the i.c.

  $$\gamma^{light} > \gamma^{heavy}$$
- initial saturation scale similar for light and heavy

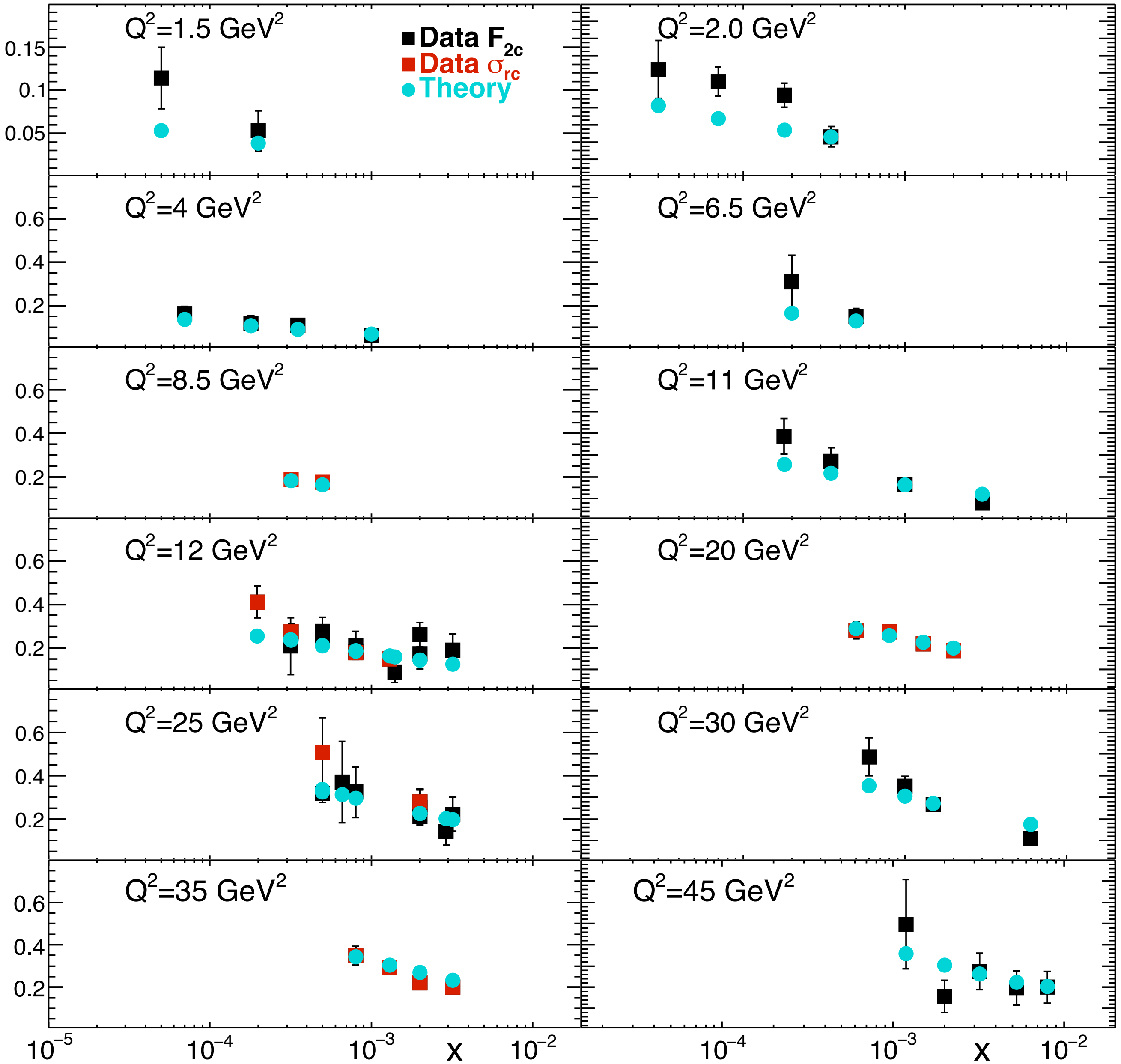
Stability w.r.t. variations in:

- IR regularization of the coupling
- reference scale to determine Λ_{QCD}

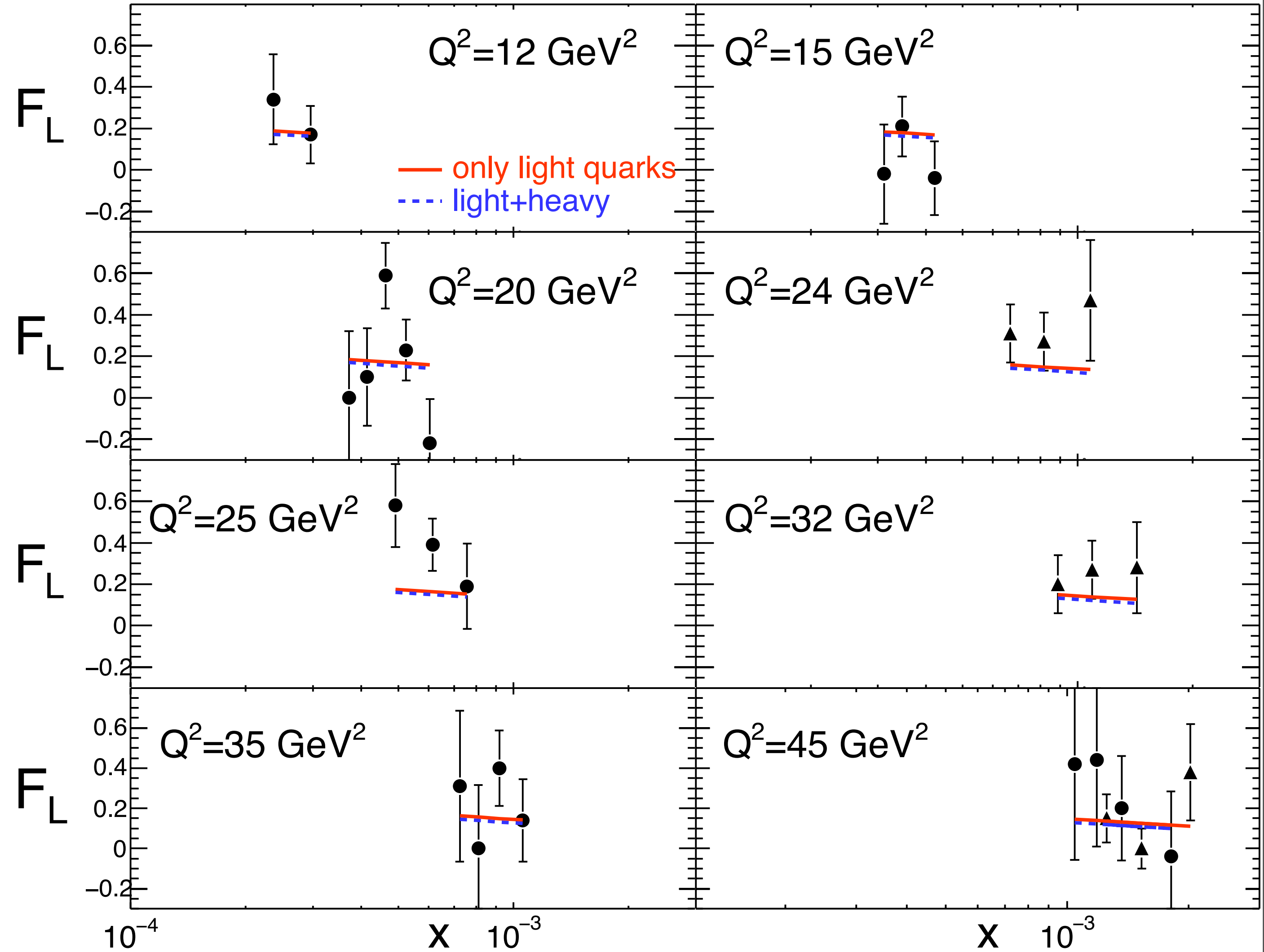
	fit	$\frac{\chi^2}{d.o.f}$	$Q^2_{S,0}$	σ_0	γ	$Q^2_{S,0,c}$	$\sigma_{0,c}$	γ_c	C	m_l^2
	GBW									
a	$\alpha_f = 0.7$	1.269	0.2294	36.953	1.259	0.2289	18.962	0.881	4.363	fixed
a'	$\alpha_f = 0.7$ (Λ_{m_τ})	1.302	0.2341	36.362	1.241	0.2249	20.380	0.919	7.858	fixed
b	$\alpha_f = 0.7$	1.231	0.2386	35.465	1.263	0.2329	18.430	0.883	3.902	1.458E-2
c	$\alpha_f = 1$	1.356	0.2373	35.861	1.270	0.2360	13.717	0.789	2.442	fixed
d	$\alpha_f = 1$	1.221	0.2295	35.037	1.195	0.2274	20.262	0.924	3.725	1.351E-2
	MV									
e	$\alpha_f = 0.7$	1.395	0.1673	36.032	1.355	0.1650	18.740	1.099	3.813	fixed
f	$\alpha_f = 0.7$	1.244	0.1687	35.449	1.369	0.1417	19.066	1.035	4.079	1.445E-2
g	$\alpha_f = 1$	1.325	0.1481	40.216	1.362	0.1378	13.577	0.914	4.850	fixed
h	$\alpha_f = 1$	1.298	0.156	37.003	1.319	0.147	19.774	1.074	4.355	1.692E-2

Comparison of F_{2C} and F_L

Comparison with data on F_{2c} and σ_{rc}



F_L data: H1 and ZEUS direct measurement



- description of the charm contribution to the inclusive structure function and the reduced cross-section
- systematic deviation between different F_{2c} and σ_{rc} data sets [larger χ^2/d.o.f.]

- Good description in both cases
- F_L NOT included in any fit **[independent test]**
- all fits good description [only 2 shown]

Non-linear deviations in DGLAP fits?

determining the saturation boundaries

With Guilherme Milhano and Juan Rojo

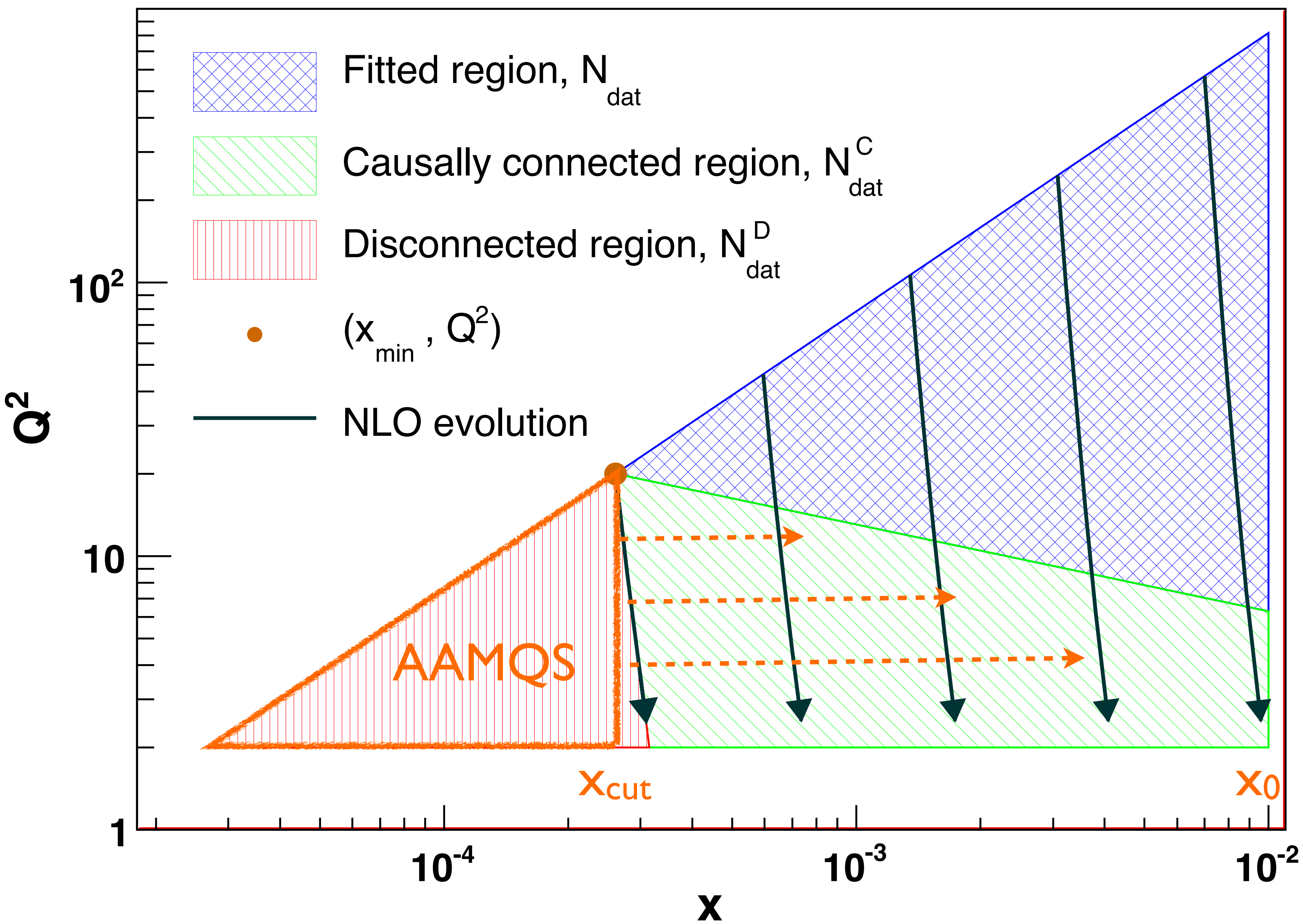
- Deviations from NLO QCD evolution in HERA structure function after systematic exclusion of low-Q^2 regions [green area]

- AAMQS fit to low $x < x_{cut}$ data [red] => extrapolation to data with $x_{cut} < x < x_0 = 10^{-2}$

Test the evolution NOT the choice of initial conditions

F.Caola, S.Forte, J.Rojo

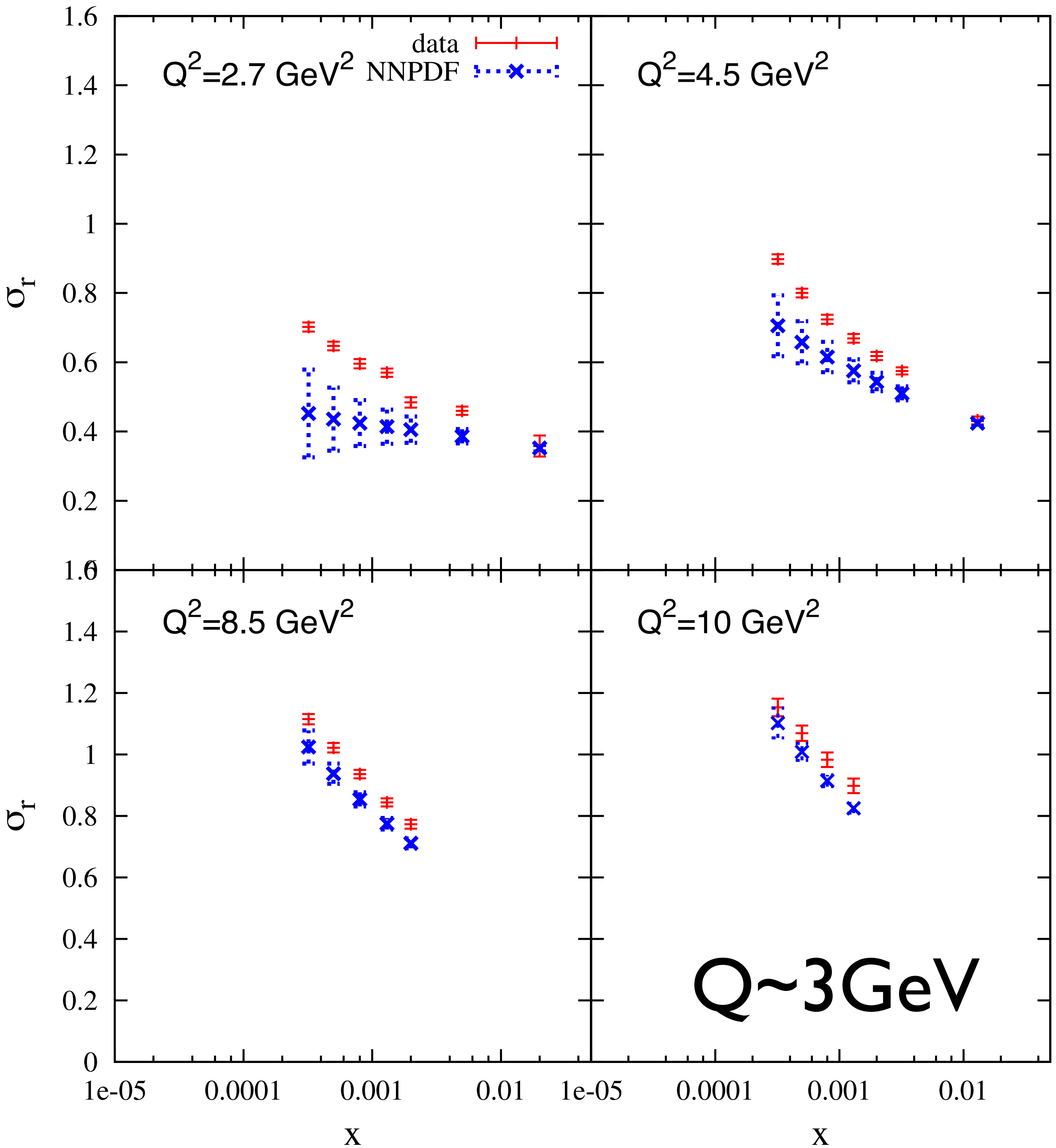
Phys.Lett.B686:127-135,2010



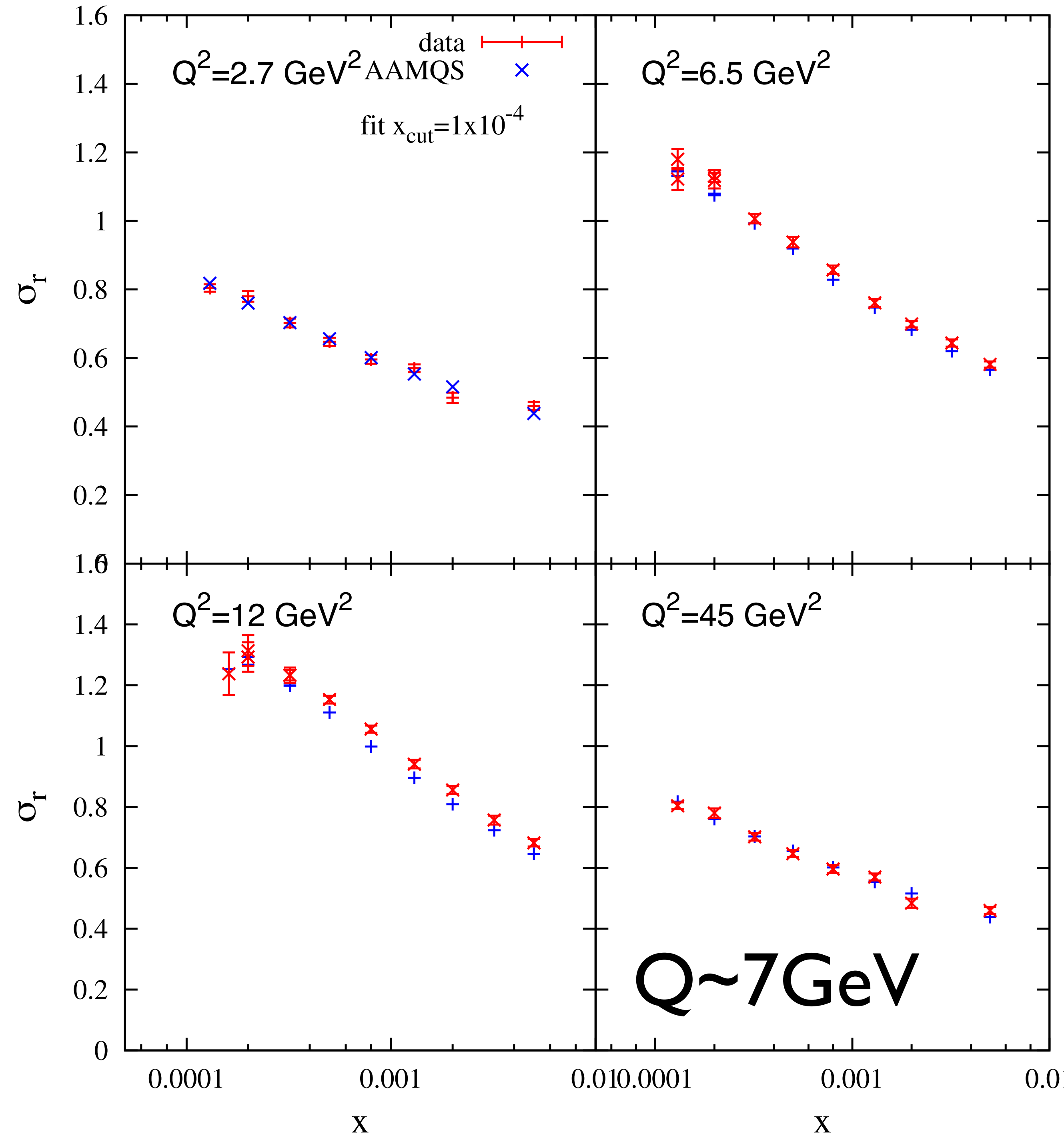
Non-linear deviations in DGLAP fits?

- NNPDF and AAMQS comparison to **same** σ_r data in the unfitted region

NNPDF



AAMQS $x_{cut}=10^{-4}$

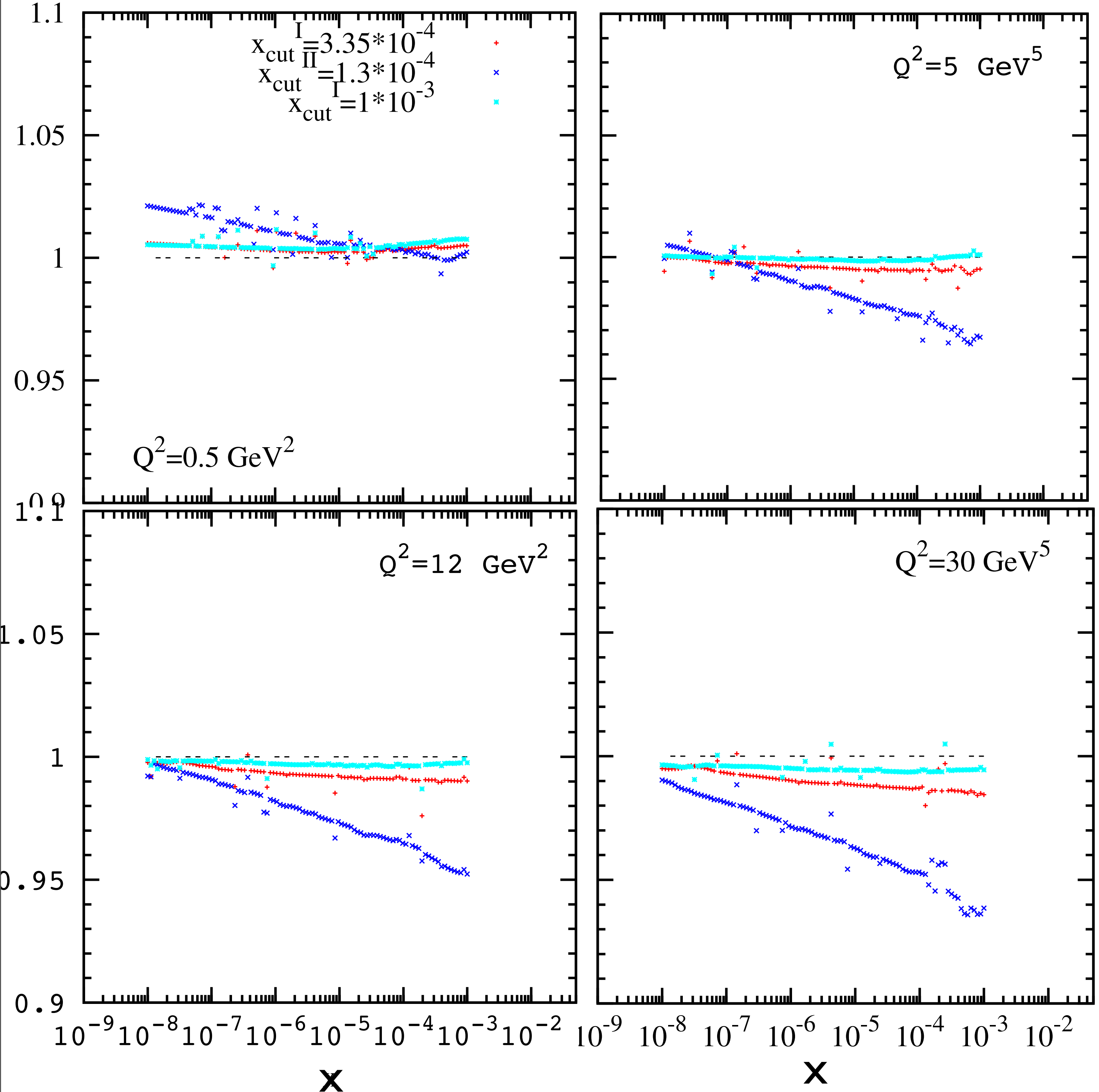


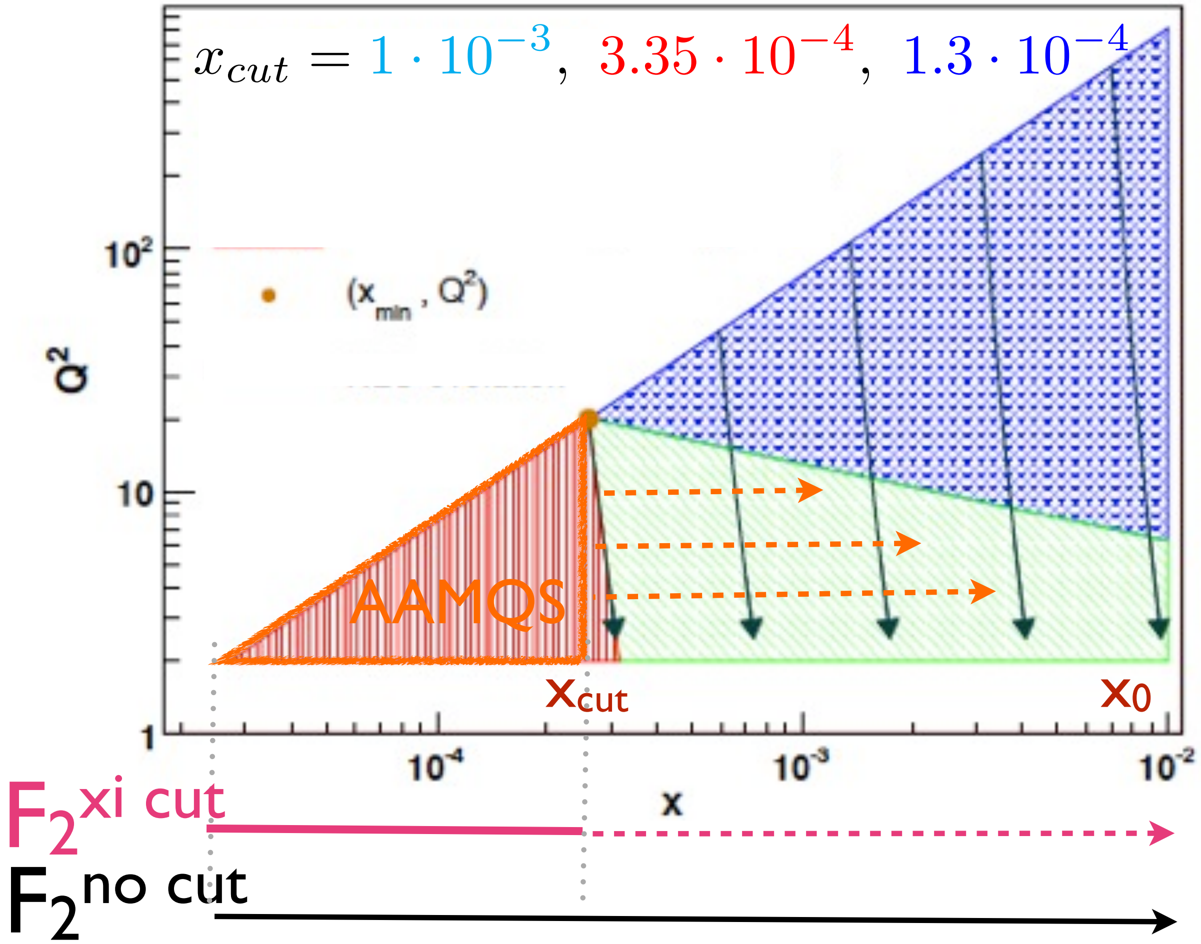
- deviation from data at low x and low Q^2

- very good description of data even with the more restrictive cut

Non-linear deviations in DGLAP fits?

$F_2^{x^i_{cut}} / F_2^{no\ cut}$





- Small deviations are found: other relevant physics [DGLAP, ...?] not included in AAMQS is relevant in such region

- Deviations increase with decreasing x_{cut} and increasing Q^2. MAKES PERFECT SENSE

Conclusions and outlook

- rcBK evolution describes correctly new data on reduced cross section at small-x from the H1+ZEUS combined analysis

[indications for the presence of non-linear saturation effects in present data]

CGC as practical phenomenological tool to approach HE QCD scattering

- Inclusion of heavy quarks naturally incorporated in the dipole formalism as long as a smaller transverse size of the heavy quark effective distribution is allowed

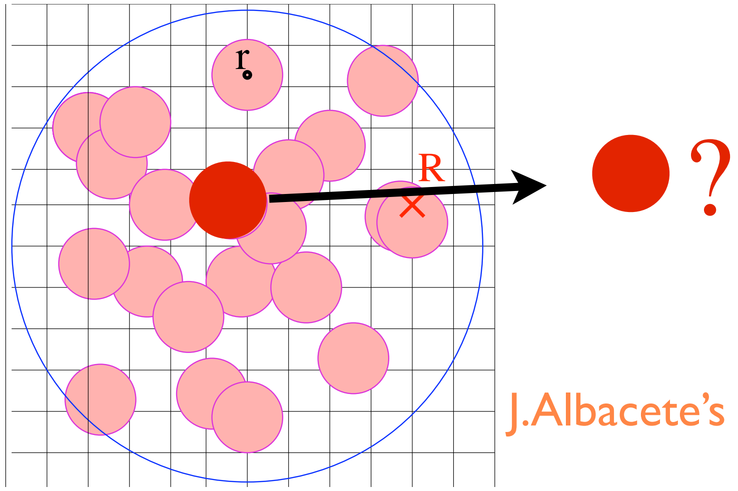
- Ongoing systematic studies to determine the **saturation boundary**

- analyses with current data: suggest breakdown of collinear factorization and onset of non-linear corrections at low-x
  - Deviations in DGLAP in the low-x region. AAMQS correctly describes the data

comparison of results in a common region of phase space calculation of K factors to DGLAP evolution in the small-x region

in progress

- AAMQS [saturation] describes data at scales relevant in heavy ion collisions



J.Albacete's talk

AAMQS: input at relevant scales

- Fits to e+A deep inelastic scattering in progress. Additional parameters $Q_s^2 = Q_{s,0}^2 \cdot c \cdot A^\delta$

AAMQS 1.0 parametrization available online

http://fpaxp1.usc.es/phenom/aamqs/aamqs.html

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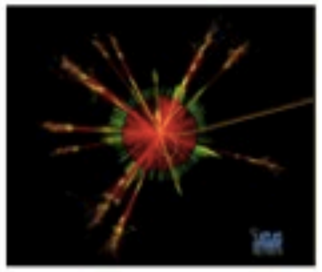
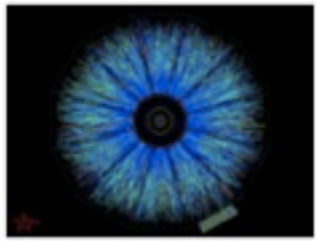
Dipole-proton cross section





The imaginary part of the dipole-proton scattering amplitude is available as a FORTRAN routine for public use. This quantity has been fitted to lepton-proton data using the Balitsky-Kovchegov evolution equations with running coupling. More details can be found at

J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga Arias and C. A. Salgado, arXiv:1012.4408

Please refer to this publication when using the routine.

In order to compute the dipole cross section, simply multiply the output from the routine by the corresponding values in Table 1 of arXiv:1012.4408 (the actual values depend on the chosen set of parameters). These values are

For the fits with only light flavors (subroutine aamqs10l):

sigma0=32.357 mb for GBW initial conditions, set a
sigma0=32.895 mb for MV initial conditions, set e

For the fits with light+heavy flavors (subroutine aamqs10h):

sigma0=35.465 mb for GBW initial conditions, light, set b
sigma0=18.430 mb for GBW initial conditions, heavy, set b
sigma0=35.449 mb for MV initial conditions, light, set f
sigma0=19.066 mb for MV initial conditions, heavy, set f

Full instructions and explanations can also be found at the headers of the routines.

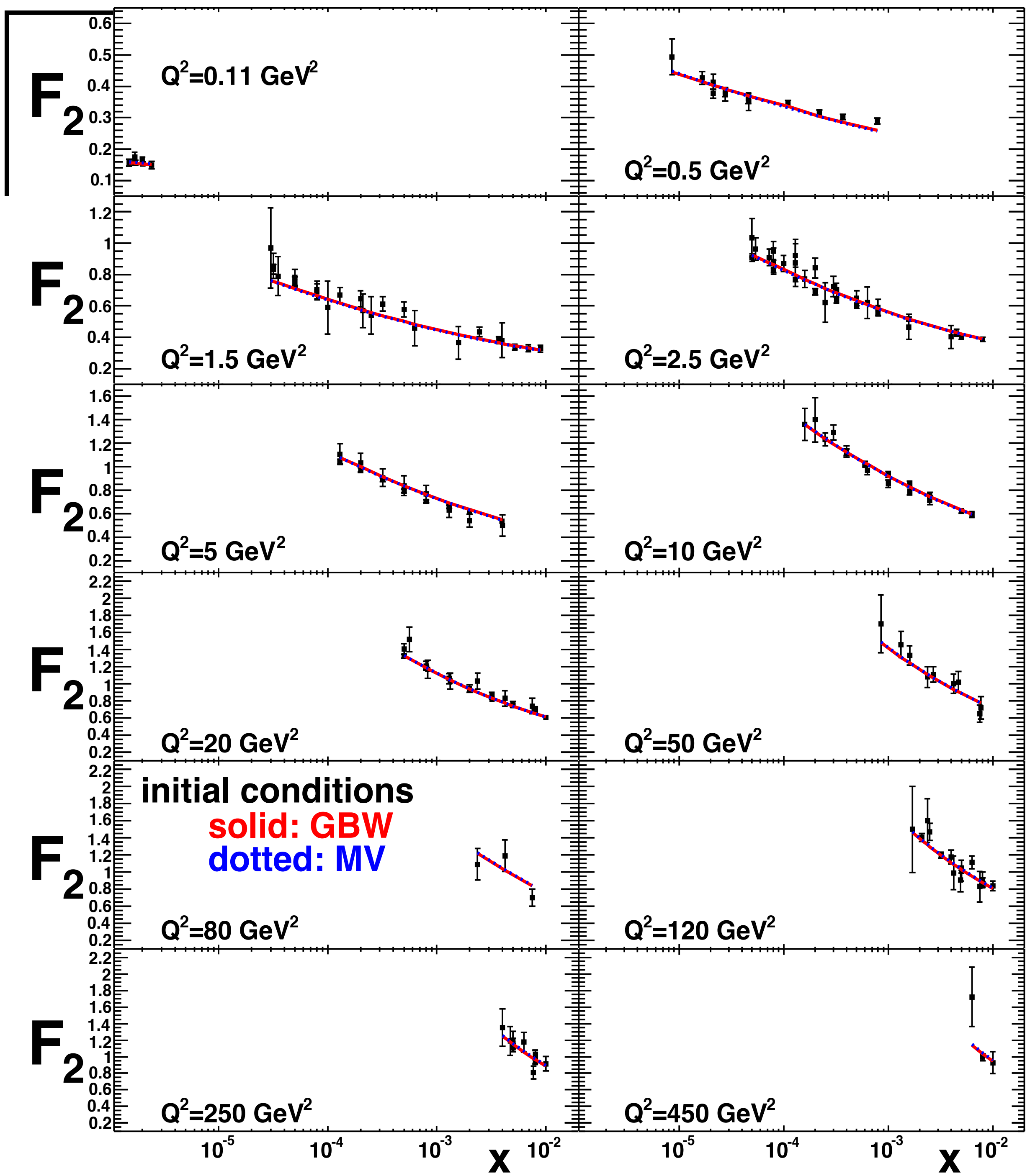
To download the code, please follow this link

The main novelties on these parametrizations with respect to our older one arXiv:0902.1112 are the use of the new (H1 and ZEUS combined) HERA data with much smaller error bars as well as the inclusion of heavy flavors in the fits.

backup slides

Fit results: AAMS [non-linear QCD meets data]

AAMS fit to 'old' H1 and ZEUS F2 data + non HERA [E665, NMC]



J.Albacete, N.Armesto, G.Milhano, C.Salgado

Phys.Rev.D80:034031,2009

Initial condition	σ_0 (mb)	Q^2_{s0} (GeV^2)	C^2	γ	χ^2/d.o.f.
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075

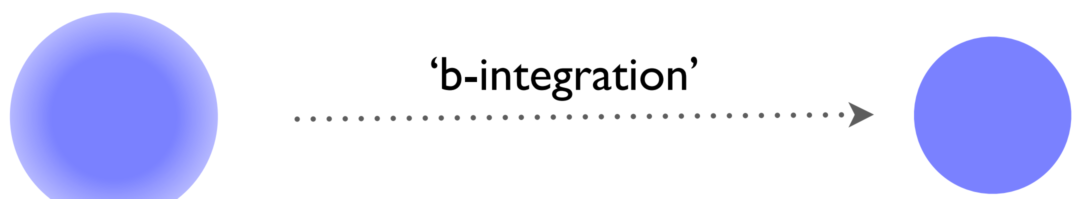
- rcBK describes data
  [inclusive and longitudinal structure functions]
- Data with large error bars
- F_2: extraction uncertainty

$$\sigma_r(y,x,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$

AAMQS setup. Impact parameter. Initial condition [light]

- b-dependence of dipole amplitude $\mathcal{N}$(b,r,x): governed by long-distance non-perturbative phenomena [extra model input]

- AAMQS resorts to translational invariance approximation



$\mathcal{N}(\mathbf{b},\mathbf{r},x)$ $\quad\quad$ $\sigma_0\mathcal{N}(\mathbf{r},x)$

average over impact parameter

$$2\int d\mathbf{b} \rightarrow \sigma_0$$

[average transv. area of quark distrib. in transv. plane]

- 2 families of initial conditions [for the rcBK evol. eq. $\frac{\partial\mathcal{N}(r,x)}{\partial\ln(x_0/x)}$] $\quad$ $x_0<0.01$: largest value of x (=0.00893)

  - GBW $\quad$ $\mathcal{N}^{GBW}(r,x_0) = 1 - e^{-\left(\frac{r^2 Q_{s,0}^2}{4}\right)^{\gamma}}$

  - MV $\quad$ $\mathcal{N}^{MV}(r,x_0) = 1 - e^{-\left(\frac{r^2 Q_{s,0}^2}{4}\right)^{\gamma}\ln\left(\frac{1}{r\Lambda_{QCD}}\right)}$

2 fit parameters:

initial saturation scale [at x_0]

anomalous dimension [steepness of dipole amplitude fall-off with decreasing r]

- Third family: 'scaling' i.c.: asymptotic solutions of rcBK are universal [independent of i.c.]

$\mathcal{N}(r,Y>>1)\rightarrow\mathcal{N}^{scal}(\tau=rQ_s(Y))$ $\quad$ evolve rcBK to high rapidity. Then rescale back to i.c. [$\tau=rQ_s(Y)\rightarrow rQ_{s,0}$]

under study with no good fits so far

evolution generates a universal shape for the dipole amplitude at asymptotically large rapidities. shape Nscal not known=>numerical implementation

AAMQS setup. Regularization of coupling. Variable flavor scheme

- **Regularization of the coupling:** phase space for all dipoles sizes explored [arbitrarily large] => need to regulate in the IR

$$\begin{cases} \alpha_s(r^2 < r_{fr}^2) = \dfrac{12\pi}{(11N_c - 2n_f)\ln\left(\frac{4C^2}{r^2\Lambda_{QCD}}\right)} \\ \alpha_s(r^2 \geq r_{fr}^2) = \alpha_{fr} \end{cases}$$

momentum space [calculation of the quark part of β]
Fourier transform ↓
coordinate space

$$\Lambda_{QCD} = 0.241\text{GeV}\ [\alpha_s(m_{Z^0})]$$

[also Λ_{QCD} corresponding to $\alpha_s(m_\tau)$]

- We use two different values of the coupling: $\alpha_{fr} = 0.7,\ 1$ [coupling frozen to such value when the dipole size is larger than the scale at which α_{fr} is reached]

- Fits including heavy quarks: variable flavor scheme

$$\alpha_s(r^2 < r_{fr}^2) = \frac{4\pi}{\beta_{0,n_f}\ln\left(\frac{4C^2}{r^2\Lambda_{n_f}}\right)}, \qquad \alpha_{s,n_{f-1}}(r_*^2) = \alpha_{s,n_f}(r_*^2), \qquad r_*^2 = \frac{4C^2}{m_f^2}$$

Match the branches of the coupling with adjacent n_f at the scale corresponding to the quark masses

$$\Lambda_{n_{f-1}} = (m_f)^{1-\frac{\beta_{0,n_f}}{\beta_{0,n_{f-1}}}}\left(\Lambda_{n_f}\right)^{\frac{\beta_{0,n_f}}{\beta_{0,n_{f-1}}}}$$

Data sets

- data on different observables:
  - inclusive structure function F_2 from E665 (FNAL) & NMC (CERN-SPS)
  - reduced cross section σ_r from combined H1+ZEUS (HERA) analysis => reduces systematic uncertainties [new w.r.t. AAMS]

σ_r is measured [no theoretical bias in extraction unlike $F_{2,L}$]

- Cuts: small enough $x(<10^{-2})$ and not too high $Q^2(<50\ \mathrm{GeV}^2)$ => 325 data points
- Kinematical redefinition of Bjorken-x to approach photoproduction region safely

$$\tilde{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right)$$

only use data where full evolution can be computed from i.c.

- F_L data from H1 and ZEUS not included in the fit [compared to AAMQS calculation]

LO BK: BFKL kernel

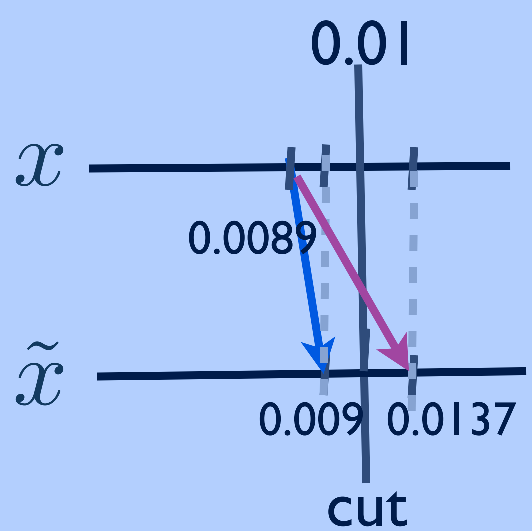
$$K^{LO}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \frac{\alpha_s N_c}{2\pi^2} \frac{r^2}{r_1^2 r_2^2}$$

Number of data points. Kinematic shift

- fits with light quarks only: n_{dat}=325
- fits with light+heavy quarks: n_{dat}=329

excluded data not fulfilling the kinematic shift cut: more restrictive for m_{charm}

$$\tilde{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right)$$



m_{light}=0.14 GeV
m_{charm}=1.27 GeV