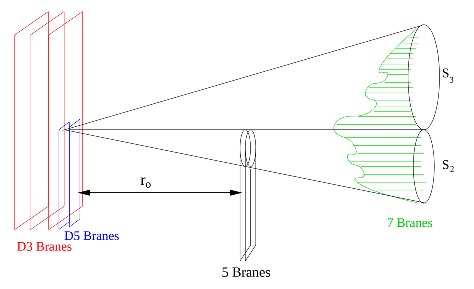

Linear Confinement and Phase Transitions in Top Down Holographic QCD.

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Introduction

- At low energies only color neutral partons exist, hence color is confined. Our goal is to analyze confinement/deconfinement phase transition in QCD.
- However, α_s is large at low energies, so pQCD breaks down and one uses lattice QCD to study confinement. But there are alternatives!
- At strong coupling, certain gauge theories can be described using classical gravity. We find the supergravity description for a non-conformal thermal gauge theory, which resembles large N QCD and study the confinement/deconfinement mechanism while estimating critical temperature.

The Gauge Theory and Dual Gravity

We construct the dual gravity of thermal field theory which confines in the IR but behaves almost conformal in the UV without any Landau poles [1]. The gauge theory arises from the following set up:

- Place N D3 branes at the tip of a conifold with base $T^{1,1}$, M D5 branes wrapping the S^2, then embed $N_f = 24$ 7 branes. The 7 brane embedding in the dual geometry is shown in fig 1.
- Place M number of (p,q) branes with p,q negative at the neighborhood of $r = r_0$ in the dual geometry.

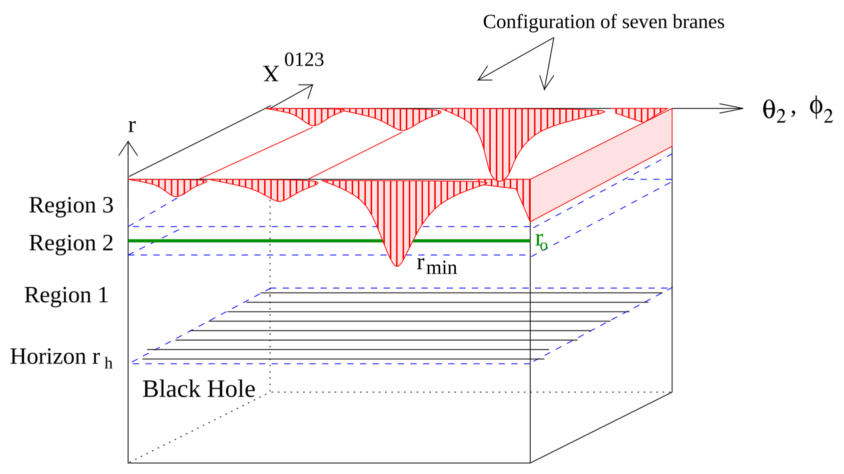

Figure 1: 7 brane embedding in the dual geometry. Most of the branes stay near the boundary resulting in almost conformal behavior of the gauge theory and strings ending on them give heavy quarks . Only a few extend deep into the interior.

At low energy, the gauge theory behaves like large N QCD with the following RG flow:

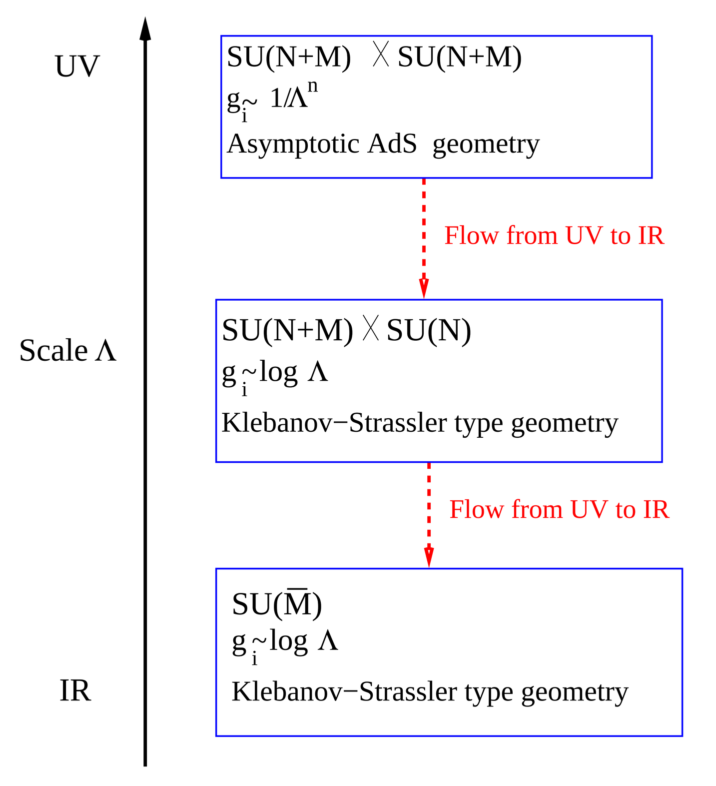

Figure 2: RG flow of the gauge theory arising from branes.

Thermodynamics of The Gauge Theory

- Entropy of the gauge theory is identified with the black hole entropy.
- While for AdS black-hole s/T^3 is constant, for our non-AdS geometry, we observe a sharp change near some T_c. For convenience, we set AdS throat radius $L \equiv 1$ in the plots.
- Free energy and pressure densities also change sharply near T_c- all these indicating a possible phase transition!

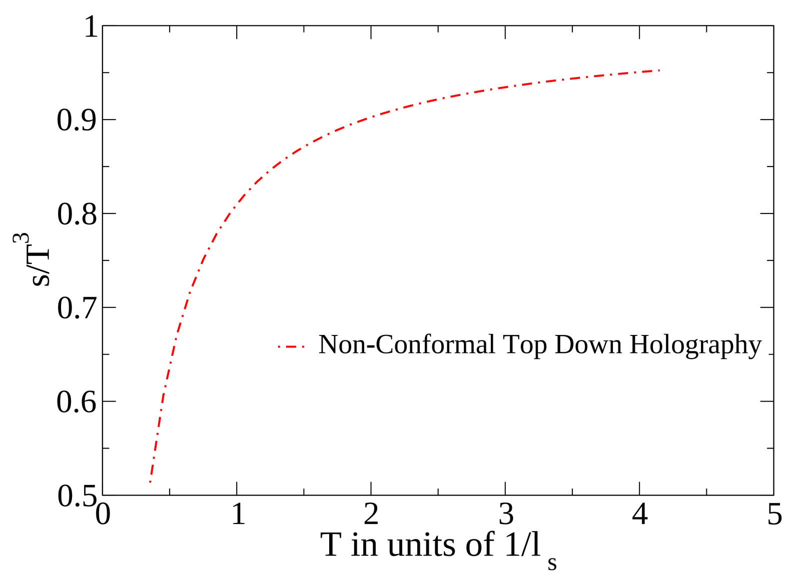

Figure 3: Entropy of the gauge theory identified with the entropy of non-AdS black hole.

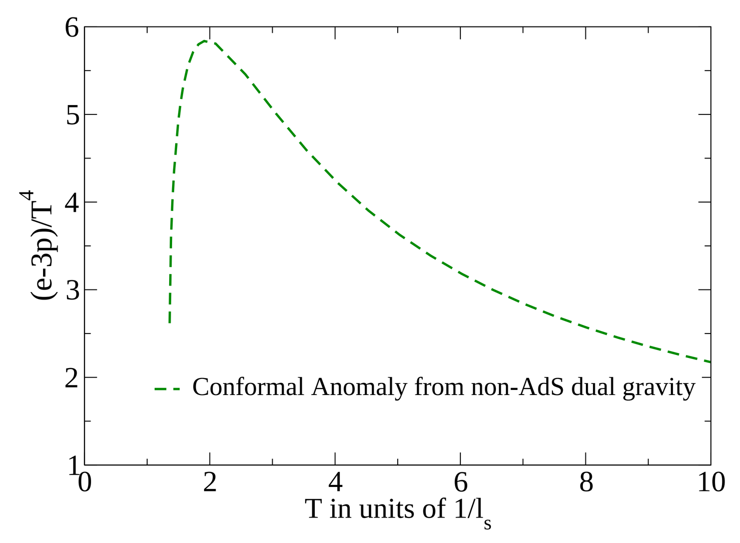

Figure 4: The conformal anomaly from non-conformal dual gravity.

Quarkonium Potential

To analyze the phase transition, we compute free energy of pair of quarks $Q\bar{Q}$ separated by a spatial distance d in thermal medium.

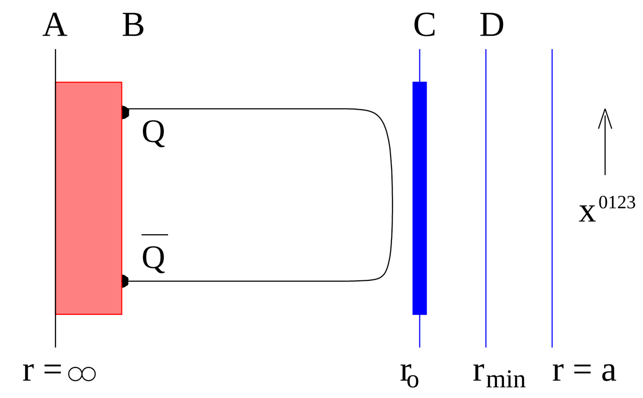

Figure 5: The U shaped string in the geometry along with the D7 and (p,q) branes. The endpoints represent quark and various shape of the string gives various energy for the Quarkonium.

We consider the Wilson loop of a rectangular path $\mathcal{C}$ with space-like width d and time-like length $\mathcal{T}$. The time-like paths are world lines of quarks $Q\bar{Q}$ and in the limit $\mathcal{T} \to \infty$, $\langle W(\mathcal{C})\rangle \sim \exp(-\mathcal{T} V_{Q\bar{Q}})$ where $V_{Q\bar{Q}}$ is the potential energy of pair of quarks. By the principle of holography [3] [4], $\langle W(\mathcal{C})\rangle \sim \exp(-S_{\rm NG}^{\rm ren})$ with $\mathcal{C}$ the boundary of string world sheet and $S_{\rm NG}^{\rm ren}$ is the action for the U shaped string attached to the quarks. We can read off the potential

$$V_{Q\bar{Q}} = \lim_{\mathcal{T}\to\infty} \frac{S_{\rm NG}^{\rm ren}}{\mathcal{T}} \quad (1)$$

At non zero temperature, we identify the free energy of the quarks with $S_{\rm NG}^{\rm ren}$, i.e $F_{Q\bar{Q}} \sim S_{\rm NG}^{\rm ren}/\beta$ where β is inverse temperature. We use the following **non-AdS black-hole metric** describing the dual geometry of **non-conformal gauge theory** arising from our brane configuration:

$$ds^2 = g_{\mu\nu}dX^\mu dX^\nu = \frac{\mathcal{F}(u)}{u^2}\left[-g(u)dt^2 + d\vec{x}^2\right] + \frac{\mathcal{H}(u)}{\mathcal{F}(u)u^2 g(u)}du^2 + \frac{1}{\mathcal{F}(u)}ds^2_{\mathcal{M}_5} \quad (2)$$

where $\mathcal{F} \sim 1/N_{\rm eff}$, with $N_{\rm eff}$ being the effective degrees of freedom at an energy scale $\Lambda \sim 1/u$. We have a regular black-hole horizon u_h with $g = 1 - \frac{u^4}{u_h^4}$. Minimizing the Nambu-Goto action gives the U shaped string and using the boundary condition for the string, one can write both the action $S_{\rm NG}^{\rm ren}$ and the inter-quark separation d as a function of the depth of string $u_{\rm max}$- which is the maximum value of u for a given distance d between the quarks. We observe that

- Inter quark distance d can be imaginary [6] [1], thus unphysical.
- For d to be always real, there exists an upper bound to $u_{\rm max} = \mathbf{x}$ obeying

$$2\mathcal{F}(\mathbf{x}) - \mathbf{x}\mathcal{F}'(\mathbf{x}) = \frac{\mathbf{x}\mathcal{F}'(\mathbf{x})g'(\mathbf{x})}{2g(\mathbf{x})} \quad (3)$$

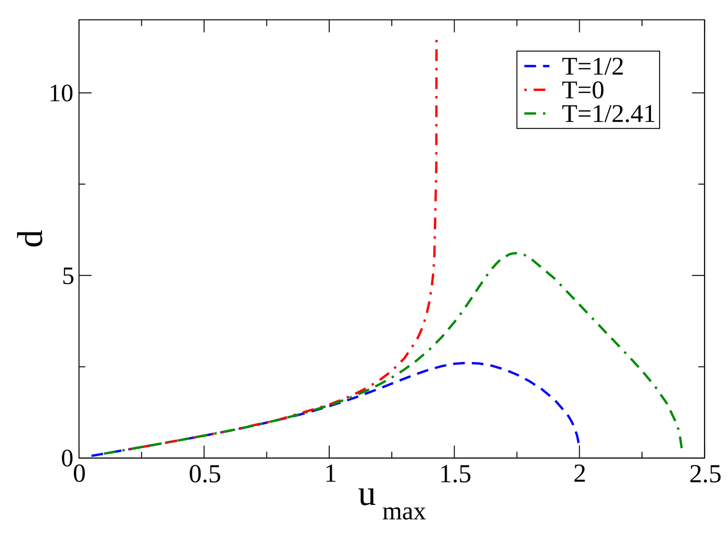

Figure 6: The distance d between the quarks as a function of $u_{\rm max}$ for zero and non-zero temperatures.

We restrict to gauge theories such that $\frac{dN_{\rm eff}}{d\Lambda} > 0$, which gives $\mathcal{F}' > 0$. Then for $T \equiv 1/u_h = 0$, there exists real positive $\mathbf{x}$.

- As $u_{\rm max} \to \mathbf{x}$, $d \to \infty$, [see Fig 6].
- However, $S_{\rm NG}^{\rm ren} \to \infty$ also and for large d, we have $S_{\rm NG}^{\rm ren} \sim \mathcal{T} d$, **Linear Confinement!** [see Fig 7].

For $T \neq 0$,

- $\mathbf{x}$ exists for small T - d can be arbitrarily large- we again have confinement at low temperature.
- For large $T > T_c$, $\mathbf{x}$ does not exist, d is finite- short range interaction. We get maximum value $d_{\rm max}$ for inter quark distance [5] [1] such that for $d > d_{\rm max}$, there is no U shaped string - **Deconfined Quarks!**.
- In deconfined phase, we have two free strings stretched between 7 brane and black-hole horizon, and energy is independent of distance between them.

From Fig 7 we observe that when d is small, we have the **Coulomb potential**. Whereas at large d and small T, we have **Linear Confinement**. For large T, the potential melts!

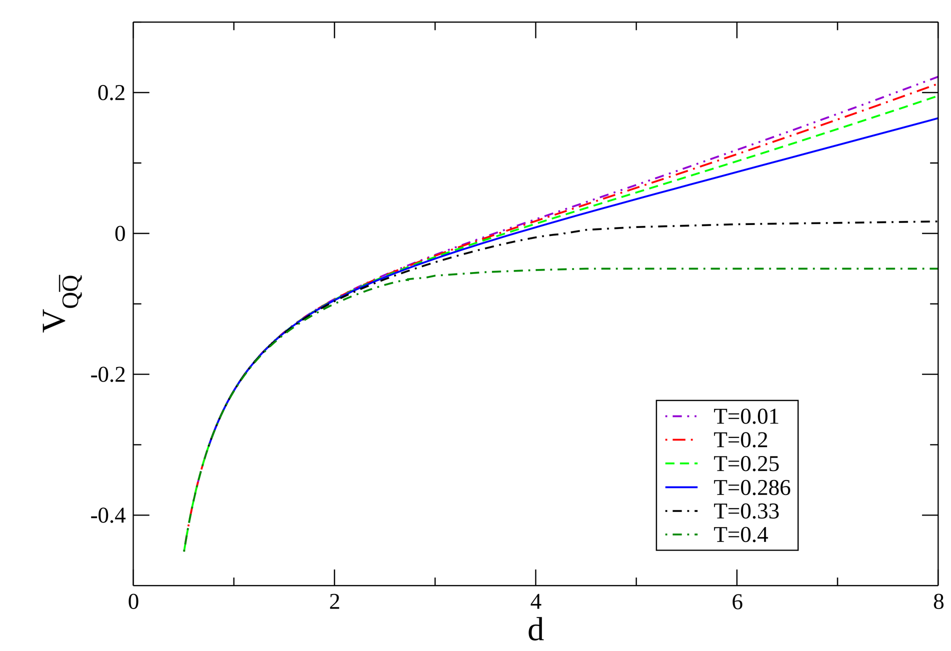

Figure 7: Quarkonium free energy as a function of inter quark distance d at various temperatures.

To estimate melting temperature T_c, we look at how the slope of linear potential change with T.

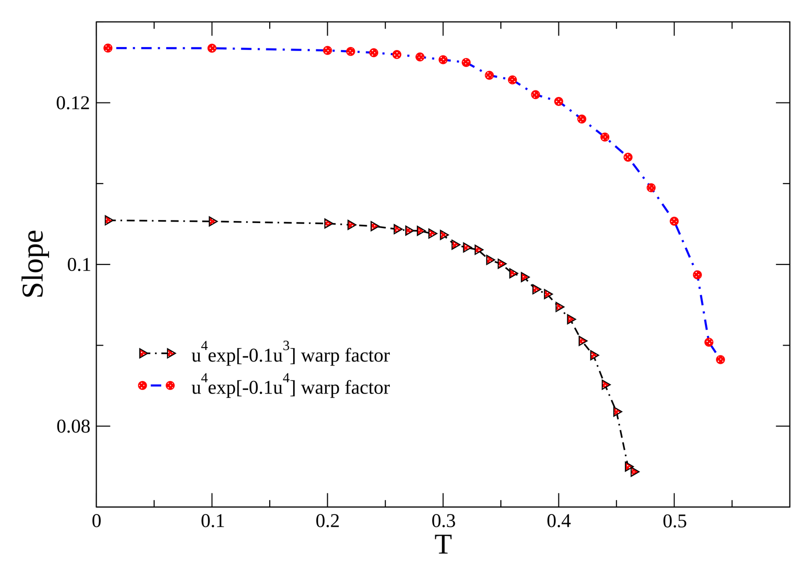

Figure 8: Slope as a function of T for two distinct asymptotically conformal gauge theories. For small variation of T near some critical value T_c, slope sharply changes.

Also looking at variation of $d_{\rm max}$ as a function of T, we see sharp change near $T \sim T_c$ and our numerical analysis leads to the following estimate for critical temperature, putting back units:

$$\frac{0.91}{\sqrt{g_s N \alpha'}} \leq T_c \leq \frac{1.06}{\sqrt{g_s N \alpha'}}$$

Summary

- Starting with a brane configuration in ten dimensions, we obtain gravity dual of a gauge theory which resembles large N QCD at low energies.
- The quarkonium potential at large distances is linear - thus we have confinement. As temperature grows, the potential melts and there is a maximum separation between the quarks after which they become deconfined.

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