

Net-baryon-, net-proton-, and net-charged particle kurtosis in heavy-ion collisions within a relativistic transport approach

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HGS-HIRe for FAIR
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How to study the QCD phase diagram...

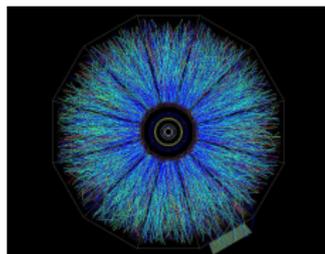
... be brave and solve

$$Z(T, \mu_B) = \int \mathcal{D}(A, q, q^\dagger) e^{-S_{\text{QCD}}^E}$$

ab initio and nonperturbatively,



... be strong and collide heavy ions at ultrarelativistic energies,



... be creative and study effective models of QCD.

\mathcal{L}_{eff}

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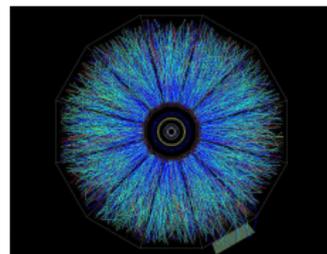
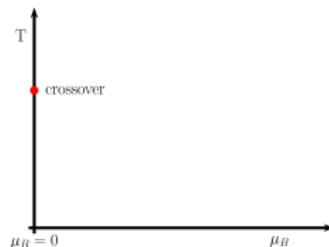
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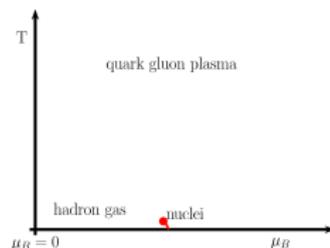
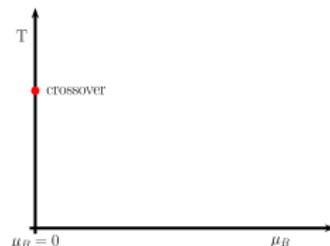
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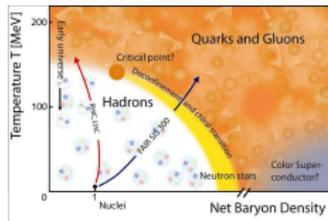
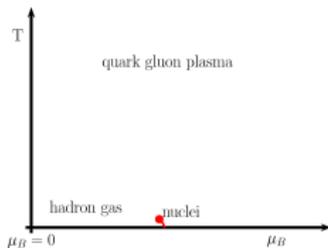
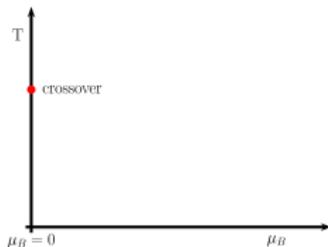
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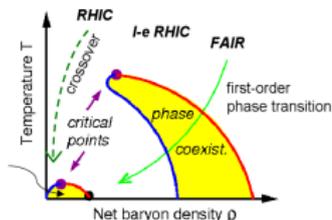
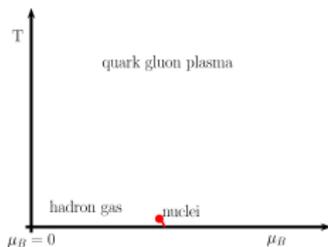
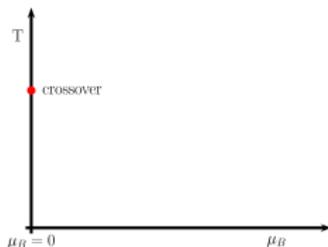
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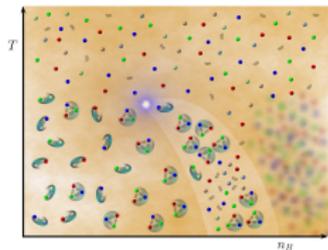
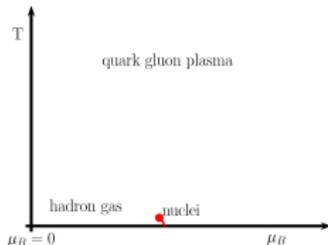
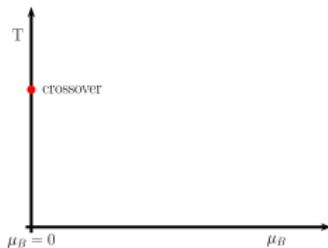
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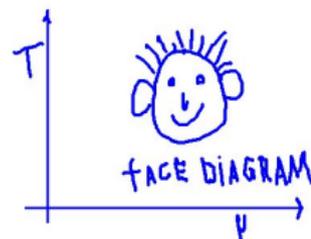
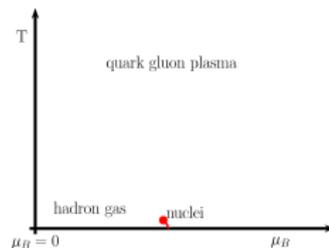
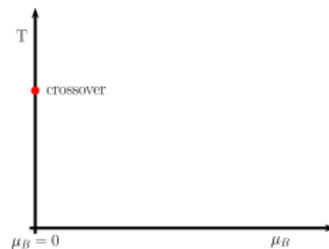
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Definition of the kurtosis

susceptibilities of conserved charges

$$\chi_n(T, \mu_N) = \frac{1}{VT^3} \frac{\partial^n \ln Z(V, T, \mu_N)}{\partial \mu_N^n} \Big|_T$$

N : net-baryon, net-proton, net-charge number
quadratic and quartic susceptibilities :

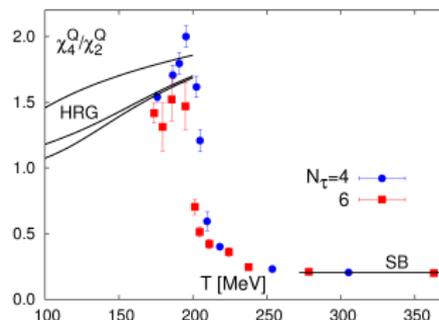
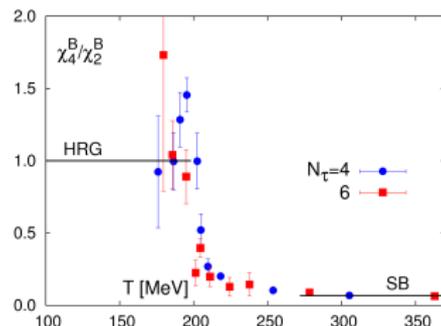
$$\chi_2 = \frac{1}{VT^3} \langle \delta N^2 \rangle$$
$$\chi_4 = \frac{1}{VT^3} (\langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2)$$

effective kurtosis:

$$K^{\text{eff}} = \frac{\chi_4}{\chi_2} = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3 \langle \delta N^2 \rangle \equiv \kappa \sigma^2 .$$

Kurtosis on the lattice

thermodynamic susceptibilities can be calculated on the lattice



reproduces the HRG below T_c and the Stefan-Boltzmann limit at high temperatures

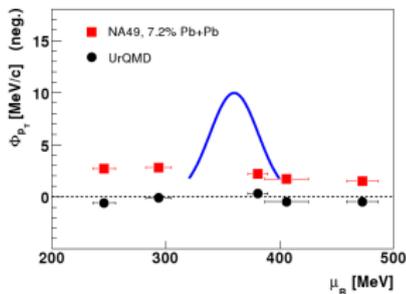
⇒ has the potential to probe the confined and the deconfined phase of QCD

Fluctuations at the critical point

non-monotonic fluctuations in pion and proton multiplicities

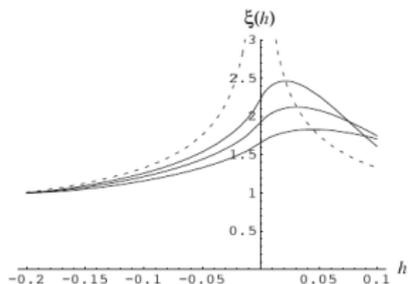
$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRD **60** (1999))



(NA49 collaboration J. Phys. G **35** (2008))

BUT: critical slowing down



(B. Berdnikov and K. Rajagopal, PRD **61** (2000))

Fluctuation measures based on the second moments do not show any critical behavior.

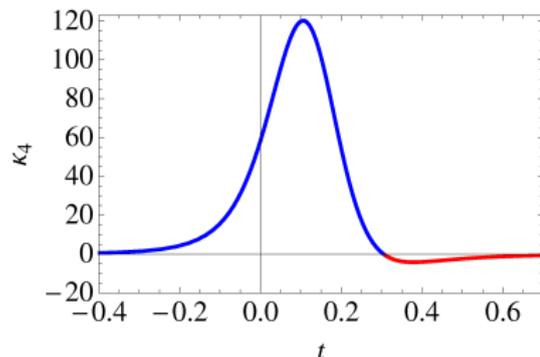
Higher moments at the critical point

higher moments of the distribution of conserved quantities are more sensitive to critical phenomena

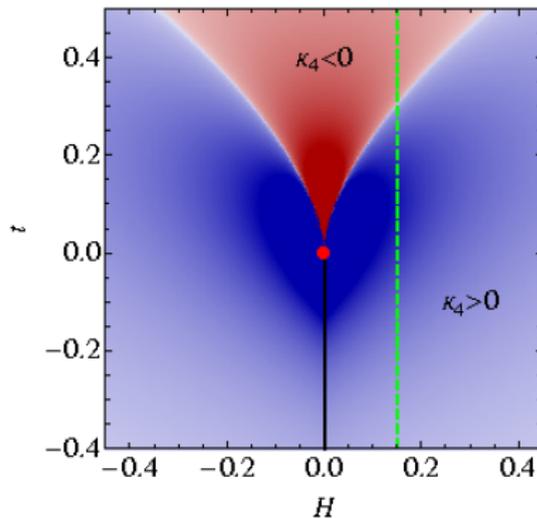
$$\kappa \propto \zeta^7$$

(M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009))

The kurtosis can be negative at the critical point!

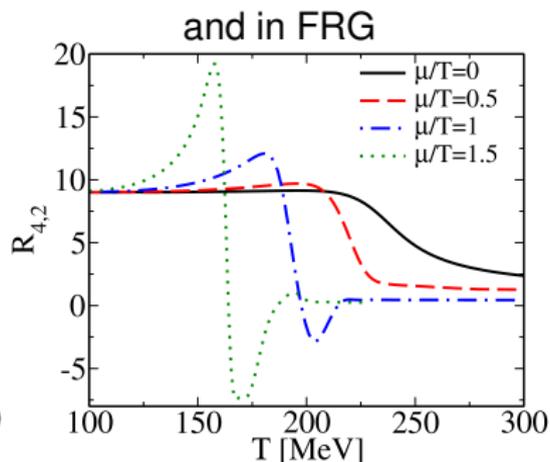
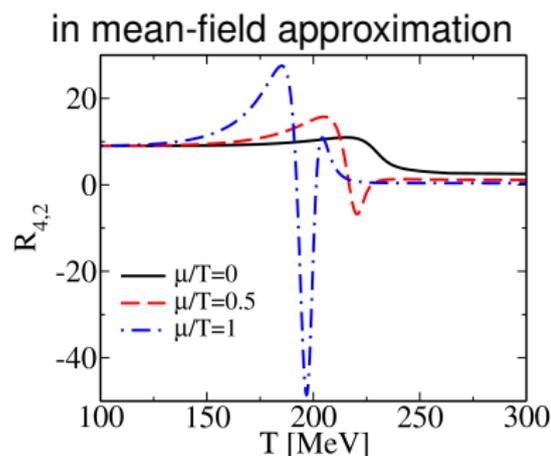


(M. A. Stephanov, arXiv:1104.1627 [hep-ph])



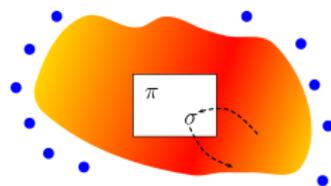
Kurtosis in effective models

The effective kurtosis can be calculated in effective models, e.g. in the Polyakov-loop extended quark-meson model:



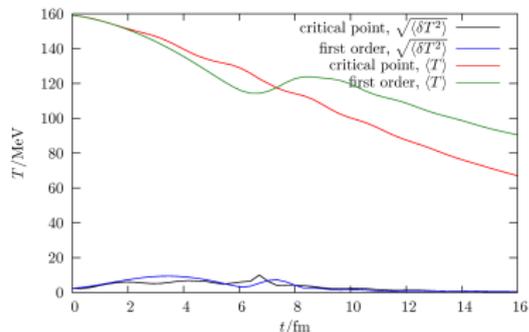
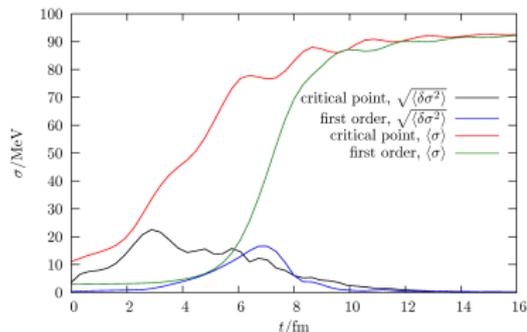
Motivation

- ▶ Fluctuations have so far been investigated in static systems.
 - ▶ However, systems created in a heavy-ion collisions are finite in size and time and inhomogeneous.
 - ▶ Necessary to propagate fluctuations explicitly!
 - ▶ e.g.: hydrodynamic fluctuations (see talk by J.Kapusta)
-
- ▶ Nonequilibrium chiral fluid dynamics:
 - ▶ fluid dynamics +
 - ▶ phase transition model +
 - ▶ explicit propagation of fluctuations +
 - ▶ dissipation and noise

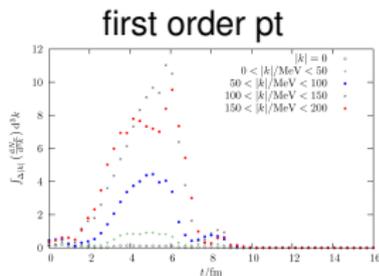
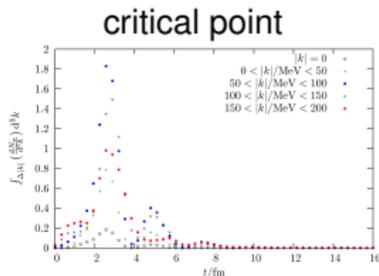
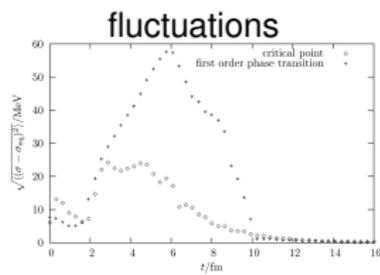


Dynamic fluctuations at the phase transition

Relaxation of the order parameter and reheating effects



intensity of σ fluctuations
$$\frac{dN_\sigma}{d^3k} = \frac{(\omega_k^2 |\sigma_k|^2 + |\partial_t \sigma_k|^2)}{(2\pi)^3 2\omega_k}$$



Relativistic Transport Approach

cover other effects in realistic simulations of heavy-ion collisions,
here: UrQMD (www.urqmd.org)

issues:

- ▶ centrality selection
- ▶ eventwise baryon number and charge conservation instead of grandcanonical ensembles
- ▶ stopping power, transport of baryon number to midrapidity, not thermal

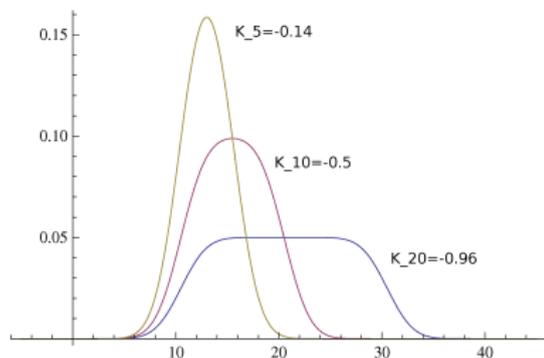
Centrality selection, e.g. by impact parameter

We investigate central collisions with $b \leq 2.75$ fm.

The superposition of two Gauss distributions (with mean $\mu_{1,2}$ and variance $\sigma_{1,2}$) has a negative kurtosis

$$K_2 = \frac{1/8\Delta\mu^4 + 3\Sigma^2\Delta\mu^2 + 6\Sigma^4}{1/8\Delta\mu^4 + \Sigma^2\Delta\mu^2 + 2\Sigma^4} - 3 < 0$$

with $\Delta\mu = |\mu_2 - \mu_1|$ and $\Sigma^2 = \sigma_1^2 + \sigma_2^2$.



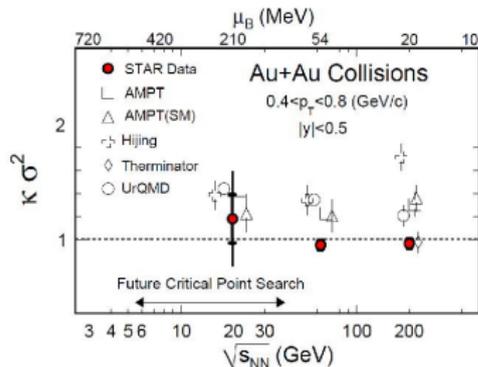
The distribution approaches a box-distribution with a $K_{\text{box}} = -1.2$.

Effects of centrality selection

Suggestion by STAR to reduce centrality bin width effects:

- ▶ calculate moments for each fixed N_{charge} in one wider centrality bin
- ▶ take the weighted average

(PhD-thesis by Xiaofeng Luo)



(STAR Collaboration, Phys. Rev. Lett. 105 (2010) 022302.)

problems:

- ▶ (anti-) protons constitute a larger fraction of all charged particles with decreasing energy
- ▶ fixing N_{charge} puts a bias on the fluctuations

Analytic toy model

Baryon number conservation limits fluctuations of net-baryon number.

$$P_{\mu}(N, C) = \mathcal{N}(\mu, C) e^{-\mu} \frac{\mu^N}{N!} \quad \text{on} \quad [\mu - C, \mu + C]$$

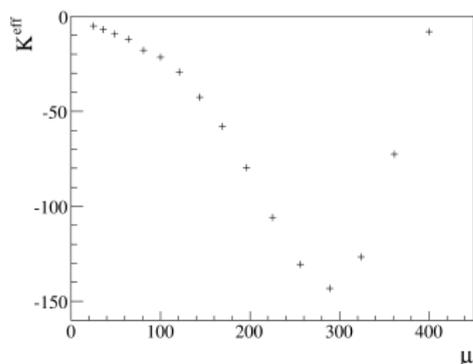
μ : the expectation value of the original Poisson distribution, $\mathcal{N}(\mu, C)$: normalization factor, $C > 0$: cut parameter

$$C = \alpha \sqrt{\mu} \left(1 - \left(\frac{\mu}{N_{\text{tot}}} \right)^2 \right).$$

$$\alpha = 3, N_{\text{tot}} = 416.$$

An increase of the average net-baryon number does not lead to stronger fluctuations.

When the upper limit of the total net-baryon number N_{tot} is approached the distribution changes to a δ -function ($K_{\delta}^{\text{eff}} = 0$).

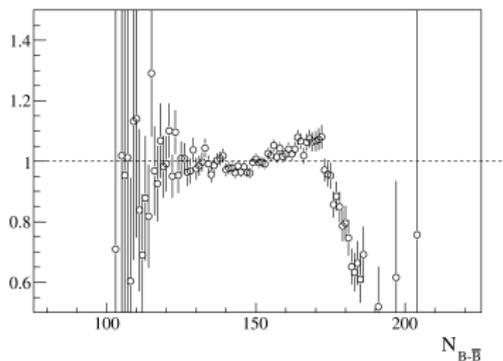
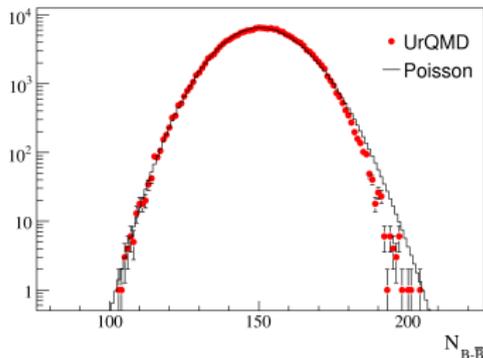


Net-baryon number distribution in UrQMD

net-baryon number distribution

- ▶ central Pb+Pb collisions at $E_{\text{lab}} = 20\text{A GeV}$
- ▶ fit to a Poisson distribution
- ▶ shoulders are enhanced
- ▶ tails are cut

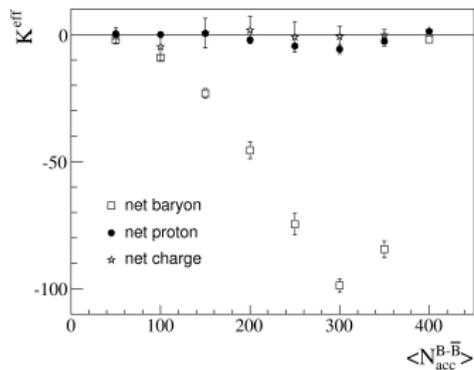
\Rightarrow decrease from $K_{\text{Poisson}}^{\text{eff}} = 1$
to $K_{\text{UrQMD}}^{\text{eff}} = -22.2$



ratio of UrQMD to Poisson
distribution

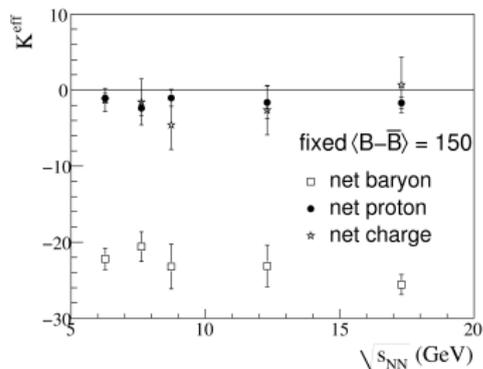
Rapidity window dependence of the effective kurtosis

- ▶ Same qualitative behavior of the net-baryon kurtosis as expected from the analytic toy model.
- ▶ $E_{\text{lab}} = 158 \text{ A GeV}$
- ▶ The net-proton kurtosis only slightly follows this trend.
- ▶ The net-charge kurtosis is not influenced, but error bars are larger.
- ▶ For small net-baryon numbers in the acceptance, the values of net-baryon, net-proton and net-charge kurtosis are compatible with values of 0 – 1.

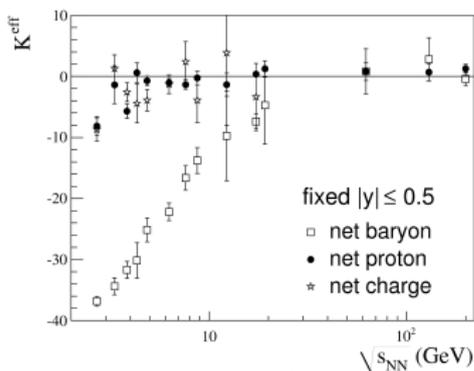


Energy dependence of the effective kurtosis

- ▶ adapting the rapidity window to fix the mean net-baryon number
- ▶ net-baryon effective kurtosis does not show an energy dependence



- ▶ fixed rapidity cut
- ▶ the net-baryon number varies with \sqrt{s}
- ▶ for lower \sqrt{s} K^{eff} becomes increasingly negative
- ▶ at $E_{\text{lab}} = 2A\text{GeV}$:
 $\langle N_{B-\bar{B}} \rangle \simeq 240$



Summary

- 
- ▶ Studied the effective kurtosis within a transport model of heavy-ion collisions.
 - ▶ dependence on energy and rapidity window
 - ▶ negative values for net-baryon K^{eff} below $\sqrt{s} = 100$ GeV
 - ▶ baryon number conservation qualitatively described by a cut Poisson distribution
 - ▶ centrality selection remains a crucial issue!

Outlook:

- ▶ further study the acceptance effects in UrQMD
- ▶ extend nonequilibrium chiral fluid dynamics to study event-by-event fluctuations

