Dissipative dynamics of highly anisotropic plasmas

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1. Abstract

We present a method to improve the description of 0+1 dimensional boost invariant dissipative dynamics in the presence of large momentum-space anisotropies. Instead of using the canonical hydrodynamical expansion of the distribution function around an isotropic equilibrium state, we expand around a state which is anisotropic in momentum space and parameterize this state in terms of proper-time and spatial-rapidity dependent parameters. At leading order the result obtained is two coupled hydro-like differential equations for the momentum-space anisotropy and typical momentum of the degrees of freedom. Within this framework, we get both the ideal hydrodynamic and free streaming expansion as asymptotic limits. In addition, we show that when linearized the differential equations reduce to 2nd order Israel-Stewart viscous hydrodynamics. Finally, we make quantitative comparisons of the evolution of the pressure anisotropy within our approach and 2nd order viscous hydrodynamics.

2. Hydrodynamics from kinetic theory

In order to approach to disipative dynamics from kinetic theory, one usually takes moments of the Boltzmann equation $p^\mu\partial_\mu f(\mathbf{x},\mathbf{p},t) = -\mathcal{C}[f]$:

$$\int_{\mathbf{p}} p^\mu\partial_\mu f(x,p) \Rightarrow \partial_\mu N^\mu = \int_{\mathbf{p}} \mathcal{C}[f]\ \text{, Particle number Eqn.}$$

$$\int_{\mathbf{p}} p^\mu p^\nu\partial_\mu f(x,p) \Rightarrow \partial_\mu T^{\mu\nu} = \int_{\mathbf{p}} p^\nu\mathcal{C}[f] \equiv 0\ \text{, Energy-momentum conservation}$$

$$\int_{\mathbf{p}} p^\mu p^\nu p^\lambda\partial_\mu f(x,p) \Rightarrow \partial_\mu I^{\mu\nu\lambda} = \int_{\mathbf{p}} p^\nu p^\lambda\mathcal{C}[f] \equiv P^{\nu\lambda}\ \text{.Dissipative fluxes Eqn.}$$

To obtain the usual equations of motion of hydrodynamics, one expands the distribution function around an equilibrated state at certain temperature T:

$$f(x,p) = f_{eq.}(p_T^2 + p_z^2, T) + \delta f_1 + \delta f_2 + \ldots .$$

The functional form of the fluctuations δf_n can be evaluated by using for instance 14 Grad's ansatz or Chapman-Enskog method.

One limitation of this approach occurs when the system is highly anisotropic in momentum space so $\delta f_n \gg f_{eq}$ $\Rightarrow$ breakdown of the perturbative expansion.

3. Kinetic theory approach to a highly anisotropic QGP

We propose to reorganize the perturbation series by expanding the distribution function around an anisotropic state in momentum-space [1]:

$$f(x,p) = f_{anis.}(p_T^2 + (\xi+1)p_z^2, p_{hard}),$$

where p_{hard} is the typical momentum scale of the system and ξ is the anisotropy parameter.

3.1. Evolution equations for p_{hard} and ξ

We consider that the collisional kernel is given by the relaxation time approach, i.e.,

$$\mathcal{C}[f_{anis.}] = p_\mu u^\mu \Gamma\left(f_{anis.}(p_T^2 + (\xi+1)p_z^2, p_{hard}) - f_{eq.}(|p|, T(\tau))\right)$$

where Γ is the collision rate.

By taking the first two moments of the Boltzmann equation for an anisotropic distribution function, we get [1]:

$$\frac{1}{1+\xi}\partial_\tau\xi - \frac{2}{\tau} - \frac{6}{p_{hard}}\partial_\tau p_{hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right],$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau\xi + \frac{4}{p_{hard}}\partial_\tau p_{hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right],$$

where $\mathcal{R}'(\xi) = \partial_\xi\mathcal{R}(\xi) = \frac{1}{4}\left(\frac{1-\xi}{\xi(1+\xi)^2} - \frac{\arctan\sqrt{\xi}}{\xi^{3/2}}\right)$.

3.2. Asymptotic limits

Three interesting analytical limits of the solutions of the evolution equations are found [1]:

- *Ideal hydrodynamical expansion:* $\Gamma \to \infty$

$$\xi(\tau) = 0$$

$$p_{hard} = p_{hard}(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{1/3}$$

- *Free streaming expansion:* $\Gamma \to 0$

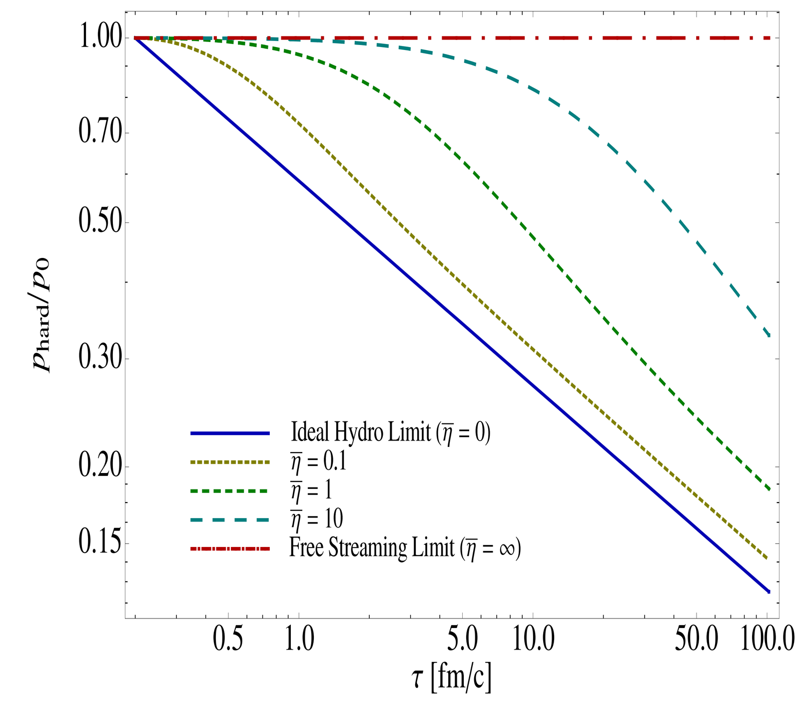
$$\xi(\tau) = (1+\xi(\tau_0))\left(\frac{\tau}{\tau_0}\right)^2 - 1,$$

$$p_{hard} = p_{hard}(\tau_0).$$

- :*2nd order Israel-Stewart theory:* in the small anisotropy parameter $\mathcal{O}(\xi)$ and matching $\Gamma = \frac{2T\mathcal{S}}{5\eta}$, one finds exactly the evolution equations of the 2nd. order Israel-Stewart theory [1, 2].

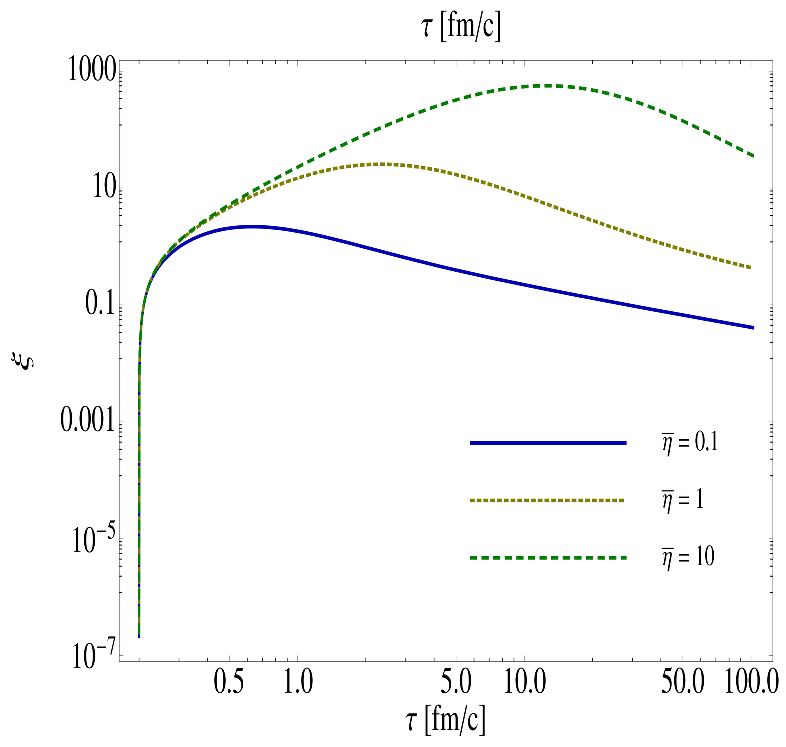
4. Results

We solve numerically the evolution equations for p_{hard} and ξ.



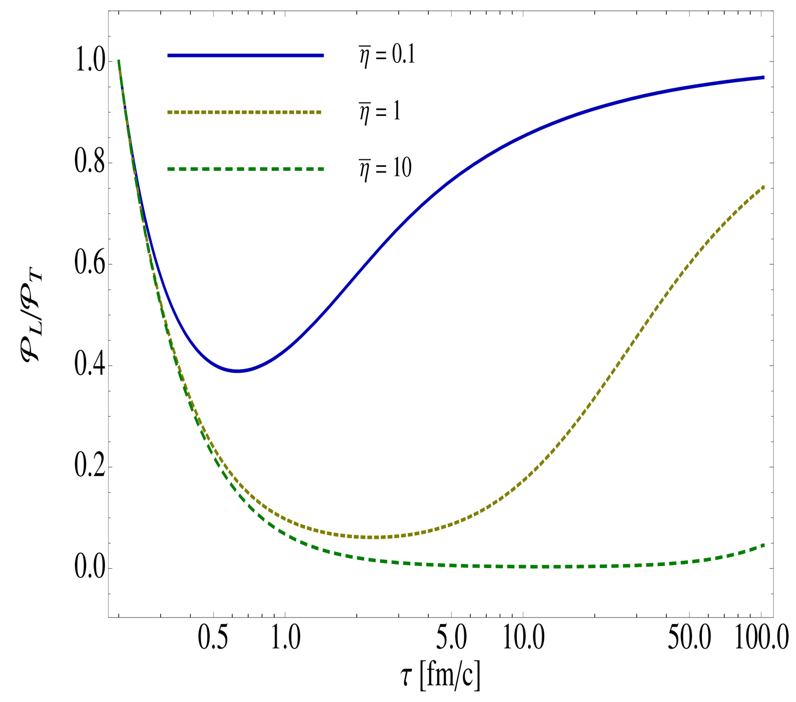
4.1. Temporal evolution of p_{hard}

We observe that the numerical solutions of $p_{hard}(\tau)$ make a smooth transition from longitudinal free streaming to ideal hydrodynamical behavior as we vary $\eta/\mathcal{S}$. The transition from early-time dynamics to late time hydro behavior can be fast or slow depending on the value of the collision rate.



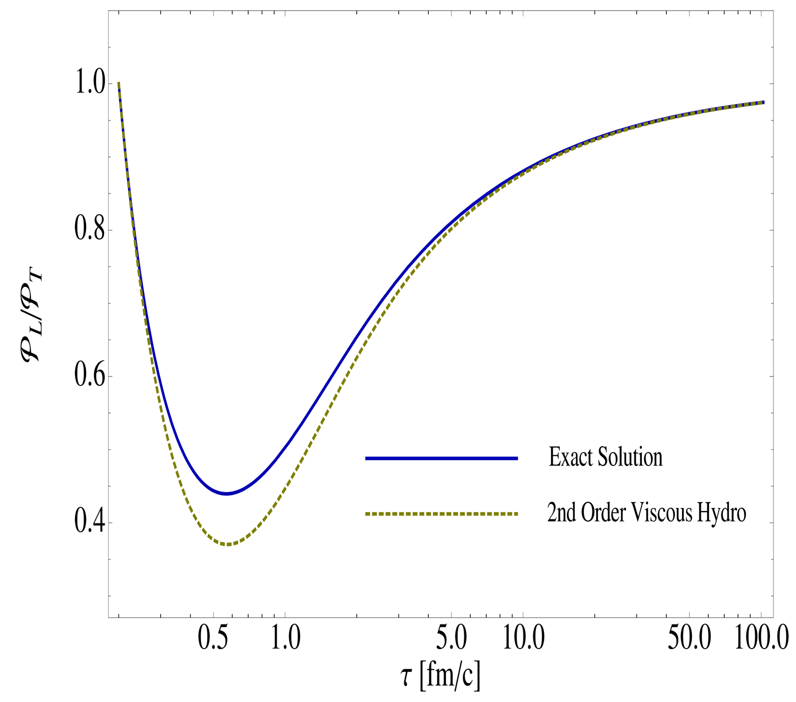
4.2. Temporal evolution of ξ

The numerical solutions of the anisotropy parameter $\xi(\tau)$ show that at very early times the behavior of the system depends weakly on $\eta/\mathcal{S}$ due to the rapid longitudinal expansion. At later times, the anisotropy has a maximum which depends on the value of $\eta/\mathcal{S}$.



4.3. Evolution of the pressure anisotropy

The ratio of the transverse to the longitudinal pressure $\mathcal{P}_L/\mathcal{P}_T$ is sensitive to momentum-space anisotropies. Our numerical solutions indicate us that as $\eta/\mathcal{S}$ is increased, the longitudinal pressure is decreased. However, at late times, the system restores isotropy in momentum-space in momentum-space for any finite value of $\eta/\mathcal{S}$. We also note that this ratio is continues and positive which means that the longitudinal pressure is positive during all the temporal evolution.



4.4. Matching 2nd. order Israel-Stewart theory

When we compare the numerical solutions for the ratio $\mathcal{P}_L/\mathcal{P}_T$ between the 2nd IS theory and our evolution equations, we observe that at late times both solutions converge. However, for $\eta/\mathcal{S} = 1/(4\pi)$, the difference between the anisotropic expansion and the IS theory is at maximum 15%. For weakly coupled values of $\eta/\mathcal{S}$, the differences are quite large$\sim$ 100% [1].

5. Conclusions

- For a 0+1 boost invariant expansion, we found solutions which smoothly connect early-time free streaming and late time viscous hydro behaviour when the expansion is around an anisotropic distribution function in momentum-space.
- By using an anisotropic distribution function and the relaxation time approach, we are able to get the ideal hydrodynamic and free streaming asymptotic limits when the collision rate goes to infinity or vanishes exactly respectively.
- When we linearize the evolution equations around small anisotropies, we recover 2nd order Israel-Stewart theory.
- The numerical solutions can be solved for extreme large values of the anisotropy in momentum-space at early-times without having negative longitudinal pressures.

References

1. M. Martinez and M. Strickland, *Nucl.Phys. A848 (2010) 183-197.*
2. W. Israel, J.M. Stewart, *Annals Phys. 118 (1979) 341-372.*