Suppression of the Repulsive Force in Nuclear Interactions near the Chiral Phase Transition

Chihiro Sasaki
Frankfurt Institute for Advanced Studies

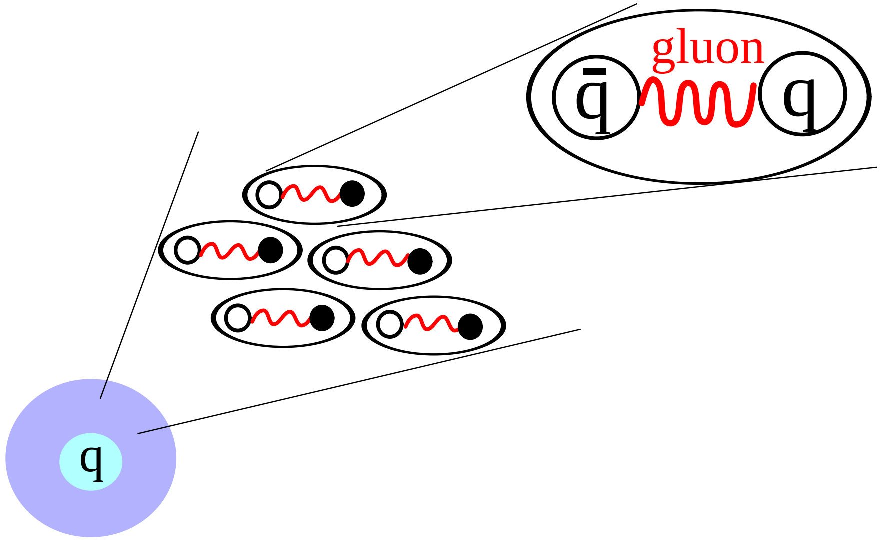
Topics

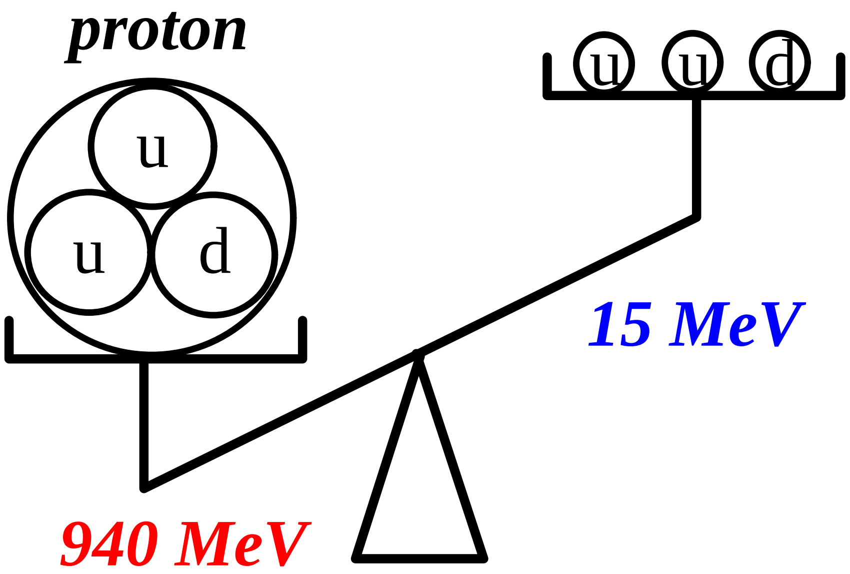
- Origin of hadron masses: role of dilaton (gluonium)
- Models for mesons, baryons and dilatons

C.S., H. K. Lee, W. G. Paeng and M. Rho, arXiv:1103.0184 [hep-ph]

Quantum Chromodynamics (QCD) underlies Hadron Physics

- origin of hadron masses? $\cdots$ dynamical chiral symmetry breaking





- emergence of a scale in QCD $\cdots$ trace anomaly
  $\mathcal{L}_{\rm QCD}$ is invariant **at classical level** under scale transf. $x^\mu \to e^\tau x^\mu$ but not **at quantum level**.

$$\partial_\mu J^\mu = T^\mu_\mu = -\left(\frac{11}{24}N_c - \frac{1}{12}N_f\right)\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu} + \sum_f m_f \bar{q}_f q_f$$

- hadron masses: not only from $\langle \bar{q}q \rangle$ but also other condensates

$$m_H = \mathcal{F}\left(m_{\rm SCSB}\,, m_{\rm non-SCSB}\right)$$

Baryons near chiral symmetry restoration

- dynamical origin of nucleon mass?
  - Gell-Mann Levy model: spontaneous χSB generates $m_N = g\langle\sigma\rangle$.

$$\mathcal{L}_{\rm GL} = i\bar{N}\partial\!\!\!/ N - g\bar{N}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N + \mathcal{L}_{\rm meson}$$

  - chiral transf. for a nucleon *assumed* to be the same as for a quark
- m_N at χ-symmetry restoration?
  - standard assignment: DχSB generates entire masses. $m_N \stackrel{\sigma\to 0}{\to} 0$

$$\psi_L \to L\,\psi_L\,, \quad \psi_R \to R\,\psi_R \quad \sim \quad \psi_L : (1/2,0) \quad \psi_R : (0,1/2)$$

  - mirror assignment: DχSB generates mass difference of parity doublers.

$m_{N_+} \stackrel{\sigma\to 0}{\to} m_{N_-} = m_0 \neq 0$ [Detar-Kunihiro (89)]

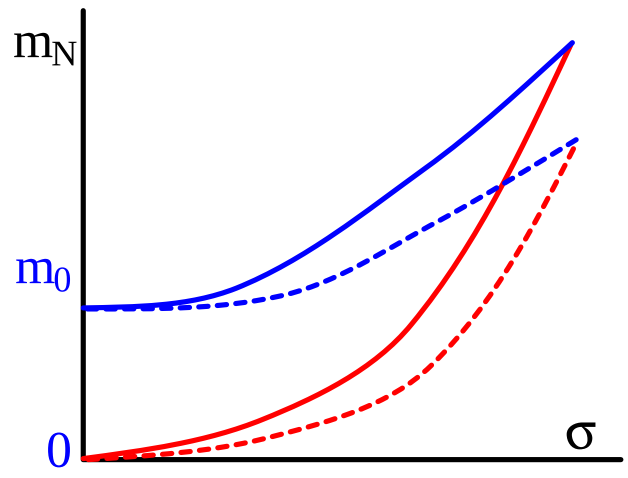
$$\begin{array}{llll} \psi_{1L} \to L\,\psi_{1L}\,, & \psi_{1R} \to R\,\psi_{1R} & \sim & \psi_{1L} : (1/2,0) \quad \psi_{1R} : (0,1/2) \\ \psi_{2L} \to R\,\psi_{2L}\,, & \psi_{2R} \to L\,\psi_{2R} & \sim & \psi_{2L} : (0,1/2) \quad \psi_{2R} : (1/2,0) \end{array}$$

$$\begin{aligned} \mathcal{L} \;=\; & \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0\left(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2\right) \\ & + a\bar{\psi}_1\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)\psi_1 + b\bar{\psi}_2\left(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)\psi_2 + \mathcal{L}_M \end{aligned}$$

- masses of positive and negative parity nucleons:

$$m_{N\pm} = \frac{1}{2}\left[\sqrt{(a+b)^2\sigma^2 + 4m_0^2} \mp (a-b)\sigma\right]$$

function of σ (SCSB) and m_0 (non-SCSB)



- how large is m_0?

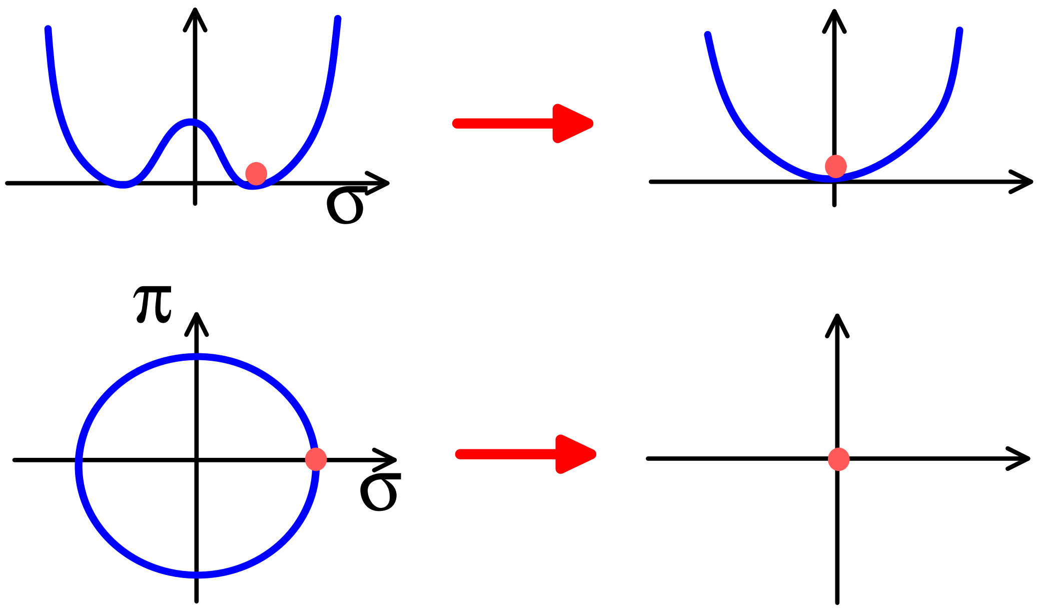
  - $m_0 \sim 270$ MeV in LσM (tree) [DeTar-Kunihiro (89)]
  - $m_0 \sim 460$ MeV in LσM (tree) [Gallas et al. (09)]
  - $m_0 \sim 330$ MeV in HBChPT (1-loop) [Nemoto et al. (98)]
  - $m_0 \sim 200$ MeV in NLσM with HLS (tree) [Paeng-Lee-Rho-CS (11)]
  - $m_0 \sim 800$ MeV in nuclear matter in LσM (mean field) [Zschiesche et al. (07)]

  $\cdots$ $m_0 \sim \langle H|G^2|H\rangle$, thus $m_0(T,\mu)$

  $\Rightarrow m_0 \sim 210$ MeV at $T_{\chi\mathrm{SR}} = 170$ MeV (mean field) [CS et al. (11)]

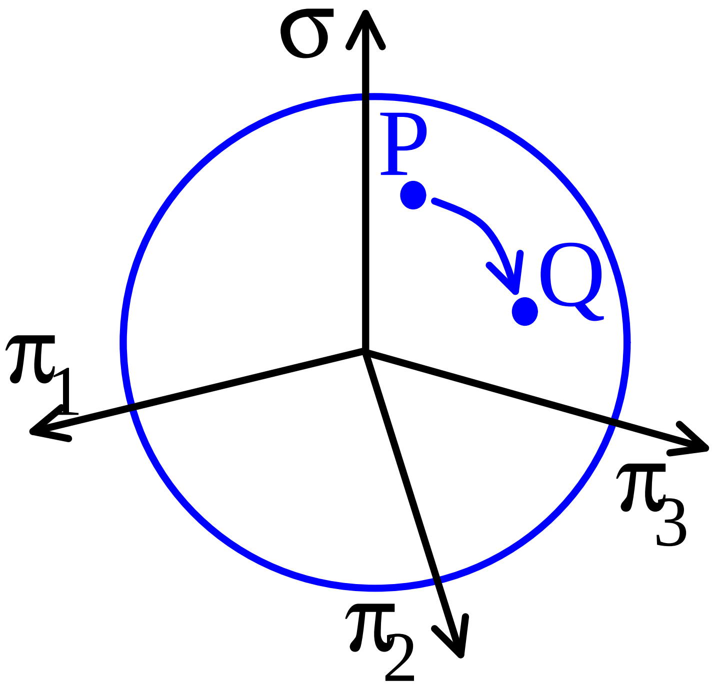
Role of scalar mesons

- nonlinear realization of chiral symmetry: pions as explicit d.o.f., scalar meson integrated out, chiral perturbation theory
- near χSR: scalar meson gets lighter and becomes massless forming an O(4) multiplet with pions, $(s, \vec{\pi})$.



switching from nonlinear SM to linear SM

- **from linear to non-linear basis, or the other way around**
  - LSM: $\Phi = (\sigma + i\vec{\tau}\cdot\vec{\pi}) \longrightarrow g_L \Phi g_R^\dagger$



$$f_\pi = \sqrt{\sigma^2 + \vec{\pi}^2}$$

$P \longrightarrow Q$: chiral transformation

$$\begin{aligned} \sigma &= f_\pi \cos\theta_1 \,, \\ \pi_1 &= f_\pi \sin\theta_1 \sin\theta_2 \cos\theta_3 \,, \\ \pi_2 &= f_\pi \sin\theta_1 \sin\theta_2 \sin\theta_3 \,, \\ \pi_3 &= f_\pi \sin\theta_1 \cos\theta_2 \,. \end{aligned}$$

  - polar decomposition: $\Phi = (\sigma_0 + \sigma)\, U$ & σ "frozen"

$$(\partial\Phi)^2 \longrightarrow (\partial U)^2 \,, \quad U(\pi) \longrightarrow g_L U(\pi) g_R^\dagger$$

  - $(\sigma\,, \vec{\pi}) \longrightarrow (\vec{\theta}; f_\pi)$: 2×2 unitary matrix $U = e^{i\vec{\tau}\cdot\vec{\theta}}$

$$\mathcal{L}(\sigma, \vec{\pi}) = \mathcal{L}(\vec{\theta}; f_\pi) = \frac{f_\pi^2}{4}\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]$$

- **two-component gluon condensate** [Miransky-Gsynin (89), Lee-Rho (09)]
  - trace anomaly in QCD

$$\partial_\mu J^\mu = T^\mu_\mu \propto \langle H|G^2|H\rangle\,, \quad H = \text{quarkonium, glueballs, etc.}$$

  - decomposition

$$\langle H|G^2|H\rangle = \underbrace{\langle G^2\rangle_{\rm soft}}_{\chi\text{SB}\,,\,N_cN_f} + \underbrace{\langle G^2\rangle_{\rm hard}}_{\text{CSB}\,,\,N_c^2}$$

  - from Lattice EoS: gluon *decondensation* at finite T [Miller (07)]

$$\langle G^2\rangle_{T_{\rm chiral}} \simeq \frac{1}{2}\langle G^2\rangle_{T=0} \quad \Rightarrow \quad \text{melting}\,\langle G^2\rangle_{\rm soft}$$

  - soft and hard dilatons

$$V(\chi) = V(\chi_s) + V(\chi_h)\,, \quad V_i = \frac{1}{4}B_i\left(\frac{\chi_i}{F_{\chi_i}}\right)^4\left[\ln\left(\frac{\chi_i}{F_{\chi_i}}\right)^4 - 1\right]$$

  - in a hot/dense medium: T & μ_B break scale symmetry explicitly.

$$V_{\rm med} = AT^4 + B\mu_B^4 + Cm^2T^2 + Dm^2\mu_B^2 + \cdots$$

- **transmutation of a scalar from NLSM to LSM** [Beane-van Kolck (94)]

1. soft dilaton (gluonium) in a NLSM: $U = \xi^2 = e^{2i\pi/F_\pi}$, $\sqrt{\kappa} = F_\pi/F_{\chi_s}$

$$\mathcal{L} = \bar{\psi} i(\not{\partial} + \not{\mathcal{V}})\psi + g_A \bar{\psi} \not{\mathcal{A}} \gamma_5 \psi - m \left(\frac{\chi_s}{F_{\chi_s}}\right) \bar{\psi}\psi + \frac{F_\pi^2}{4}\left(\frac{\chi_s}{F_{\chi_s}}\right)^2 \text{tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \mathcal{L}(\chi_s)$$

$$\mathcal{A}_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right), \quad \mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right)$$

$$V_{\chi_s} = \frac{\kappa m_{\chi_s}^2}{8F_\pi^2}\chi_s^4\left[\ln\left(\frac{\kappa \chi_s^2}{F_\pi^2}\right) - \frac{1}{2}\right], \quad B_s = \frac{1}{4} m_{\chi_s}^2 F_{\chi_s}^2$$

2. linearization: $\Sigma = \sqrt{\kappa} U \chi = s + i\vec{\tau}\cdot\vec{\pi}$ & $B = \frac{1}{2}[(\xi + \xi^\dagger) - \gamma_5(\xi - \xi^\dagger)]\psi$

$$\mathcal{L} = \bar{B} i\not{\partial} B - \frac{m_N}{2F_\pi}\bar{B}\left[\Sigma + \Sigma^\dagger + \gamma_5\left(\Sigma - \Sigma^\dagger\right)\right]B + \frac{1}{4}\text{tr}\left[\partial_\mu \Sigma \cdot \partial^\mu \Sigma^\dagger\right]$$
$$+ \frac{m_s^2}{64F_\pi^2}\left(\text{tr}\left[\Sigma\Sigma^\dagger\right]\right)^2 - \frac{m_s^2}{32F_\pi^2}\left(\text{tr}\left[\Sigma\Sigma^\dagger\right]\right)^2 \ln\left(\frac{\text{tr}\left[\Sigma\Sigma^\dagger\right]}{2F_\pi^2}\right) + \mathcal{L}_{\text{sing}}(1/\text{tr}[\Sigma\Sigma^\dagger]; \kappa, g_A)$$

3. a LSM $\mathcal{L}(s, \vec{\pi}, B)$ emerges when $\kappa \to 1$ & $g_A \to 1$ (dilaton limit).

$$\mathcal{L}_{\text{sing}} = (1-\kappa)\mathcal{F}(1/\text{tr}[\Sigma\Sigma^\dagger]) + (1-g_A)\mathcal{G}(1/\text{tr}[\Sigma\Sigma^\dagger]) \to 0$$

- **introduce vector mesons:** $(N\,,\pi\,,\rho\,,\omega\,,\chi)$ [CS-Lee-Paeng-Rho (2011)]

  hidden local symmetry (HLS) [Bando-Kugo-Uehara-Yamawaki-Yanagida (85)]

  – an extension of non-linear chiral Lagrangian: $U = \xi^2 \longrightarrow \xi_L^\dagger \xi_R$

  – vector mesons $V = \rho, \omega$ introduced as gauge bosons of HLS (redundancy in decomposition of U)

  – chiral perturbation theory: $m_V \sim \mathcal{O}(p)$ [Georgi, Tanabashi, Harada-Yamawaki]

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_N^{(\text{“naive''/mirror})} + \mathcal{L}_M^{(\text{HLS})} + \mathcal{L}_\chi \\ &= \cdots + \frac{a}{2i}\text{tr}\left[\left(\Sigma\partial_\mu\Sigma^\dagger + \Sigma^\dagger\partial_\mu\Sigma\right)V^\mu\right] + \frac{a}{2}\text{tr}\left[\Sigma\Sigma^\dagger\right]\text{tr}\left[V_\mu V^\mu\right] - \frac{1}{2g^2}\text{tr}\left[V_{\mu\nu}V^{\mu\nu}\right]\end{aligned}$$

  – dilaton limit: $\kappa = 1$ and $g_A = g_V$ (common to “naive” and mirror)

  – $g_A = 1$ in large N_c: Regge asym. behavior $J \propto m^2$ [Weinberg (90)]

  – consequence: VN repulsion suppressed $g_{VN} = g\,(1 - g_V) \rightarrow 0$

  $\Rightarrow$ **softer equation of state at high density**

- **a rough estimate of m_0**
  - mean-field thermodynamic potential around χ-SR: $\langle s \rangle \simeq 0\,, m_\pm \simeq m_0$

$$\Omega = 8\int \frac{d^3p}{(2\pi)^3} T\left[\ln\left(1-n_0\right) + \ln\left(1-\bar{n}_0\right)\right] + V(\chi_h) - \int \frac{d^3p}{(2\pi)^3} T \ln\left(1+n_h(m_{\chi_h})\right)\,,$$

$$V = \frac{1}{4}B_h\left(\frac{\chi_h}{F_{\chi_h}}\right)^4\left[\ln\left(\frac{\chi_h}{F_{\chi_h}}\right)^4 - 1\right]\,, \quad m_{\chi_h}^2 = \frac{\partial^2 V}{\partial \chi_h^2}\,.$$

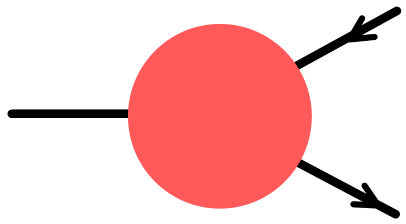
  - $T_{\rm ch} \simeq 170$ MeV & $\mu_B = 0$: $\partial\Omega/\partial\chi_h = 0$ $(\chi_h\,, m_0)$ & m_{χ_h} (χ_h)
  - from lattice EoS: $\langle G_{\mu\nu}G^{\mu\nu}\rangle_{T_{\rm ch}} \simeq \frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle_{T=0}$ [Miller (07)]

$$B_h(T_{\rm ch}) = \frac{1}{2}B(T=0)\,, \quad m_{\chi_h}^2 = \frac{1}{2}m_G^2\,.$$

known numbers: $\langle G_{\mu\nu}G^{\mu\nu}\rangle_{T=0} = 0.012$ GeV^4 [Shifman et al. (79)], $B = (0.4\,\text{GeV})^4$ [Narison (97)], $m_G = 1.7$ GeV [Sexton et al. (95)]

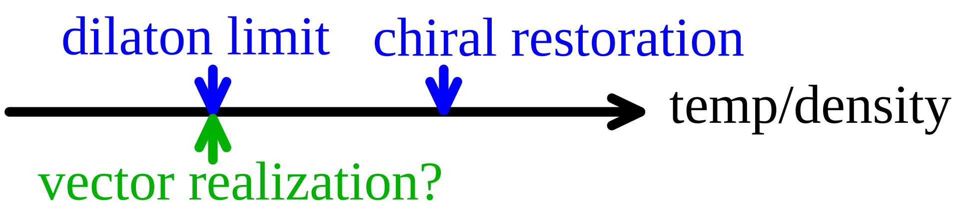
  - thermodynamically favored solution: $m_0 = 210$ MeV

Dilaton limit protected at quantum level [Paeng-Lee-Rho-CS, in preparation]



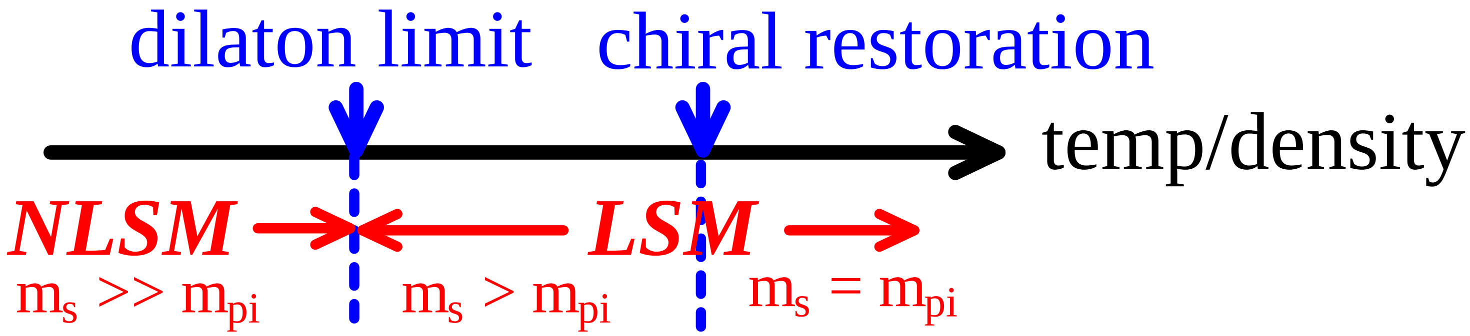
$\Rightarrow$ renormalization group equations of g_A and g_V

- IR fixed point: $(g_A - g_V\,, 1 - g_V) = (0\,, 0)$
  $\Rightarrow$ dilaton limit $(g_A = g_V = 1)$ remains the same at 1-loop.
- why $g_A = g_V = 1$?
  - similar evolution of $g_{A/V}(N_c \to \infty)$ to $g_{A/V}(\rho_B)$?
  - chiral expansion justified in large N_c: $m^2_{\rm meson}/(4\pi f_\pi)^2 \sim 1/N_c$
  - justified also if m_V becomes small (i.e. small g).
- what protects it?
  vector realization ($\rho_V \simeq \rho_A$ but $f_\pi \neq 0$)? [Georgi (89,90)]
  enlarged symmetry: $SU(2) \times SU(2) \times SU(2) \to [SU(2)]^4$



Conclusions

- **an effective chiral Lagrangian with scale invariance**



  - dilaton limit associated with IR fixed point at quantum level
  - consequence: VN repulsive forces suppressed
  - if mirror assignment: $m_0 \sim 210$ MeV at $T_{\chi SR} = 170$ MeV

- **at which T or ρ does dilaton limit set in?**
- **mixed scaler modes: quarkonium, tetraquarks, glueballs**
- **how to make a reliable estimate of m_0 in dense matter?**
- **what are phenomenological consequences on thermodynamics?**