Thermalization of fluctuations in strongly coupled plasmas

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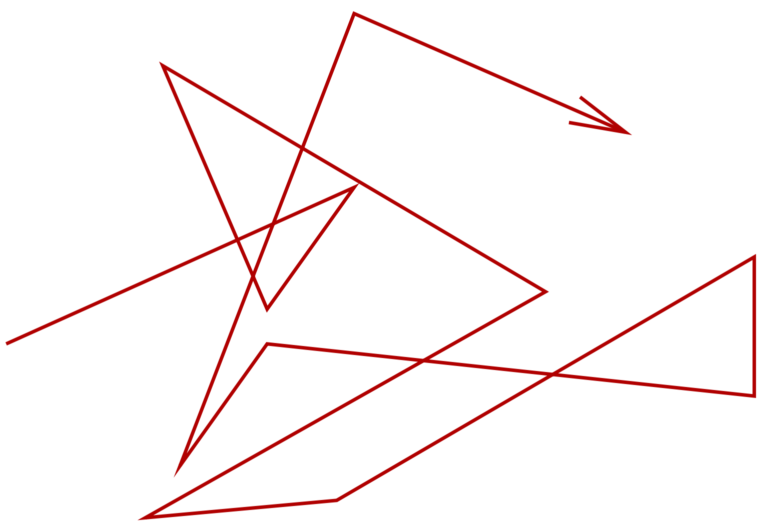
- Dam T. Son, DT; JHEP. arXiv:0901.2338
- Simon Caron-Huot, DT, Paul Chesler; PRD in press, arXiv:1102.1073

Heavy Quarks in equilibrium Quantum Field Theory

$$M\frac{d^2\mathbf{x}}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{\mathbf{x}} + \underbrace{\xi}_{\text{Noise}}$$



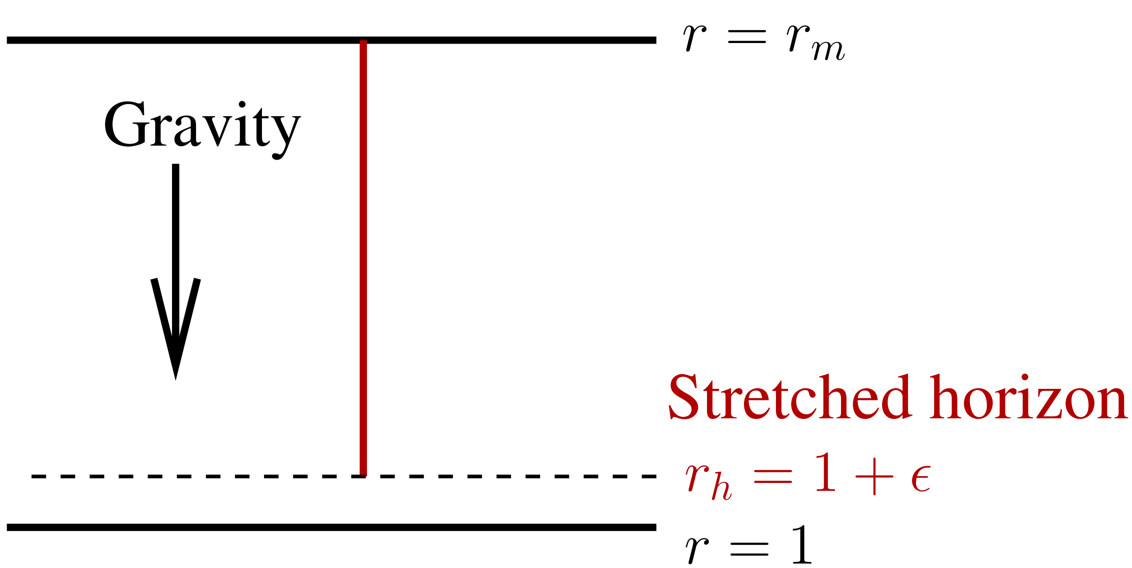
$\Longleftarrow$ "Artist's" conception of Brownian Motion

1. In equilbrium the drag and noise are balanced

$$\langle \xi(t)\xi(t')\rangle = 2T\eta\,\delta(t-t') \Leftarrow \text{Fluctuation Dissipation Theorem}$$

Heavy Quarks in equilibrium AdS

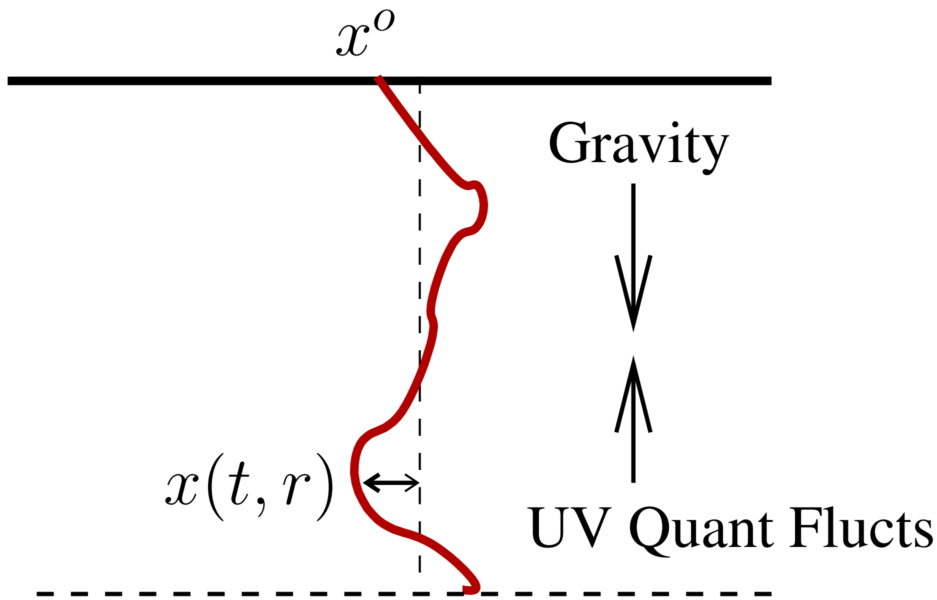
- Heavy quarks are classical strings in the 5d equilibrium AdS black hole geometry
- Solve classical string EOM and find:



Not the dual of an equilibrated quark!

Detailed Balance and Hawking Radiation:

$$M \frac{d^2 x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}} \dot{x}^o + \underbrace{\xi}_{\text{Noise}}$$



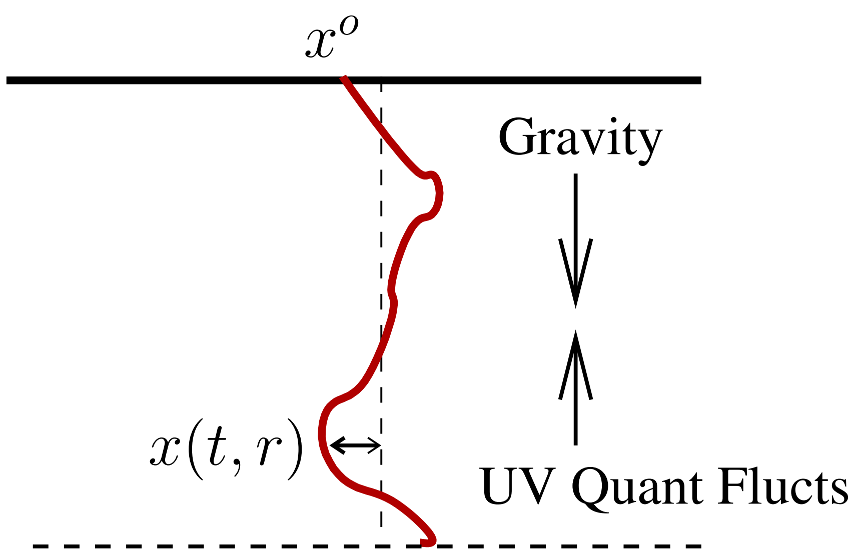
Evolves to Classical Prob Dist: (Son,DT;Iancu)

$$P[x, \pi_x] \propto e^{-\beta H[x,\pi_x]}$$

Goals:

1. Will show that Hawking Radiation is balanced by gravity
2. Generalize to non-equilibrium

Detailed Balance and Hawking Radiation (Technical Discussion)



1. Fluctuations:

$$G_{rr} \equiv \frac{1}{2} \langle \{ \hat{x}(t_1, r_1), \hat{x}(t_2, r_2) \} \rangle \ ,$$

2. Dissipation (Spectral Density)

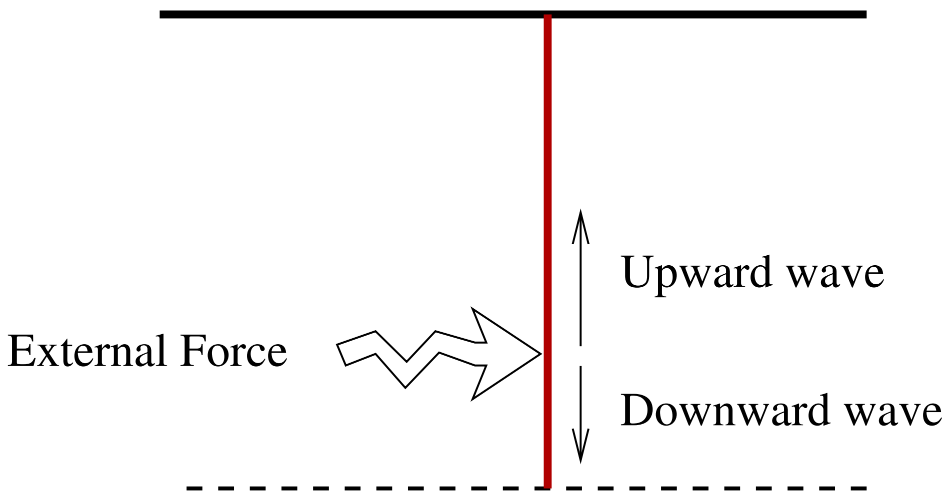
$$\rho_{ra-ar} \equiv \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle \ .$$

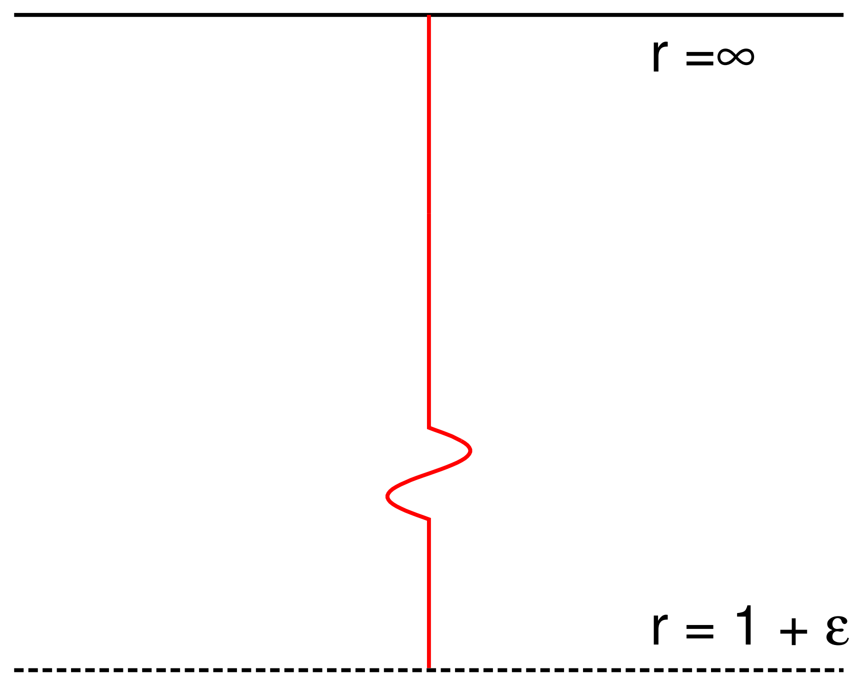
- Equilibrium $\equiv$ Fluctuation Dissipation Theorem

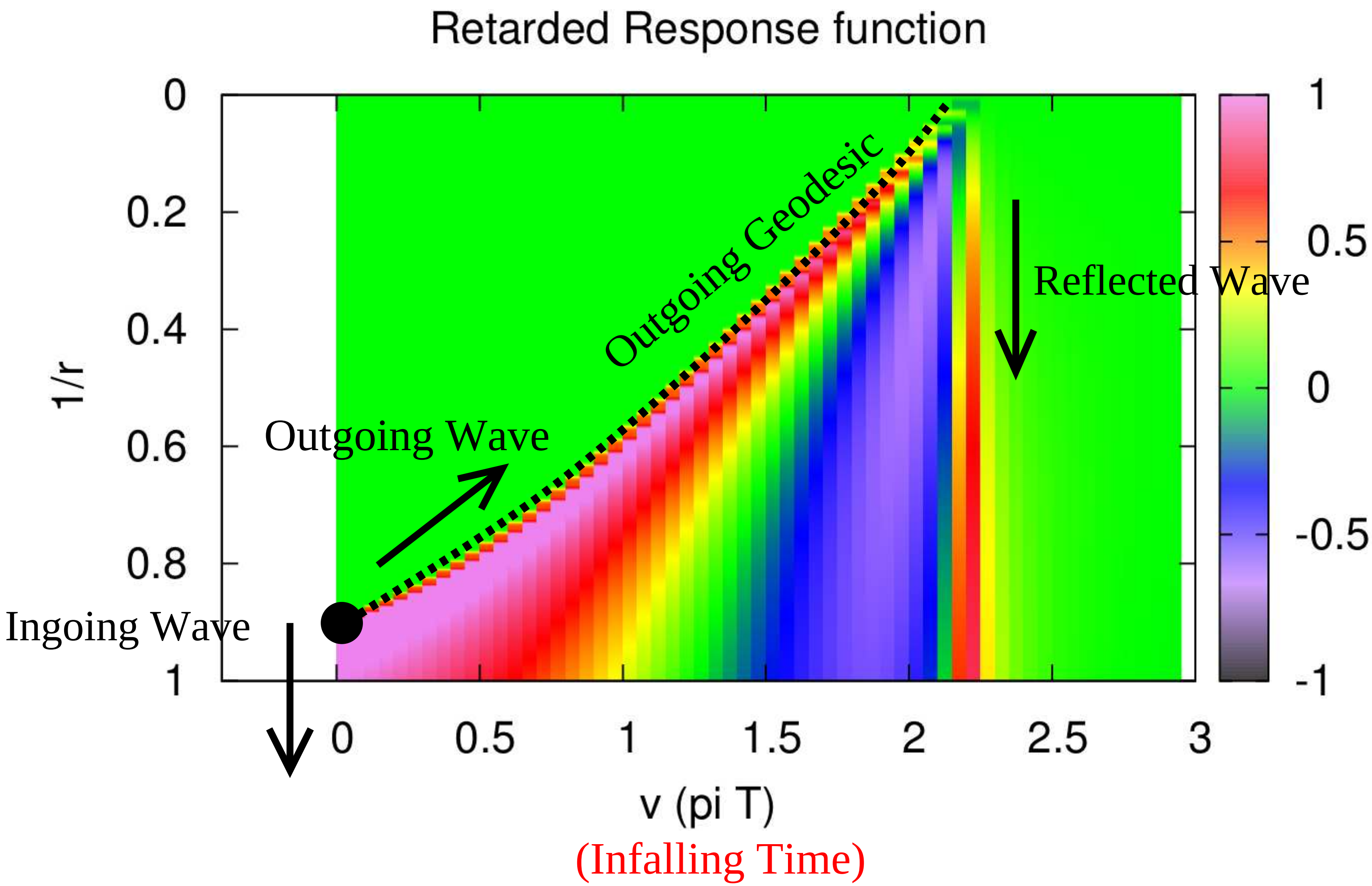
$$G_{rr}(\omega, r_1, r_2) = \left(\frac{1}{2} + n_B(\omega) \right) \rho_{ra-ar}(\omega, r_1, r_2) \qquad n(\omega) \equiv \frac{1}{e^{\omega/T} - 1}$$

The classical Green Function or response to a force:

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G = \mathcal{F}\,\delta(t_1 - t_2)\delta(r_1 - r_2)\,,$$

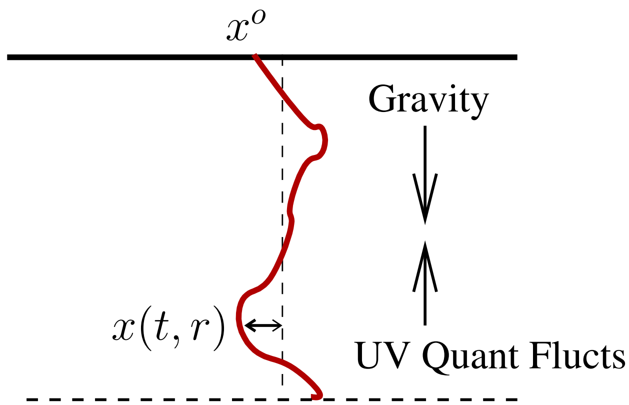






$$v = t - \frac{1}{2\pi T}\left[\tan^{-1}(r) + \tanh^{-1}(r)\right] \qquad v = \text{Eddington time}$$

Statistical Fluctuations



$$G_{rr} = \frac{1}{2} \langle \{ x(t_1, r_1), x(t_2, r_2) \} \rangle$$

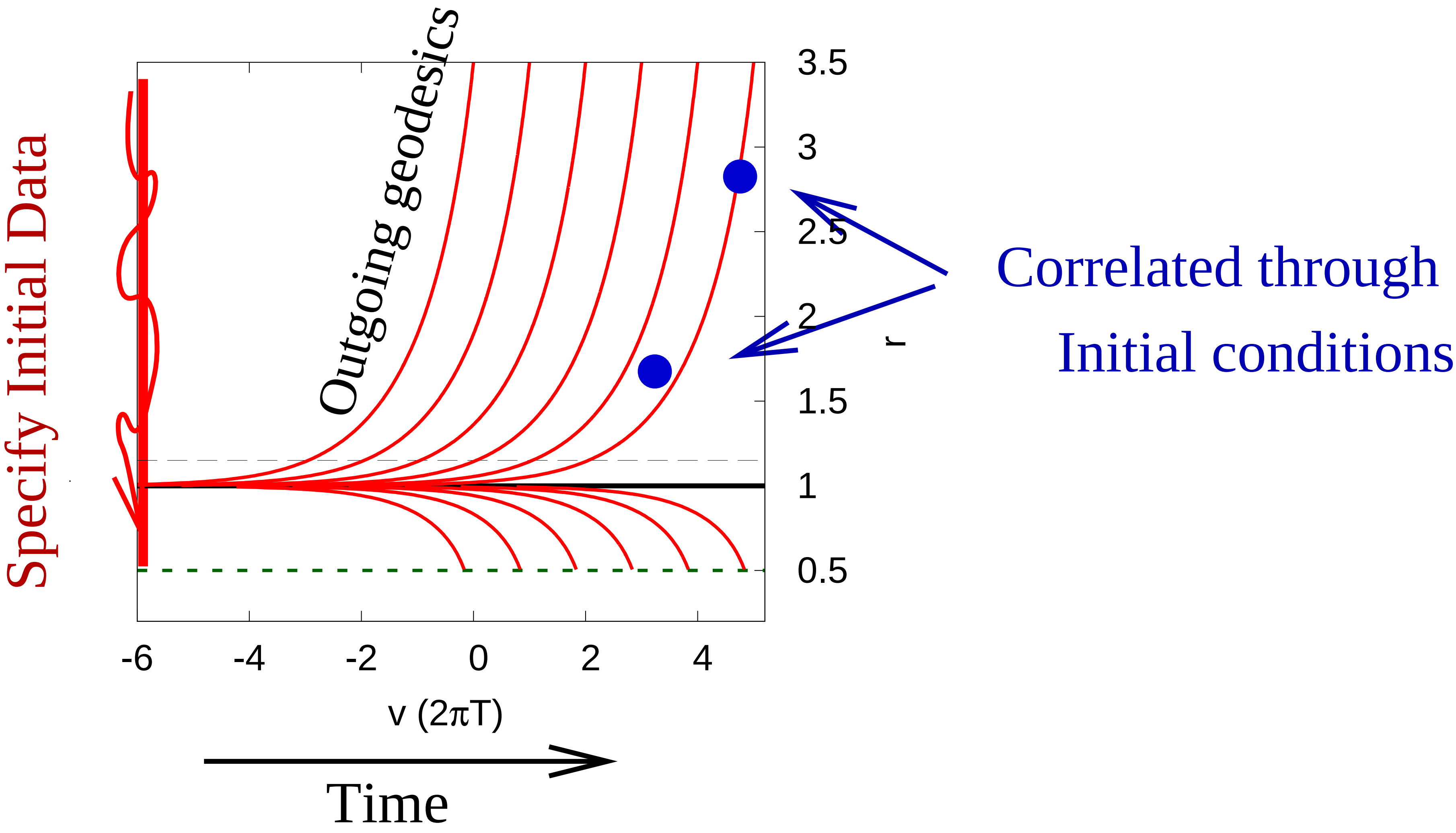
- The statistical correlator obeys the homogeneous EOM

$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] G_{rr}(t_1 r_1 | t_2 r_2) = 0$$

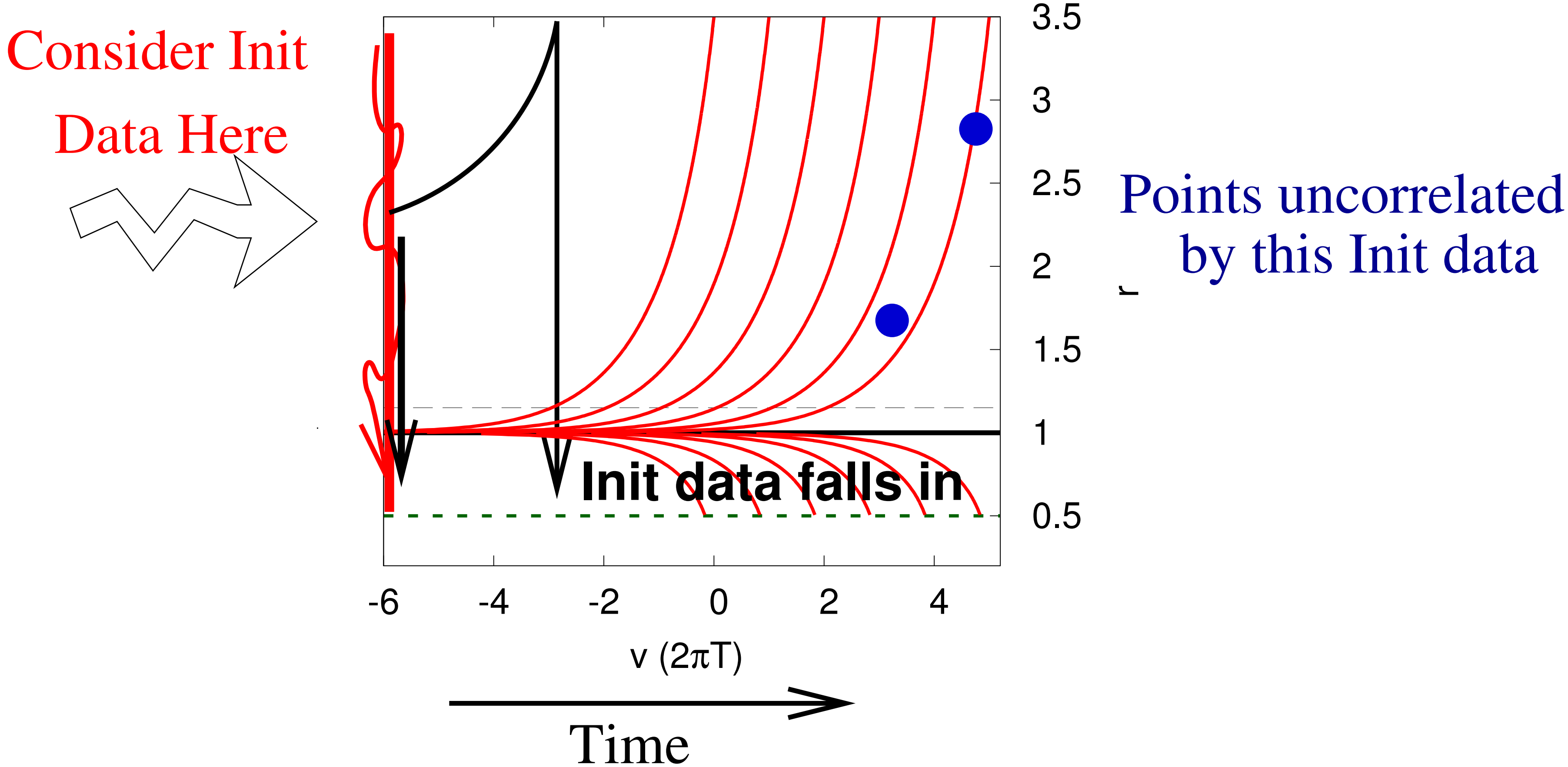
- So:

1. Specify the correlations (or density matrix) in the past
2. Final state fluctuations are correlated only through initial conditions

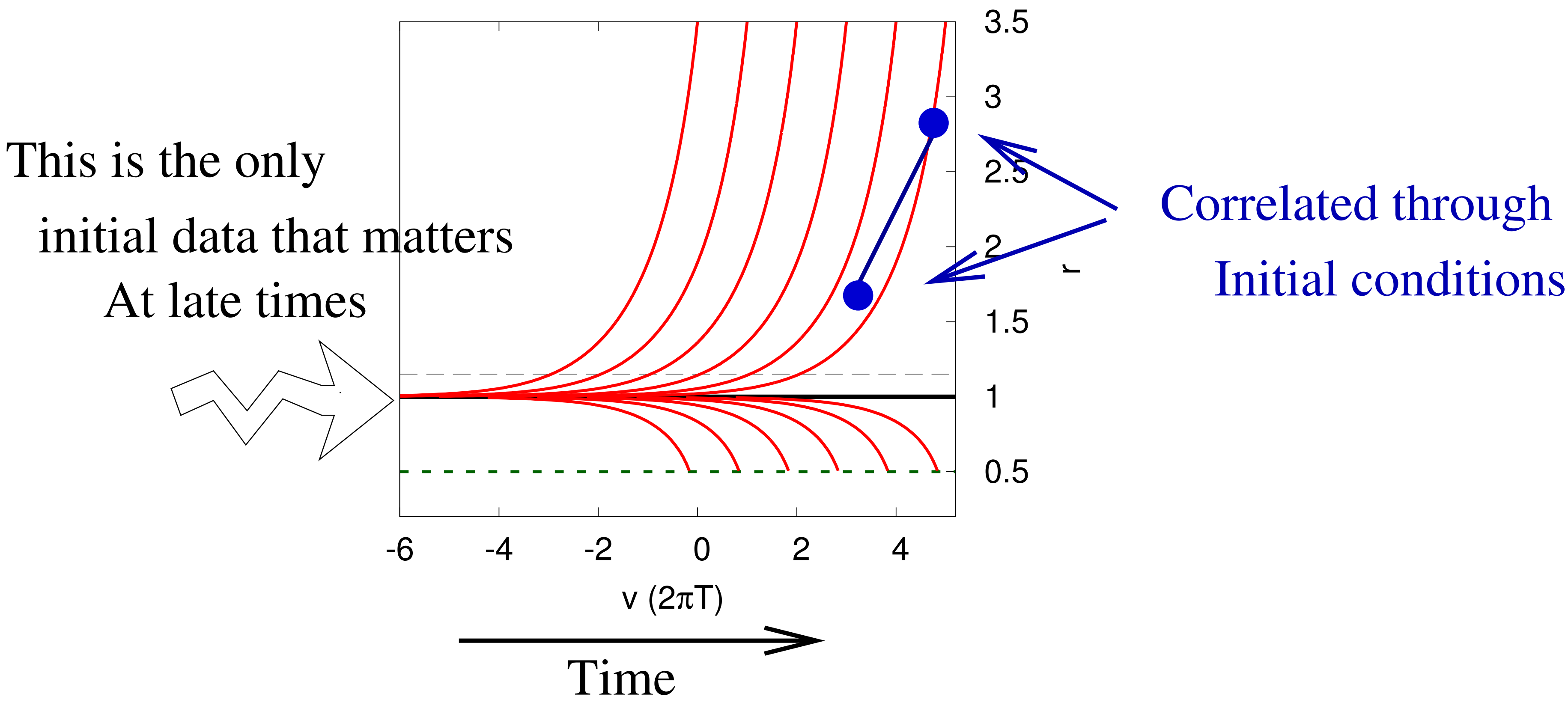
Correlations through Initial conditions



Correlations through Initial conditions



Correlations through Initial conditions



1. Final correlation come from correlated initial data very near horizon
   - Short Wavelength
2. Initial data is inflated by near horizon geometry

Initial Data from Quantum Fluctuations

1. Initial data is determined at short distance = Flat Space Physics

2. Scalar Field in 1+1D vacuum flat space

$$\frac{1}{2}\langle\{\phi(X_1),\phi(X_2)\}\rangle = -\frac{1}{4\pi K}\log|\mu\,\eta_{\mu\nu}\Delta X^\mu \Delta X^\nu|$$

K=norm of action

3. String flucts in near horizon geometry

$$S^{\rm near-horizon} = \eta\int {\rm d}t{\rm d}r\,\left[-\frac{1}{2}\sqrt{h}h^{\mu\nu}\partial_\mu x\partial_\nu x\right]$$

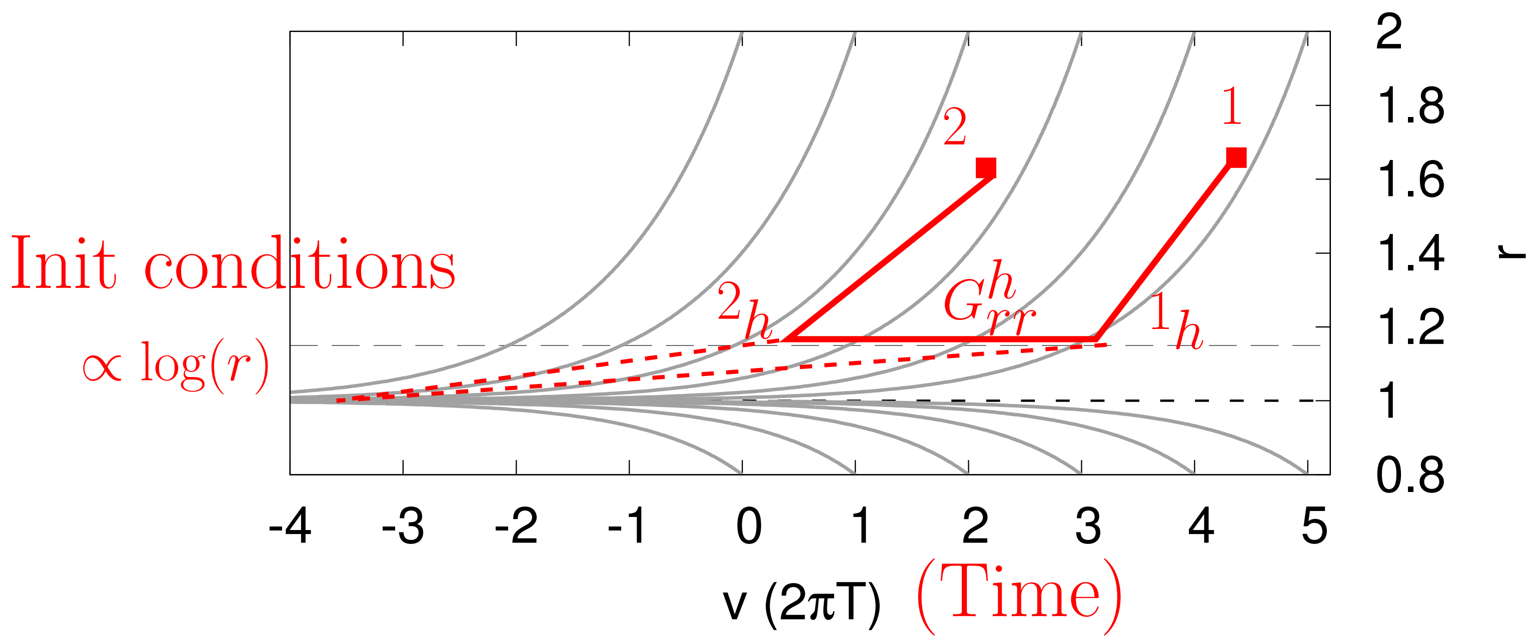
$\eta =$ Drag Coefficient

The near horizon initial condition is:

$$G_{rr}(v_1 r_1|v_2 r_2) \to -\frac{1}{4\pi\eta}\log\left|\mu\,\overbrace{2\Delta v\,\Delta r}^{\text{local }\Delta s^2}\right|$$

Fluctuations from Equations of Motion

$$\underbrace{G_{rr}(1|2)}_{\text{bulk flucts}} = \int dt_{1h} dt_{2h} \; \underbrace{G_R(1|1_h)\; G_R(2|2_h)}_{\text{outgoing Green fcns}} \; \underbrace{G^h_{rr}(1_h|2_h)}_{\text{horizon flucts}} \, ,$$

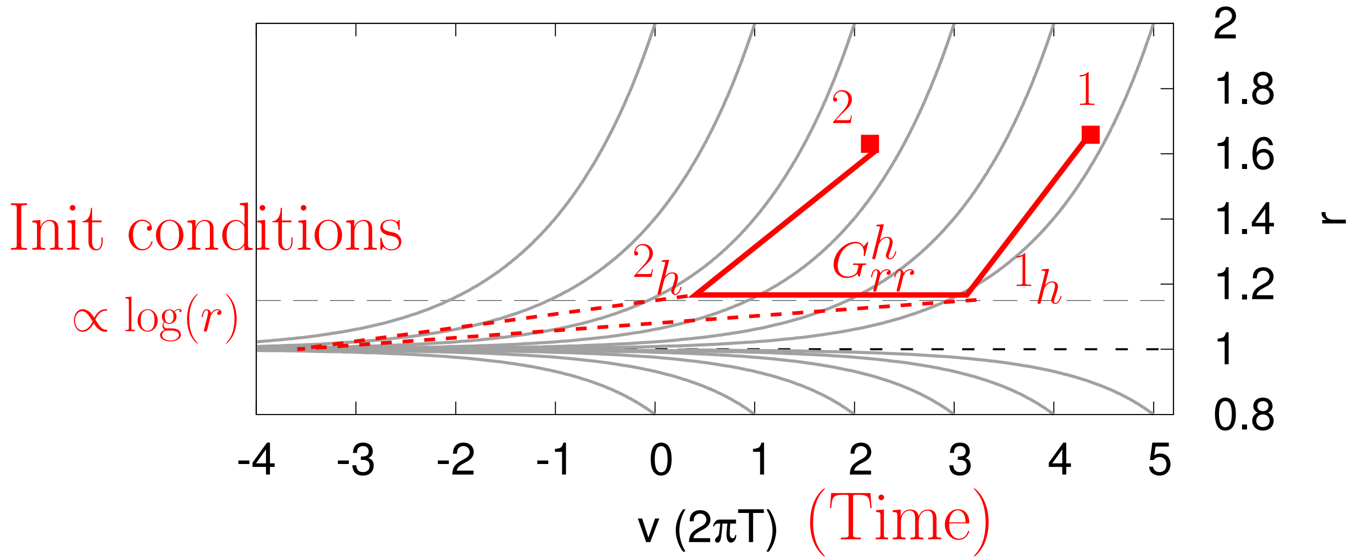


The fluctuations on the stretched horizon are from UV vacuum flucts in past

$$G^h_{rr}(t_1|t_2) = \text{Blow-up of initial data} \propto \log(r)$$

$$= -\frac{\eta}{\pi} \partial_{t_1} \partial_{t_2} \log \left| 1 - e^{-2\pi T (t_1 - t_2)} \right| .$$

The horizon fluctuations and the Lyapunov exponent



1. Thermal looking:

$$G^h_{rr}(\omega) = \text{Fourier-Trans of } -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2}\log|1-e^{-2\pi T(t_1-t_2)}|$$

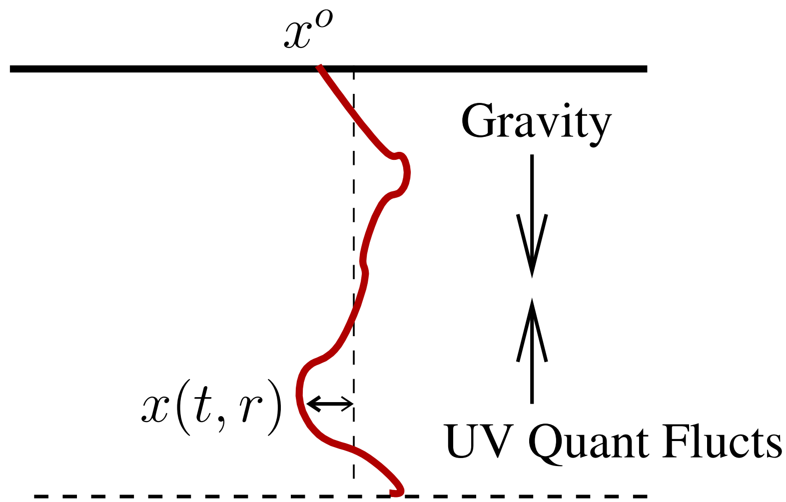
$$= \left(\tfrac{1}{2} + n(\omega)\right) 2\omega\eta \qquad\qquad n(\omega) \equiv \frac{1}{e^{\omega/T}-1}$$

2. Temperature $\propto$ inflation rate

$$2\pi T = \text{Lyapunov exponent of diverging geodesics}$$

Detailed Balance

$$G_{rr}(\omega, r_1, r_2) = \left(\tfrac{1}{2} + n(\omega)\right) \rho(\omega, r_1, r_2)$$



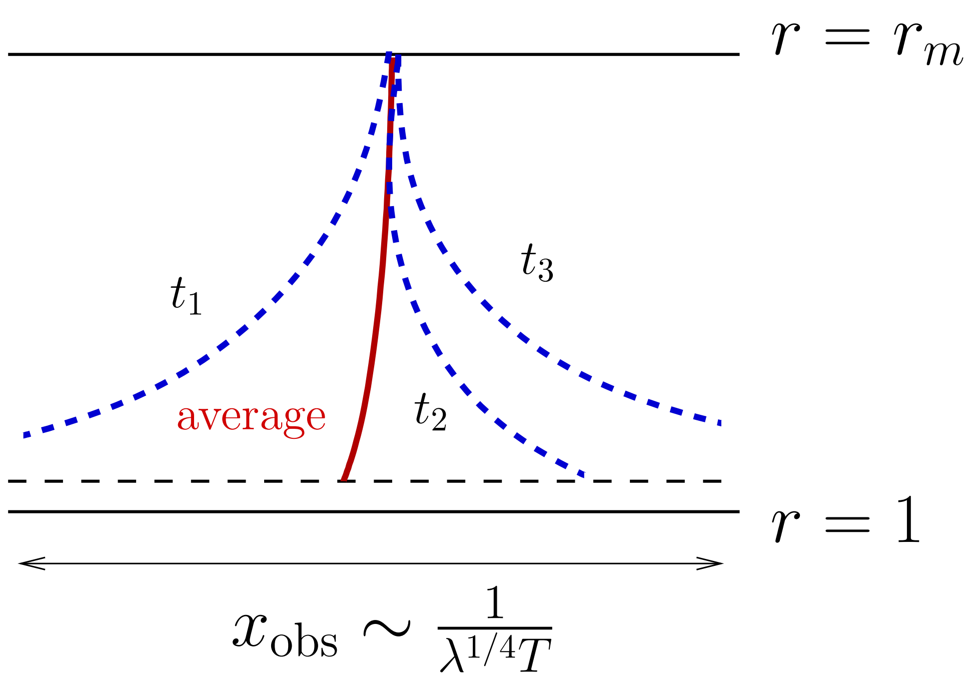
1. Fluctuations (Anti-commutator)

$$\underbrace{G_{rr}(\omega, r_1, r_2)}_{\text{bulk flucts}} = \underbrace{G_R(\omega, r_1|r_h)\; G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \; \underbrace{\left(\tfrac{1}{2} + n(\omega)\right) 2\omega\eta}_{\text{Horizon-flucts}}$$

2. Dissipation: (Commutator)

$$\underbrace{\rho_{ra-ar}(\omega, r_1, r_2)}_{\text{bulk spec dense}} = \underbrace{G_R(\omega, r_1|r_h)\; G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \; \underbrace{2\omega\eta}_{\text{Horizon spec dense}}$$

Fluctuation dissipation and stochastic dynamics



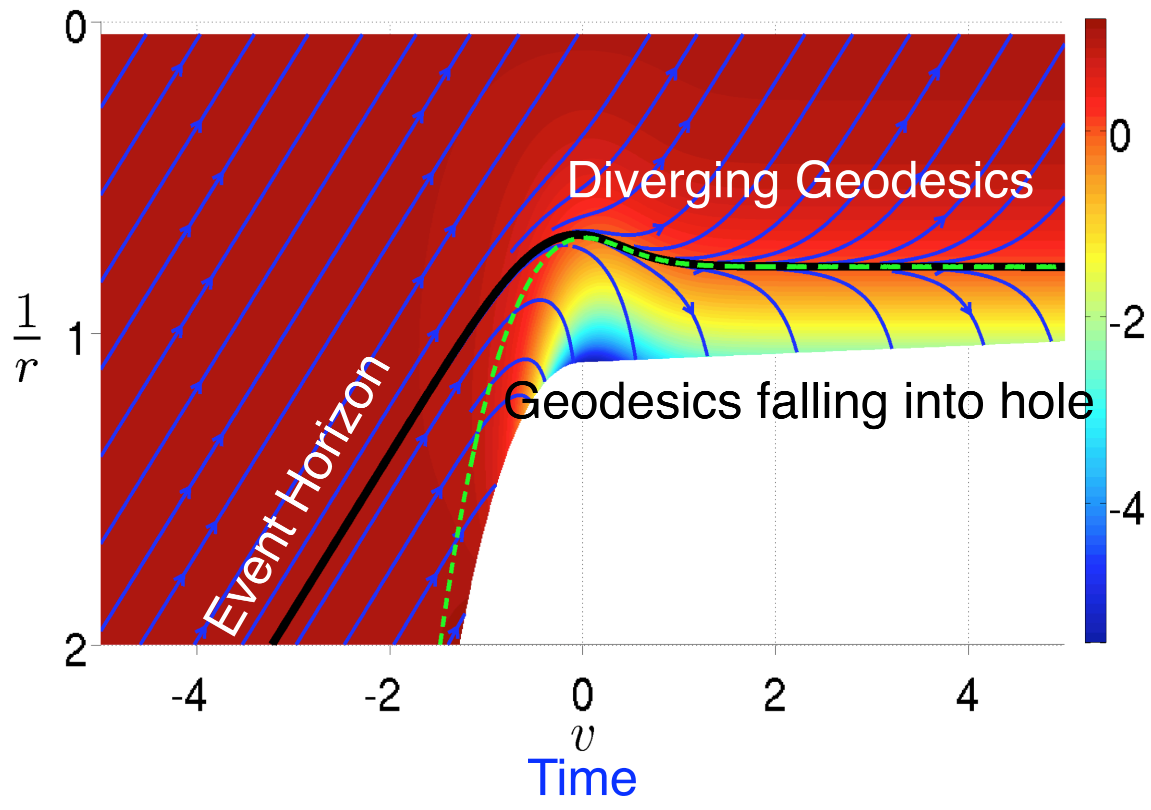
1. Every step t_1, t_2, t_3 fluctuates to a new trailing string – $\longrightarrow$ random force

2. The average of the trailing strings gives the drag – average string $\rightarrow$ drag

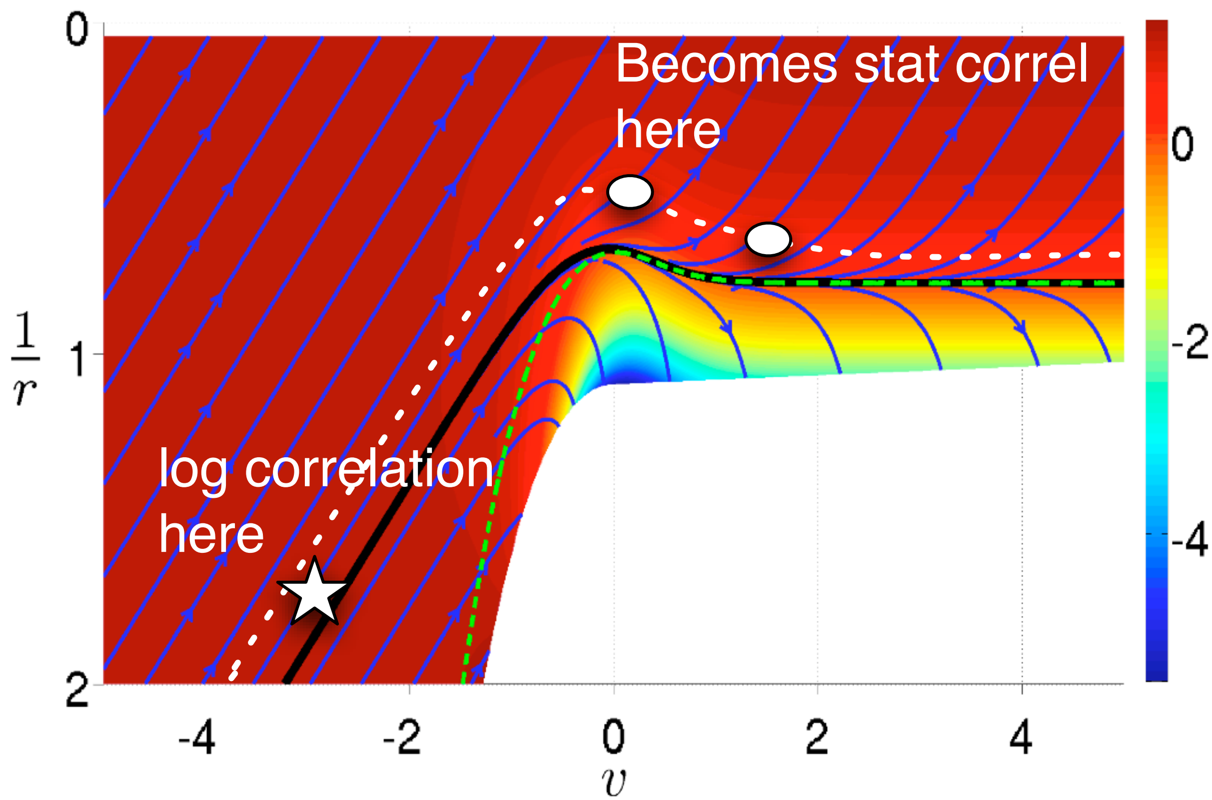
Non-equilibrium

1. Chesler&Yaffe create QGP by turning a gravitational pulse in vacuum

2. Corresponds to non-equilbrium geometry with BH formation in AdS_5



Fluctuations in non-equilibrium



- Surface Properties – on event horizon

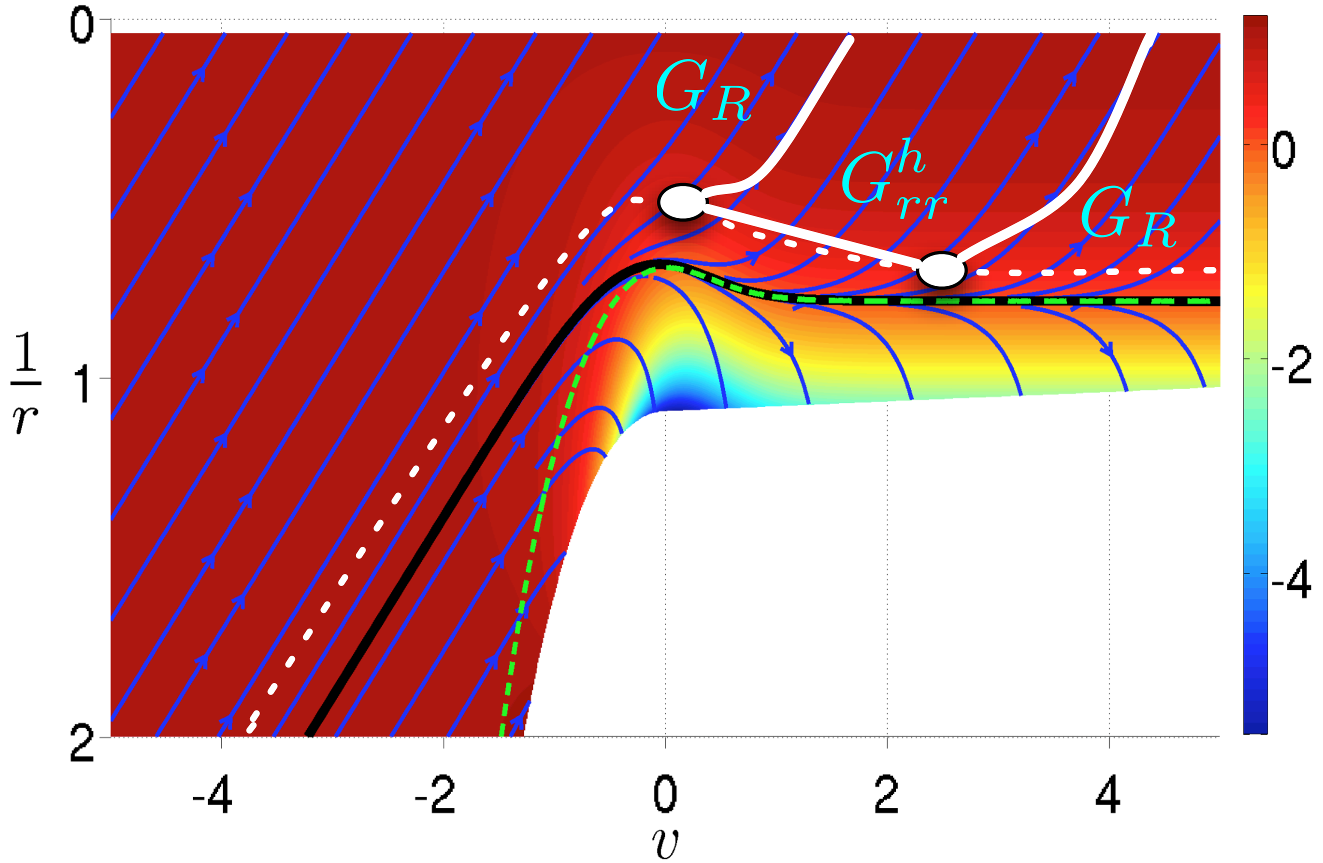
$$\underbrace{2\pi T_{\rm eff}(v)}_{\text{Lyapunov exponent}} \equiv \overbrace{\frac{1}{2}\frac{\partial A(r,v)}{\partial r}}^{\rm Metric-coeff}\Bigg|_{r=r_h(v)} \propto \text{extrinsic curvature}$$

Result:

- General form of near horizon fluctuations in non-equilibrium

$$G^h_{rr}(v_1|v_2) = -\frac{\sqrt{\eta(v_1)\eta(v_2)}}{\pi}\partial_{v_1}\partial_{v_2}\log\left|1 - e^{-\int_{v_1}^{v_2} 2\pi T_{\rm eff}(v')dv'}\right|.$$

- Can map the near horizon fluctuations up to boundary (numerics in progress)



Equilibration (on the stretched horizon)

- In non-equilbrium spectral density is a fcn of two time arguements

$$\rho(v_1, v_2) = \underbrace{\rho(\bar{v} + \Delta v/2, \bar{v} - \Delta v/2)}_{\text{Func of average and difference}}$$

- Take Wigner transforms of horizon correlator

$$\rho^h(\bar{v}, \omega) = \int_{-\infty}^{\infty} \mathrm{d}\Delta v \; e^{+i\omega\Delta v} \; \rho^h(\bar{v} + \Delta v/2, \bar{v} - \Delta v/2)\,,$$

$$= 2\eta(\bar{v})\,\omega \;\Leftarrow \text{Reflects commutation relations}$$

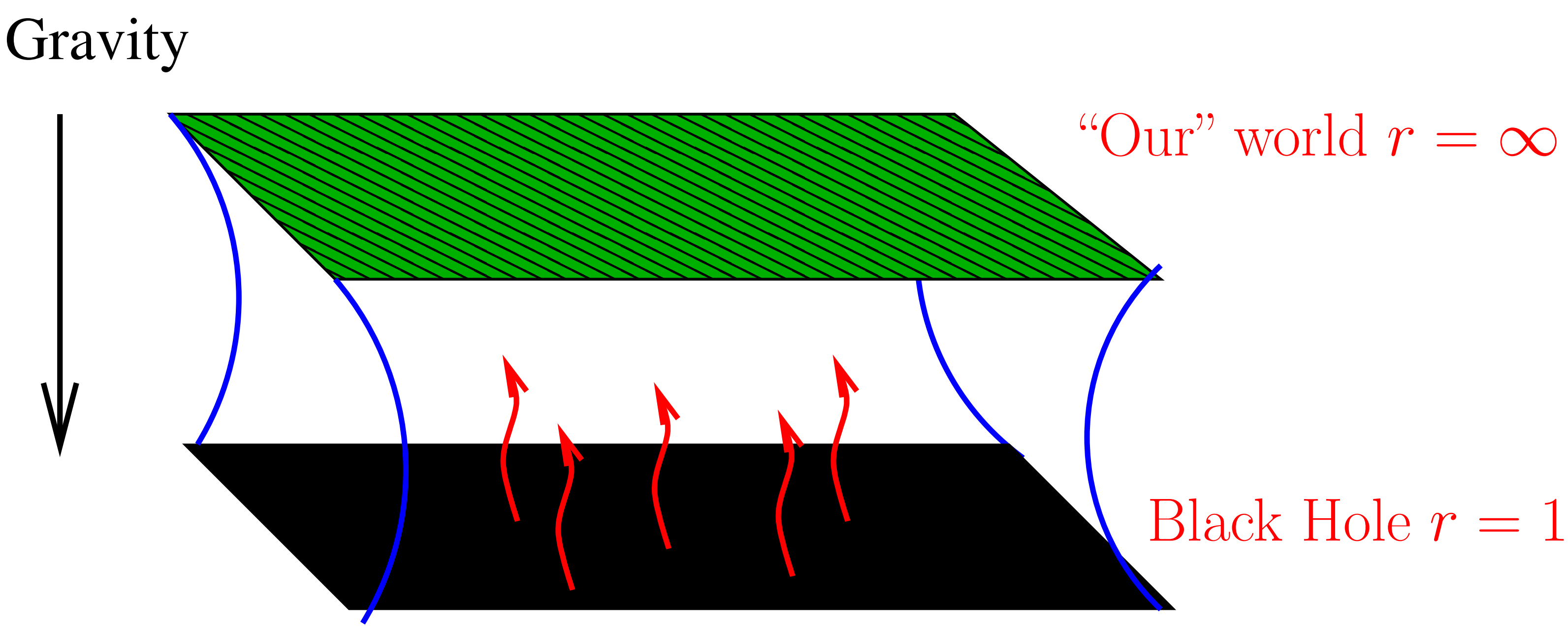
- At high frequency $\omega\tau \gg 1$ we have

$$G^h_{rr}(\bar{v}, \omega) \simeq \left(\frac{1}{2} + n(\omega/T_{\rm eff}(\bar{v}))\right) \rho^h(\bar{v}, \omega) \; + \; O\left(\frac{1}{\tau^2\omega^2}\right) \; .$$

High frequencies are born into equilibrium on the event horizon!

(see also Berndt Mueller's talk)

Not conclusions, but picture:



Gravity pulls down, but quantum fields fluctuate, reaching equilibrium