Highly–anisotropic and strongly–dissipative hydrodynamics for early stages of relativistic heavy-ion collisions

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Introduction

The soft-hadronic observables measured in ultra-relativistic heavy-ion collisions at RHIC are very well described by the perfect-fluid hydrodynamics or by viscous hydrodynamics with small viscosity. The implicit in this success is the use of a rather early thermalization time $\tau_i \ll 1$ fm. Choice of such early thermalization time is a debatable problem. Thus there are attempts to develop models wich are able to describe the very early highly anisotropic stage. One of such models is ADHYDRO (highly-Anisotropic and strongly-Dissipative HYDROdynamics) which interpolates between a highly-anisotropic initial state and the regime of perfect-fluid hydrodynamics.

Anisotropic system of particles

- The basis of ADHYDRO is the assumption that the system of particles produced just after the collision of two nuclei is highly anisotropic. Thus the the phase-space distribution function of the system is assymetric in the momentum space

$$\mathbf{f} = \mathbf{f}\left(\mathbf{p}_\perp/\lambda_\perp, \mathbf{p}_\parallel/\lambda_\parallel\right)$$

Explicit form of the distribution function may be proposed as the equlibrium distribution that has been stretched (or squeezed) in the longitudinal direction

$$\mathbf{f} = \mathbf{g_0}\left[\exp\left(\sqrt{\mathbf{p}_\perp^2/\lambda_\perp^2 + \mathbf{p}_\parallel^2/\lambda_\parallel^2}\right) \pm 1\right]^{-1}$$

$\lambda_\perp, \lambda_\parallel$ - transverse and longitudinal temperature

Anisotropic hydrodynamics with dissipation

- Form of the energy-momentum tensor following from the assumption about the distribution function allows for anisotropy in pressures

$$\mathbf{T}^{\mu\nu} = (\varepsilon + \mathbf{P}_\perp)\,\mathbf{U}^\mu\mathbf{U}^\nu - \mathbf{P}_\perp\,\mathbf{g}^{\mu\nu} - (\mathbf{P}_\perp - \mathbf{P}_\parallel)\mathbf{V}^\mu\mathbf{V}^\nu$$

  - ▷ hydrodynamic flow

$$\mathbf{U}^\mu = \gamma(\mathbf{1}, \mathbf{v_x}, \mathbf{v_y}, \mathbf{v_z}), \quad \gamma = (\mathbf{1} - \mathbf{v}^2)^{-1/2}$$

  - ▷ beam direction

$$\mathbf{V}^\mu = \gamma_z(\mathbf{v_z}, \mathbf{0}, \mathbf{0}, \mathbf{1}), \quad \gamma_z = (\mathbf{1} - \mathbf{v_z^2})^{-1/2}$$

- Space-time dynamics of the system is governed by the equations expressing energy-momentum conservation and entropy production

$$\partial_\mu \mathbf{T}^{\mu\nu} = \mathbf{0}$$
$$\partial_\mu \sigma^\mu = \mathbf{\Sigma}$$

with the entropy flux $\sigma^\mu = \sigma \mathbf{U}^\mu$

Generalized equation of state

- The equations of motion form a system of 5 equations for 5 unknown functions (velocity vector $\vec{\mathbf{v}}$, $\mathbf{P}_\perp$ and $\mathbf{P}_\parallel$). They may be solved numerically if they are supplemented by the appropriate **EOS** giving relations between all thermodynamic-like quantities including $\mathbf{P}_\perp$ and $\mathbf{P}_\parallel$. Such relations play a role of the generalized **EOS**. Instead of $\mathbf{P}_\perp$ and $\mathbf{P}_\parallel$ we use anisotropy parameter $\mathbf{x} = (\lambda_\perp/\lambda_\parallel)^2$ and non-equilibrium entropy density σ, as independent variables. Close to equlibrium $(\mathbf{x} \approx \mathbf{1})$ we expect to achieve the perfect-fluid limit $\mathbf{P}_\parallel = \mathbf{P}_\perp = \mathbf{P_{qgp}}$. In this regime, the matter should be described by the realistic **EOS**. Thus we propose

$$\varepsilon = \varepsilon_{\rm qgp}(\sigma)\mathbf{r}(\mathbf{x})$$
$$\mathbf{P}_\perp = \mathbf{P}_{\rm qgp}(\sigma)\,[\mathbf{r}(\mathbf{x}) + \mathbf{3xr}'(\mathbf{x})]$$
$$\mathbf{P}_\parallel = \mathbf{P}_{\rm qgp}(\sigma)\,[\mathbf{r}(\mathbf{x}) - \mathbf{6xr}'(\mathbf{x})]$$

where

$$\mathbf{r}(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{x}^{-\frac{1}{3}}\left[\mathbf{1} + \frac{\mathbf{x}\arctan\sqrt{\mathbf{x}-\mathbf{1}}}{\sqrt{\mathbf{x}-\mathbf{1}}}\right]$$

This form has two attractive limits. In the very early stage the system consist of anisotropic fluid of massless gluons $\mathbf{P}_\perp \neq \mathbf{P}_\parallel$. For isotropic case $\mathbf{r}'(\mathbf{x} = \mathbf{1}) = \mathbf{0}$ and we obtain realistic **EOS** used in standard relativistic hydrodynamics $\mathbf{P}_\perp = \mathbf{P}_\parallel$.

Entropy source

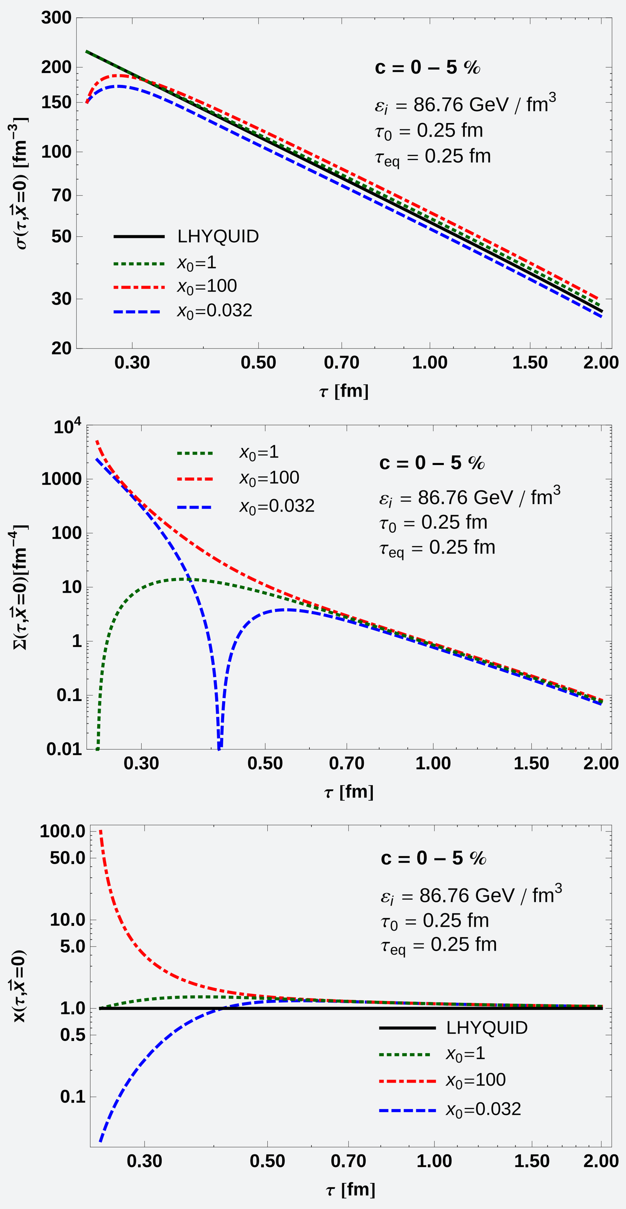
- Structure of the entropy source is an external input. The simplest form satisfying general physical requirements can be proposed as

$$\mathbf{\Sigma} = \frac{(\lambda_\perp - \lambda_\parallel)^2}{\lambda_\perp\lambda_\parallel}\frac{\sigma}{\tau_{\rm eq}}$$

Far from equilibrium, the Israel-Stewart theory is not applicable and our model becomes an alternative for much more advanced kinetic calculations. For small anisotropies $\mathbf{x} \approx \mathbf{1}$ and for purely longitudinal, boost-invariant systems ADHYDRO agrees with the Israel-Stewart theory. Some hints for the form of $\mathbf{\Sigma}$ can be found from the microscopic models or AdS/CFT correspondence.

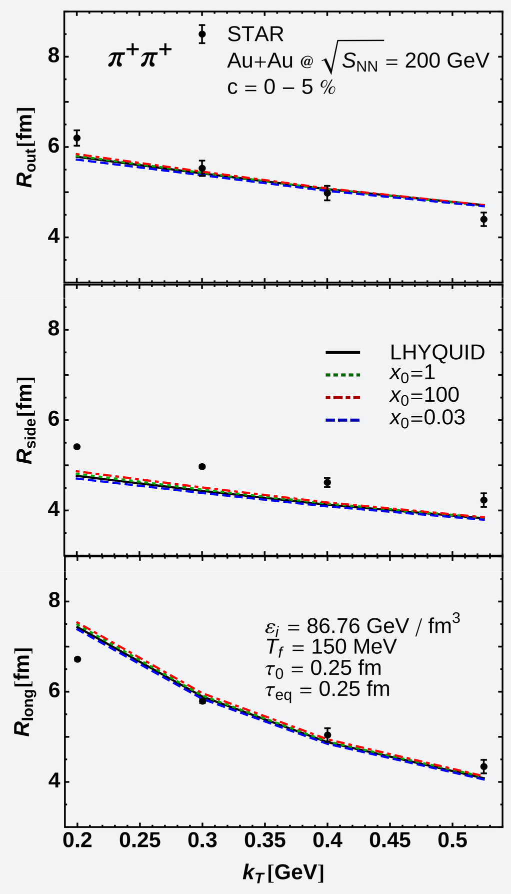
Pressure relaxation

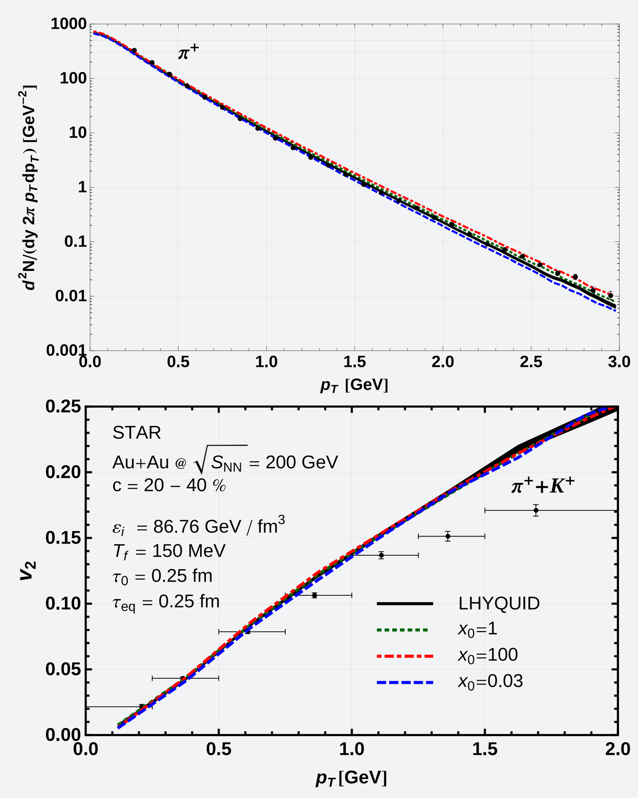
Various choices of the initial asymmetry are analysed. In the cases where $\mathbf{x} \gg \mathbf{1}$ $(\mathbf{P}_\perp \gg \mathbf{P}_\parallel)$ and $\mathbf{x} \ll \mathbf{1}$ $(\mathbf{P}_\perp \ll \mathbf{P}_\parallel)$, strong production of the entropy connected with the particle production is observed. For $\tau > \mathbf{1}$ fm various quantities behave similarly to perfect-fluid case. Initially anisotropic systems equlibrate at $\tau \approx \mathbf{1}$ fm. We compare our results obtained with ADHYDRO with perfect-fluid calculations done with LHYQUID code wich serves as a reference point.



Results

For $\tau > \mathbf{1}$ fm the matter is practically in local equlibrium and we may specify the freeze-out condition by fixing the freeze-out temperature. If the freeze-out is determined, we use it as an input for THERMINATOR 2 Monte-Carlo code to generate physical events. We are able to describe succesfully particle $\mathbf{p_T}$ spectra, elliptic flow, and HBT radii. We observe a small dependence of the model results on the initial value of anisotropy.





Conclusions

We formulated a model of highly–anisotropic and strongly–dissipative hydrodynamics (ADHYDRO) to study evolution of matter far from equilibrium and to analyse the effects of anisotropies of pressure on soft hadronic observables at RHIC. We found that initial conditions with different anisotropies lead to similar results, provided the initial energy density profile in transverse plane is fixed.

References

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