Understanding the equilibration of matter from time-dependent correlators

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Motivation

Rapid thermalization required by hydrodynamical description of elliptic flow ⇒ thermalization time $\tau \sim 0.5$fm/c

Rapid thermalization is attributed to strong coupling, but the mechanism is not fully understood.

This work is to use gauge/gravity duality to study the mechanism of thermalization in QCD-like theory($\mathcal{N}$=4 SYM) in strongly coupled regime.

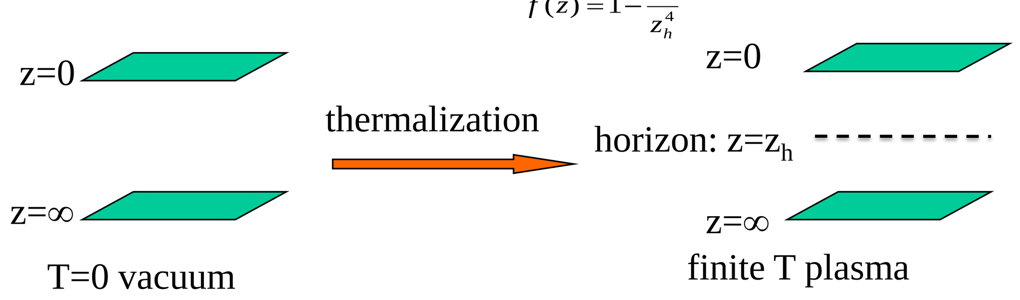
Gravitational Collapse Model

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

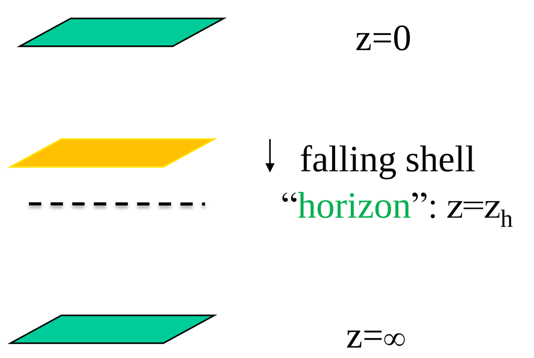
AdS Black hole

$$ds^2 = \frac{-f(z)dt^2 + d\vec{x}^2 + dz^2/f(z)}{z^2}$$

$$f(z) = 1 - \frac{z^4}{z_h^4}$$



Formation of black hole via gravitational collapse, dual to thermaliztion of plasma



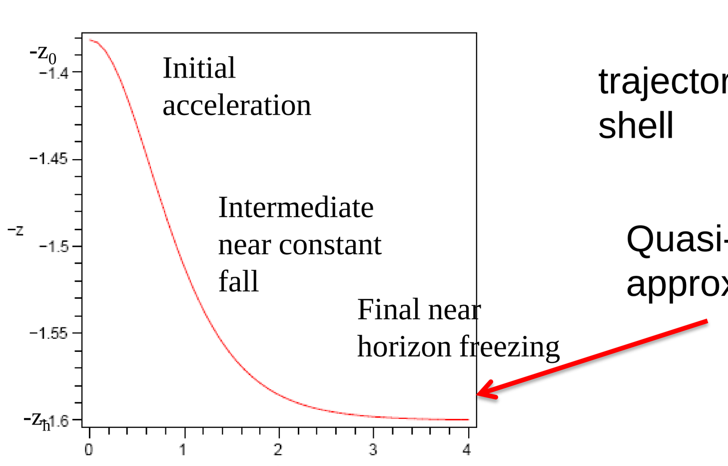
SL, E. Shuryak **Phys.Rev.D78:125018 ,2008**.

Quasi-equilibrium

one-point function of stress energy tensor the same as thermal case $< T_{mn}(x) >_{shell} = < T_{mn}(x) >_{thermal}$

two-point function deviates from thermal case

$$< T_{mn}(x) T_{kl}(0) >_{shell} \neq < T_{mn}(x) T_{kl}(0) >_{thermal}$$

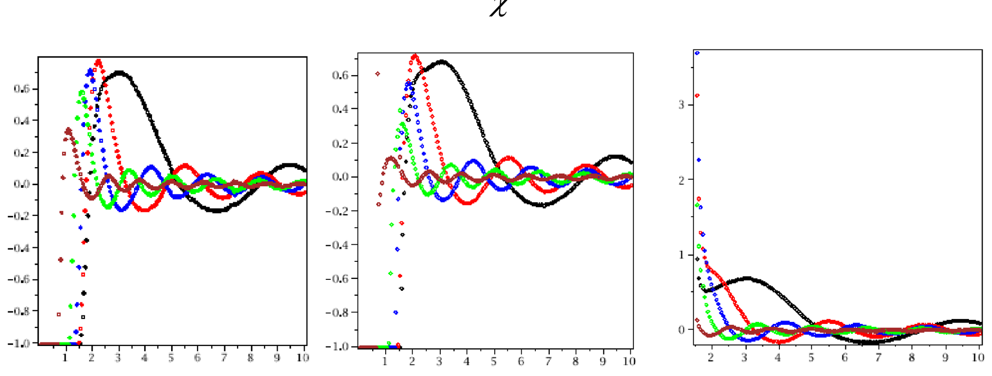


trajectory of the falling shell

Quasi-static approximation

Study deviation of two point function(spectral function) in the Quasi-static approximation: Plasma near equilibrium.

$$R = \frac{\chi^{shell} - \chi^{thermal}}{\chi^{thermal}}$$



χ versus ω /(2πT) at q/(2 πT) =1.5

χ: stress(left), momentum density(middle), energy density(right)
Colors indicate different stages of the thermalization.

$$\chi_{shell} = \chi_{thermal} + oscillation$$

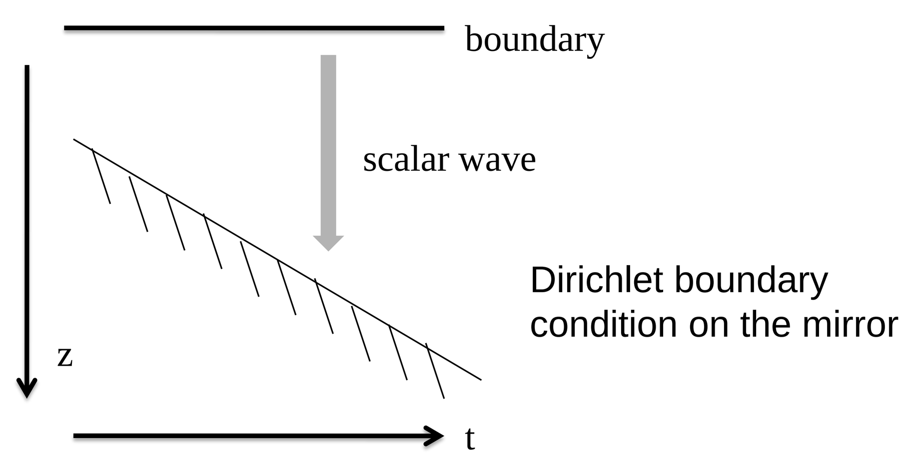
As gauge fields thermalize, the oscillation damps in amplitude and grows in frequency (reciprocal of ω, coherent time scale).

Beyond Quasi-static limit

Quasi-static limit tells nothing about far from equilibrium dynamics, which is essential for thermalization.

Going beyond Quasi-static limit is difficult:
One possibility is to study equal-time correlators, see Balasubramanian et al **arXiv:1012.4753** [hep-th]
To study correlator involving two times, generally requires solving an PDE.

Swtich to a toy model: a Moving Mirror in AdS
Thermalization not realized, corresponds to the decay of an excitation in strong coupled gauge theory(t--> ∞, vacuum).



For mirror with scaling trajectory z=t/u0. Wave equation exact ly sovable with arbitrary boudanry condition

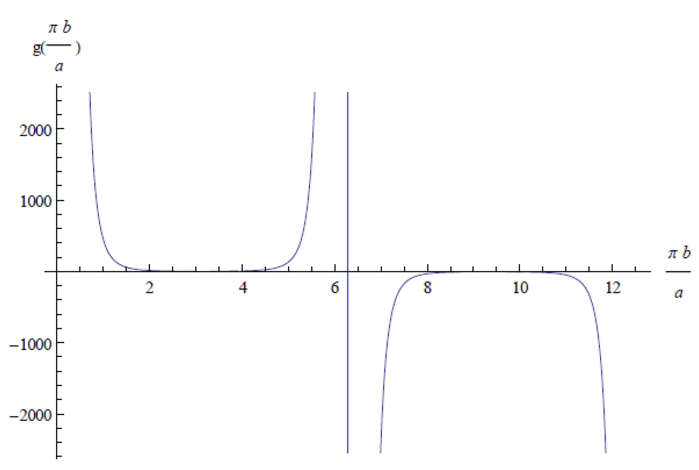
Results

Spatially integrated two point correlator

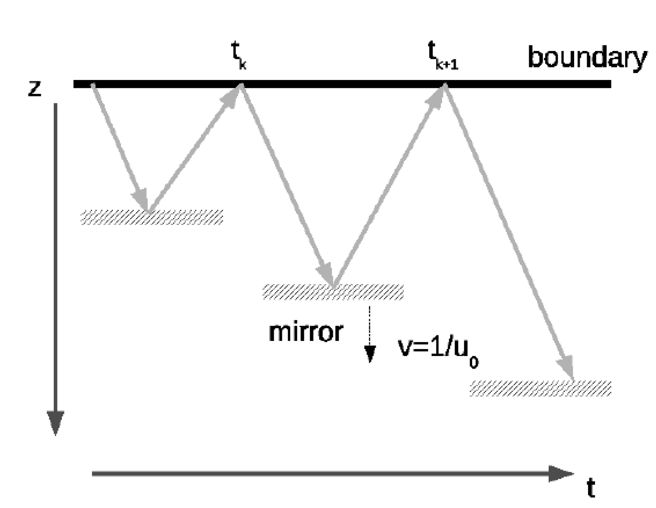
$$\langle d^{d-1}x\, O(t,x) O(t',0)\rangle_R = -\theta(t-t')\sum_{+,-}(tt')^{\frac{-d-1}{2}} d\left(\frac{2}{a}\right)^d \frac{1-e^{\mp i\pi d}}{a}\frac{\Gamma(-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{1-d}{2})\Gamma(d)}$$
$$\times e^{\pm\frac{i\pi(d-1)}{2}(\frac{b}{a}-c)}\left[\frac{d!}{(-c)^{d+1}(\pm i2\pi)} - \sum_{r=0}^{\infty}\frac{B_{d+r+1}(\frac{d-1}{4})c^r(\pm i2\pi)^{d+r}}{r!(d+r+1)}\right]$$
$$\equiv -\theta(t-t')\frac{d}{a^{d+1}}(tt')^{\frac{-d-1}{2}} g\left(\frac{\pi b}{a}\right)$$

singular regular

dimension d $a = \ln\frac{u_0+1}{u_0-1}$ and $b = \ln\frac{t}{t'}$ $e^{i2\pi c} = e^{\frac{i2\pi b}{a}}, \quad |c| < \frac{1}{2}$



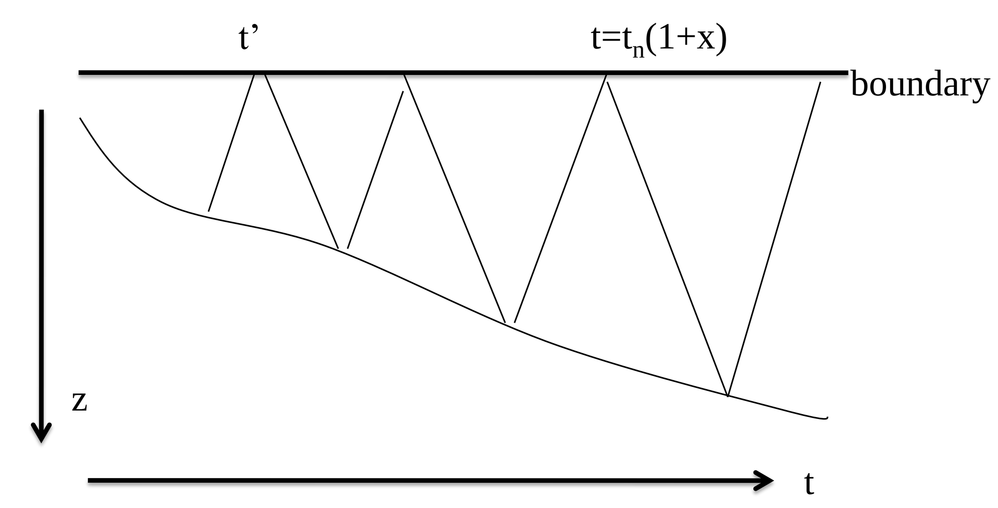
J. Erdmenger, SL, H. Ngo **JHEP 1104:035,2011**



Two point correlator becomes singular when t and t' are connected by a light ray bouncing between the mirror and the boundary. Coherent time scale increases as the exitation decays. In particular $\frac{t}{t'} = \left(\frac{u_0+1}{u_0-1}\right)^n$

A special case of bulk-cone singularities conjecture
V. Hubeny, H. Liu, M. Rangamani **JHEP 0701:009,2007**

LO divergences



x: how far is the separation of two boundary times "measured" by a light ray

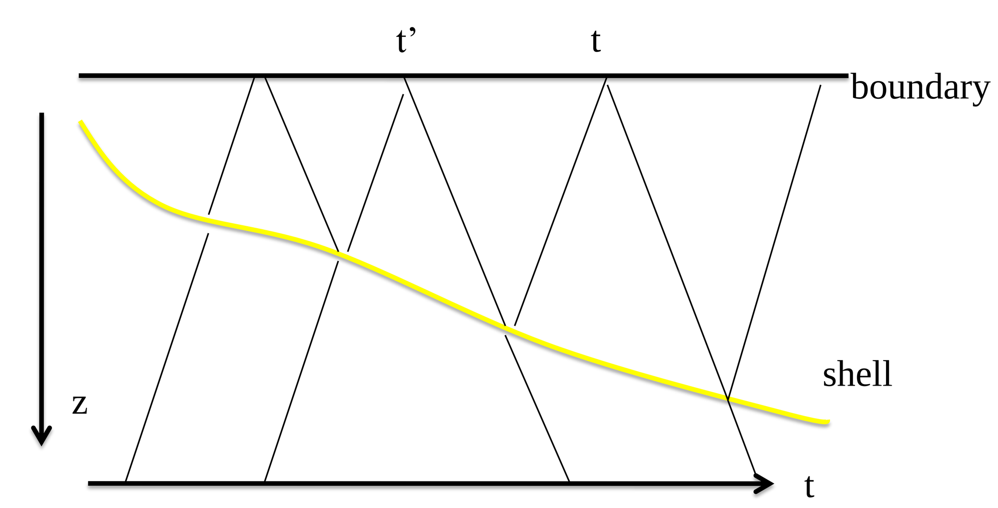
LO divergences of spatially integrated correlator:

$$\left\langle d^{d-1}x O(t,x) O(t',0)\right\rangle_{div} \sim \frac{g(t',t_n)}{x^{d+1}}$$

- LO divergence is quite general, valid for arbitrary time like mirror trajectory.
- Dependence on mirror trajectory only enters through g(t',t_n).
- A recursion relation among g(t',t_n) allows for complete determination of them in terms of mirror trajectory

J. Erdmenger, C. Hoyos, SL, H. Ngo. In preparation

Generalization to gravitational collapse model?



Much more complicated

Conclusion

- We used the quasi-static approximation to a gravitational collapse model to study the near equilibrium dynamics of stongly coupled QCD-like theory. The deviation of the spectral densities from the equilibrium shows that the coherent time scale grows as the plasma thermalizes
- We used a moving mirror model to study the decay of an excitation in strong coupled QCD-like theory. The coherent time scale diverges as the excitation decays.
- The result of the two-point function shows a structure of singularities consistent with bulk-cone singularities conjecture. The LO singularities have a generic form, which are insensitive to the trajectory of the mirror.