Running coupling corrections to inclusive gluon production

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based on arXiv:1009.0545 [hep-ph]

in collaboration with William Horowitz

Outline

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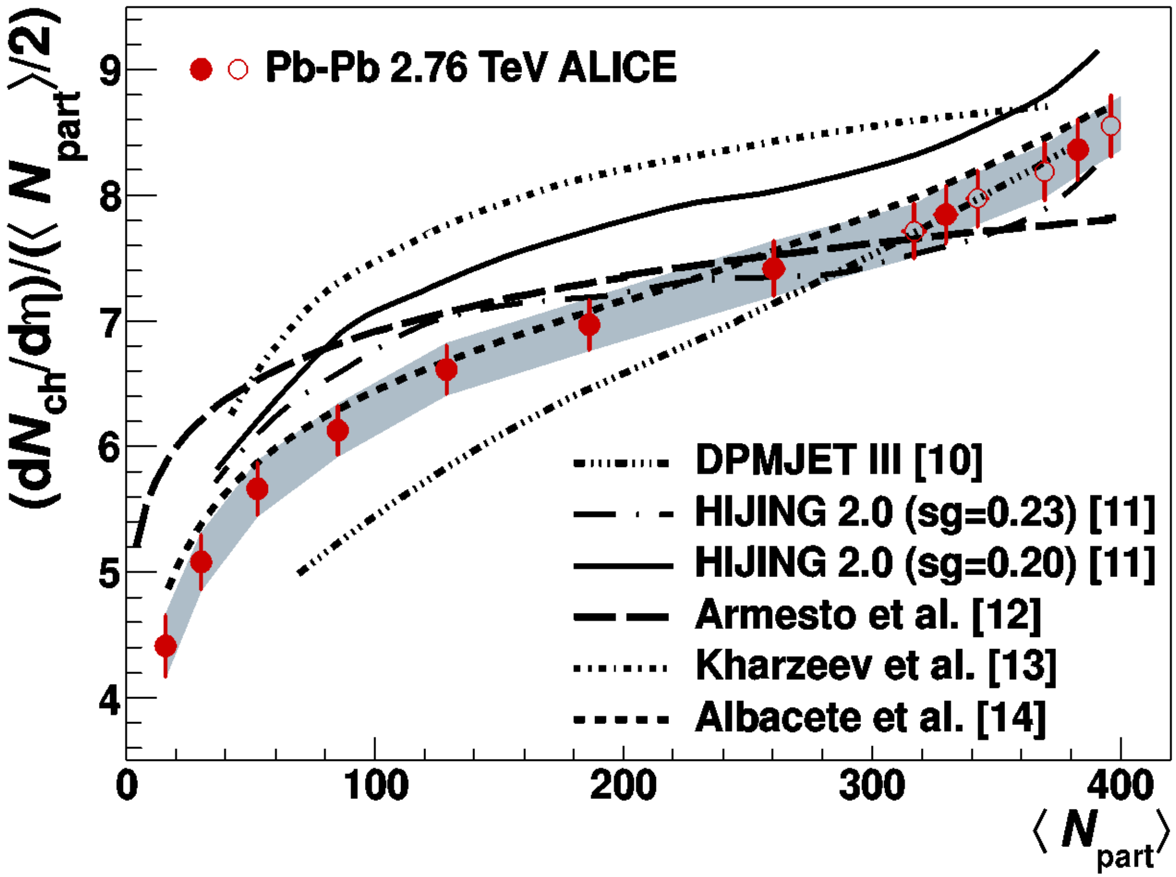
- Calculation
- Results

Motivation

CGC Multiplicity Prediction



CGC prediction by Albacete and Dumitru '10 for LHC multiplicity and its centrality dependence was quite successful.

But what was the theory behind this prediction?

Gluon Production in pA

Gluon production cross section is given by a k_{T}-factorization expression (Yu. K., Tuchin, '01; cf. Gribov, Levin, Ryskin '83):

$$\frac{d\sigma^{pA}}{d^2k\,dy} = \frac{2\alpha_S}{C_F}\frac{1}{k^2}\int d^2q\,\phi_p(\underline{q}, Y-y)\,\phi_A(\underline{k}-\underline{q}, y)$$

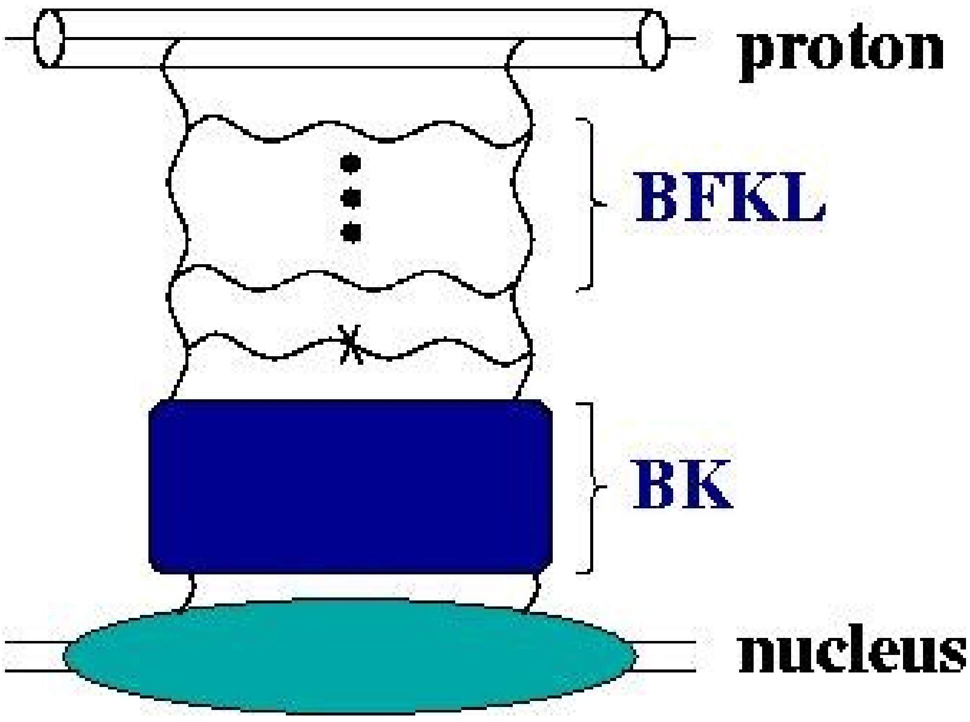
with the proton and nucleus unintegrated gluon distributions defined by

$$\phi^{p,A}(k,y) = \frac{C_F}{\alpha_S(2\pi)^3}\int d^2b\,d^2x\,e^{-i\underline{k}\cdot\underline{x}}\,\nabla_x^2\,N_G^{p,A}(x,b,y)$$

with $N_G^{p,A}$ the scattering amplitude of a GG dipole on p or A. (Includes multiple rescatterings and small-x evolution.)

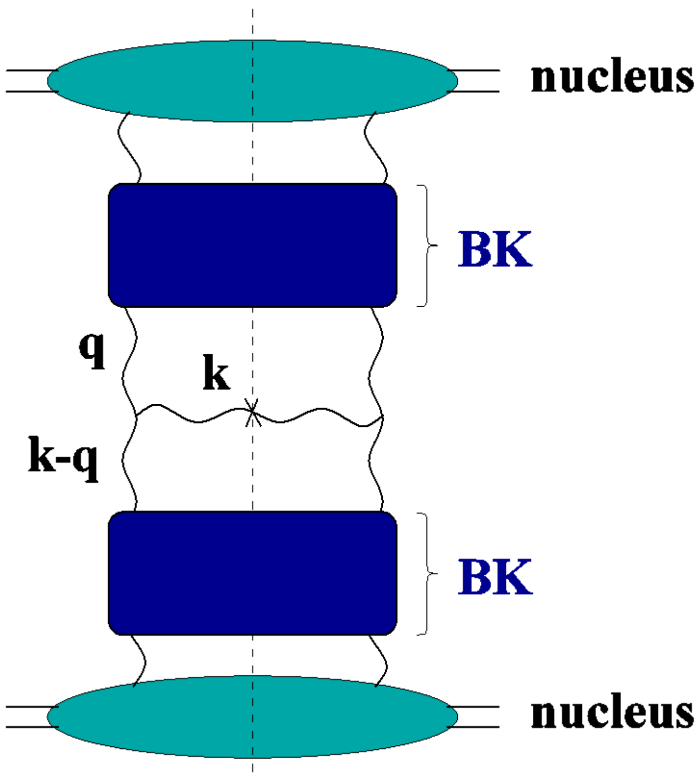
Gluon Production in pA

A simplified diagrammatic way of thinking about this result is



CGC Gluon Production in AA

This is an unsolved problem. Albacete and Dumitru, following KLN, approximate the full unknown solution by



$$= \frac{2\alpha_S}{C_F}\frac{1}{k^2}\int d^2q\, \phi_A(\underline{q}, Y-y)\, \phi_A(\underline{k}-\underline{q}, y)$$

Running Coupling BK

Here's the BK equation with the running coupling corrections (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{N_C}{2\pi^2}\int d^2 x_2$$

$$\times\left[\frac{\alpha_S(1/x_{02}^2)}{x_{02}^2} + \frac{\alpha_S(1/x_{12}^2)}{x_{12}^2} - 2\,\frac{\alpha_S(1/x_{02}^2)\,\alpha_S(1/x_{12}^2)}{\alpha_S(1/R^2)}\,\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{02}^2\, x_{12}^2}\right]$$

$$\times[N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y)\, N(x_2, x_1, Y)]$$

where (actually AD use Balitsky's prescription)

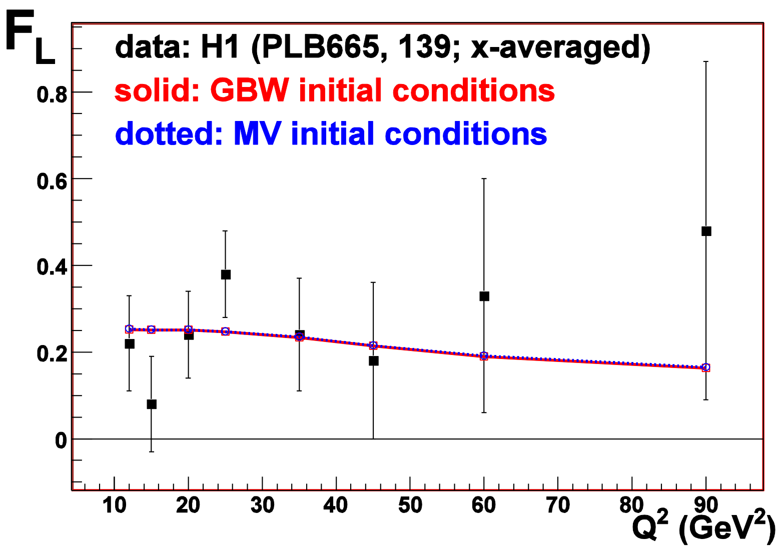
$$\ln R^2\mu^2 = \frac{x_{20}^2 \ln(x_{21}^2\,\mu^2) - x_{21}^2 \ln(x_{20}^2\,\mu^2)}{x_{20}^2 - x_{21}^2} + \frac{x_{20}^2\, x_{21}^2}{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}\,\frac{\ln(x_{20}^2 / x_{21}^2)}{x_{20}^2 - x_{21}^2}$$

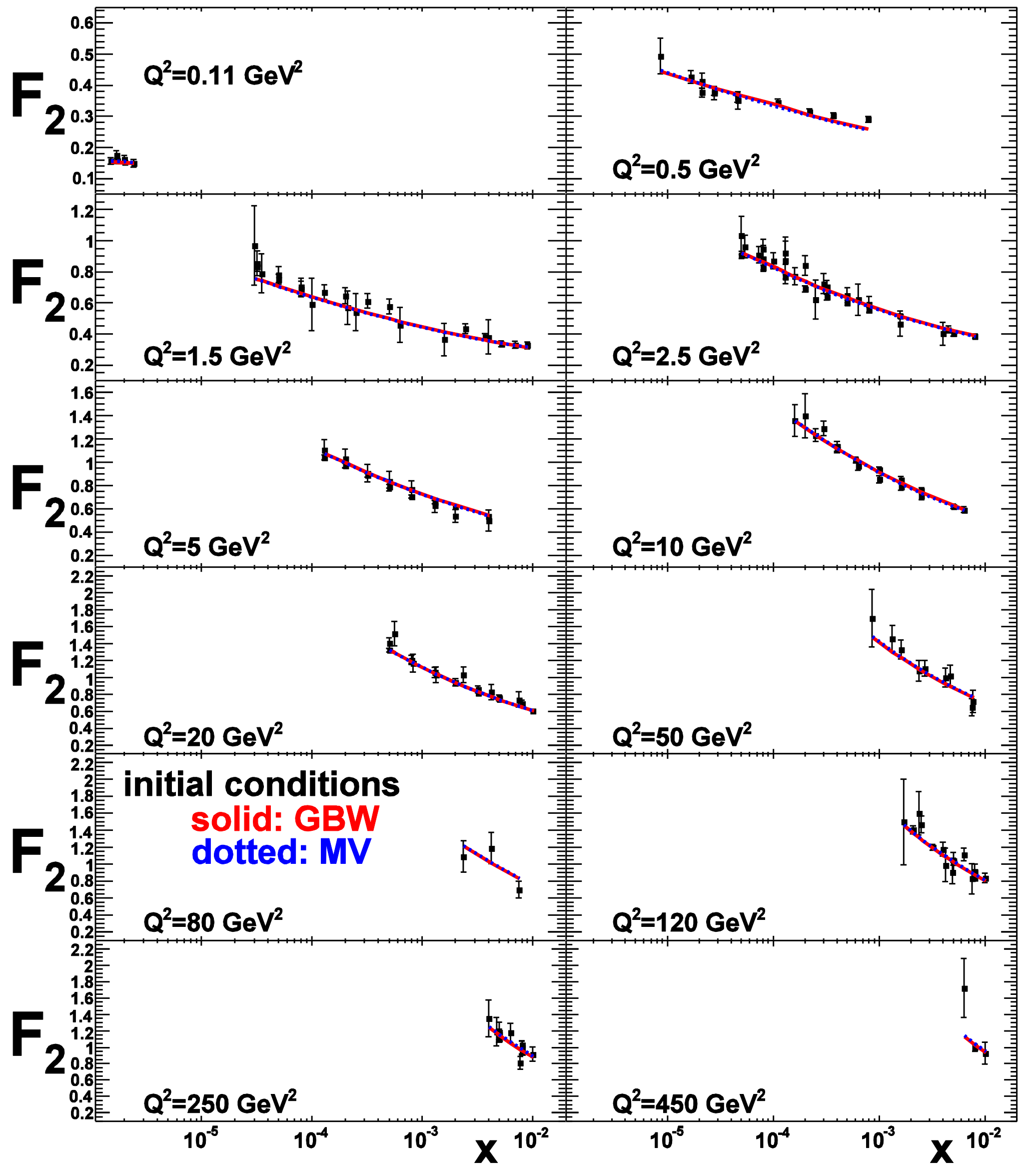
The gluon dipole amplitude is

$$N_G(x_0, x_1, Y) = 2\,N(x_0, x_1, Y) - [N(x_0, x_1, Y)]^2$$

Comparison of rcBK with HERA F2 Data

from Albacete, Armesto, Milhano, Salgado '09





Problem

We have running-coupling N_G , which we inserted into fixed coupling formulas:

$$\phi^{A}(k,y) = \frac{C_F}{\alpha_S (2\pi)^3} \int d^2 b\, d^2 x\, e^{-i \underline{k} \cdot \underline{x}}\, \nabla_x^2 N_G^A(x,b,y)$$

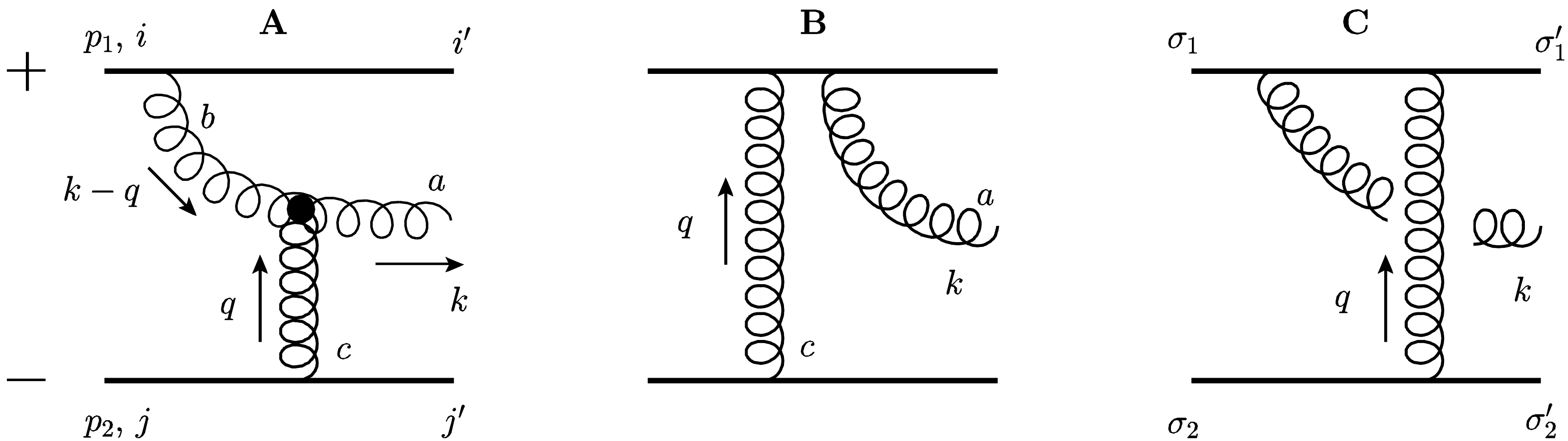
$$\frac{d\sigma^{AA}}{d^2 k\, dy} = \frac{2\alpha_S}{C_F} \frac{1}{k^2} \int d^2 q\, \phi_A(\underline{q}, Y-y)\, \phi_A(\underline{k}-\underline{q}, y)$$

Does this formalism survive if we include running-coupling corrections? If yes, what are the scales of the couplings?

Calculation

LO Gluon Production

First consider LO gluon production ($A^+=0$ light-cone gauge):



The answer is
$$\frac{d\sigma}{d^2k_T\, dy} = \frac{2\,\alpha_s^3\, C_F}{\pi^2}\,\frac{1}{\boldsymbol{k}^2}\int\frac{d^2q}{\boldsymbol{q}^2\,(\boldsymbol{k}-\boldsymbol{q})^2}$$

which is the same kT-factorization formula if we notice that at LO
$$\phi(\boldsymbol{k},y) = \frac{\alpha_s\, C_F}{\pi}\,\frac{1}{\boldsymbol{k}^2}$$

Main Principle

To set the scale of the coupling constant we will first calculate the $\alpha_S N_f$ (quark loops) corrections to LO gluon production cross section to <u>all orders</u>.

We then will complete N_f to the full QCD beta-function

$$\beta_2 = \frac{11\,N_C - 2\,N_f}{12\,\pi}$$

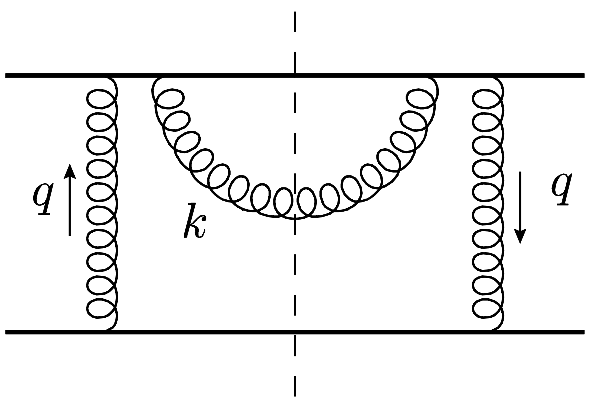
by replacing

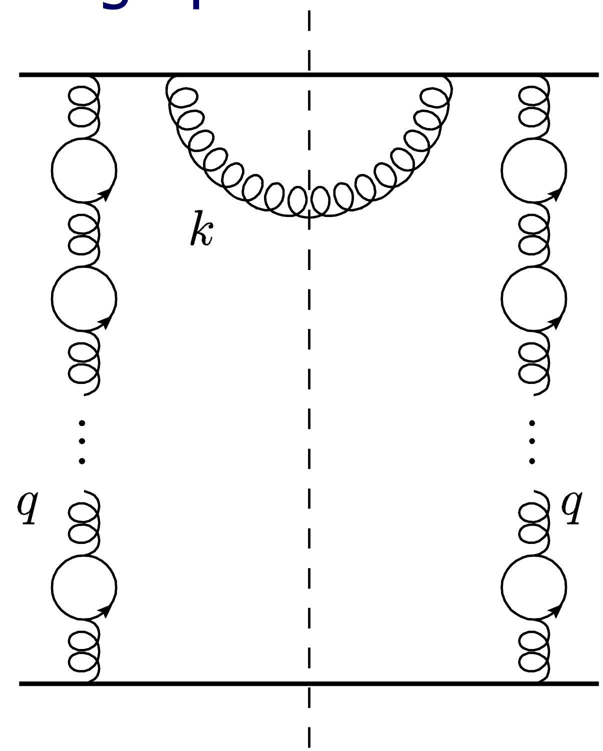
$$N_f \to -6\,\pi\,\beta_2$$

(Brodsky, Lepage, Mackenzie '83 – BLM prescription) .

Bremsstrahlung graphs

Start by putting quark bubbles on this graph:





$$\alpha_\mu^3 \rightarrow \frac{\alpha_\mu^3}{\left[1+\alpha_\mu\,\beta_2\,\ln\frac{\boldsymbol{q}^2}{\mu^2_{\overline{MS}}}\right]^2} = \alpha_\mu\,\alpha_s^2\left(\boldsymbol{q}^2\right)$$

problem: no scale here!?

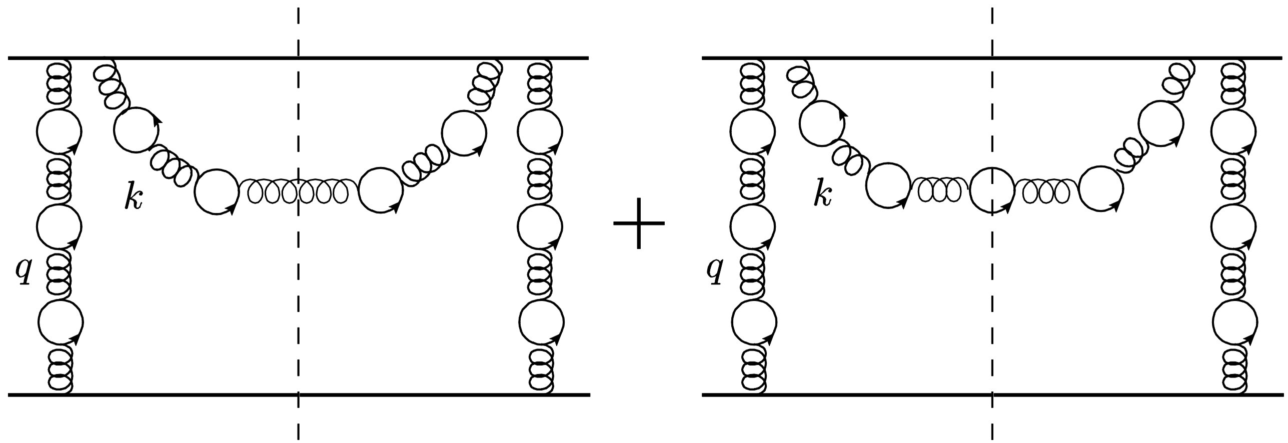
Gluon Resolution Scale

Resolution: redefine "gluon" production to include qqbar (GG) pairs with the invariant mass $k^2 \leq \Lambda_{coll}^2$

Then $$\alpha_\mu^3 \to \alpha_s\left(\Lambda_{\mathrm{coll}}^2\right)\,\alpha_s^2\left(\boldsymbol{q}^2\right)$$

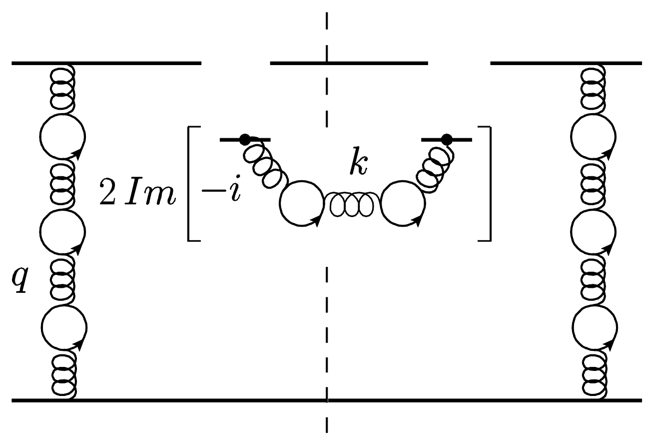
The scale is still largely arbitrary, but now it has a physical meaning.

Redefined produced gluon

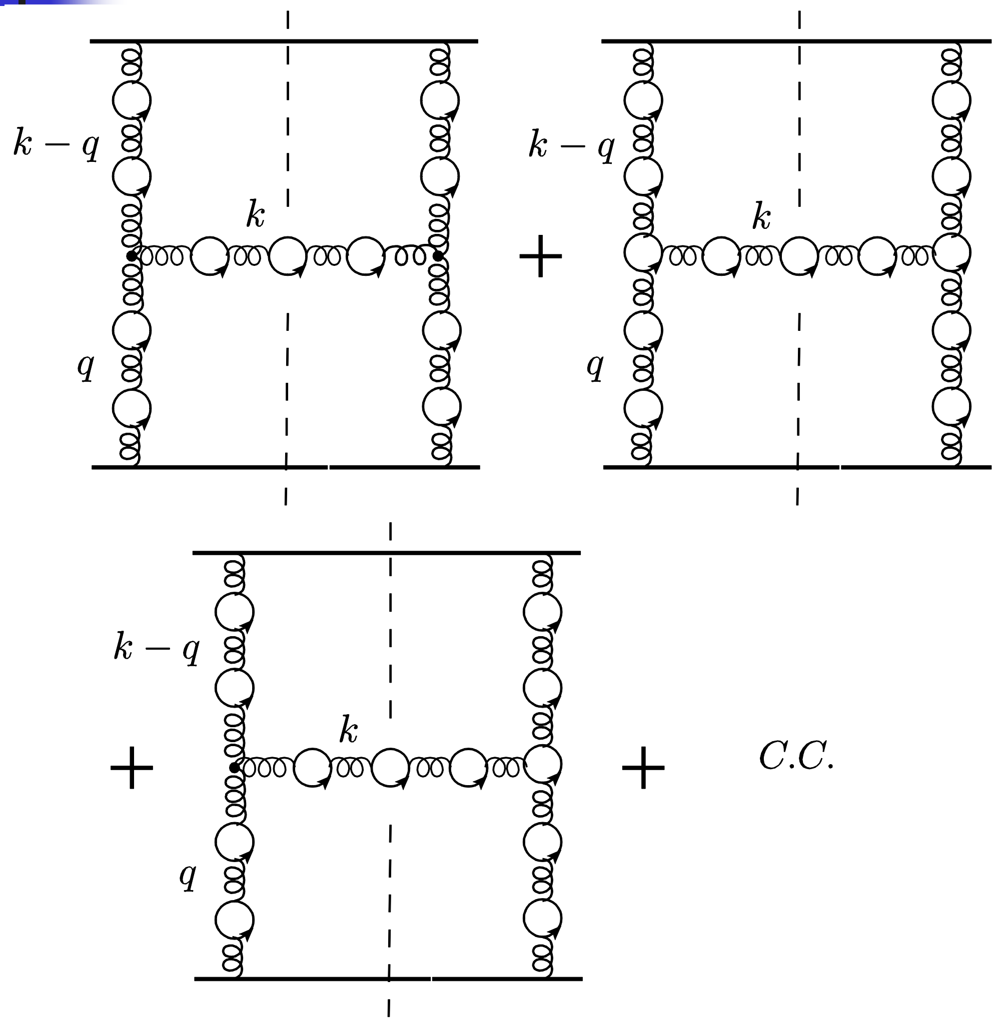


$$\alpha_\mu^3 \to \alpha_s\left(\Lambda^2_{\text{coll}}\right)\,\alpha_s^2\left(\boldsymbol{q}^2\right)$$

$=$

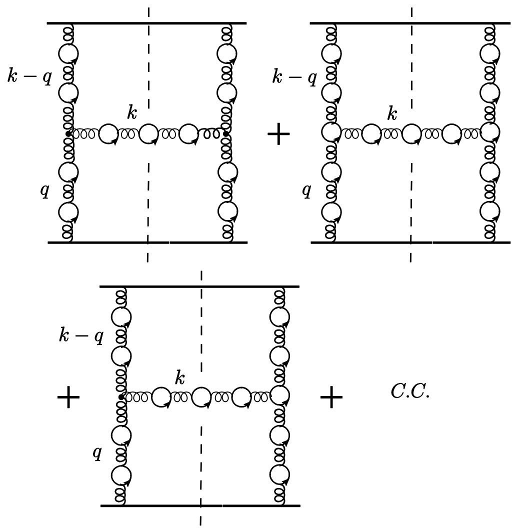


3-Gluon vertex graphs



Need to include corrections to graphs like this too.

3-Gluon vertex graphs



Quark bubbles on propagators give

$$\alpha_\mu^3 \;\to\; \alpha_s\left(\Lambda_{\rm coll}^2\right) \frac{\alpha_\mu^2}{\left[1+\alpha_\mu\,\beta_2\,\ln\frac{\boldsymbol{q}^2}{\mu^2_{\overline{MS}}}\right]^2 \left[1+\alpha_\mu\,\beta_2\,\ln\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{\mu^2_{\overline{MS}}}\right]^2}$$

$$= \frac{\alpha_s\left(\Lambda_{\rm coll}^2\right)\,\alpha_s^2\left(\boldsymbol{q}^2\right)\,\alpha_s^2\left((\boldsymbol{k}-\boldsymbol{q})^2\right)}{\alpha_\mu^2}$$

while the vertex bubbles give two new scales Q_A and Q_B :

$$\frac{1}{\alpha_\mu^2} \;\to\; \frac{1}{\alpha_\mu^2}\left[1+\alpha_\mu\,\beta_2\,\ln\frac{Q_A^2}{\mu^2_{\overline{MS}}}\right]\left[1+\alpha_\mu\,\beta_2\,\ln\frac{Q_B^2}{\mu^2_{\overline{MS}}}\right] \;\to\; \frac{1}{\alpha_s(Q_A^2)\,\alpha_s(Q_B^2)}$$

Results

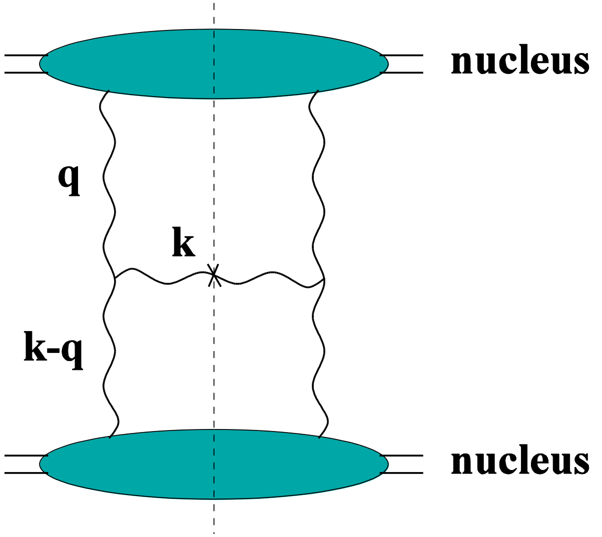
The answer for LO+rc

The final result can be written rather compactly as:

$$\frac{d\sigma}{d^2k_T\,dy} = \frac{2\,C_F}{\pi^2}\,\frac{\alpha_s\left(\Lambda^2_{\text{coll}}\right)}{\boldsymbol{k}^2}\int\frac{d^2q}{\boldsymbol{q}^2\,(\boldsymbol{k}-\boldsymbol{q})^2}\,\frac{\alpha_s^2\left(\boldsymbol{q}^2\right)\,\alpha_s^2\left((\boldsymbol{k}-\boldsymbol{q})^2\right)}{\alpha_s\left(Q^2\right)\,\alpha_s\left(Q^{*\,2}\right)}$$

with the “septumvirate” of running couplings,

except for…



The monster scale

$$
\begin{aligned}
\ln \frac{Q^2}{\mu^2_{\overline{\mathrm{MS}}}} = \frac{1}{2} \ln \frac{\boldsymbol{q}^2 (\boldsymbol{k}-\boldsymbol{q})^2}{\mu^4_{\overline{\mathrm{MS}}}} - \frac{1}{4\,\boldsymbol{q}^2 (\boldsymbol{k}-\boldsymbol{q})^2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right]^6} \Bigg\{ & \boldsymbol{k}^2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right]^3 \\
\times \Bigg\{ & \Big[\left[(\boldsymbol{k}-\boldsymbol{q})^2\right]^2 - \left(\boldsymbol{q}^2\right)^2 \Big] \Big[\left(\boldsymbol{k}^2\right)^2 + \left((\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right)^2 \Big] + 2\,\boldsymbol{k}^2 \Big[\left(\boldsymbol{q}^2\right)^3 - \left[(\boldsymbol{k}-\boldsymbol{q})^2\right]^3 \Big] \\
& - \boldsymbol{q}^2 (\boldsymbol{k}-\boldsymbol{q})^2 \Big[2 \left(\boldsymbol{k}^2\right)^2 + 3 \left[(\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right]^2 - 3\,\boldsymbol{k}^2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 + \boldsymbol{q}^2\right] \Big] \ln\left(\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{\boldsymbol{q}^2} \right) \Bigg\} \\
+ i \left[(\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right]^3 \Bigg\{ & \boldsymbol{k}^2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right] \Big[\boldsymbol{k}^2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 + \boldsymbol{q}^2\right] - \left(\boldsymbol{q}^2\right)^2 - \left[(\boldsymbol{k}-\boldsymbol{q})^2\right]^2 \Big] \\
& + \boldsymbol{q}^2 (\boldsymbol{k}-\boldsymbol{q})^2 \Big(\boldsymbol{k}^2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 + \boldsymbol{q}^2\right] - 2 \left(\boldsymbol{k}^2\right)^2 - 2 \left[(\boldsymbol{k}-\boldsymbol{q})^2 - \boldsymbol{q}^2\right]^2 \Big) \ln\left(\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{\boldsymbol{q}^2} \right) \Bigg\} \\
& \times \sqrt{2\,\boldsymbol{q}^2 (\boldsymbol{k}-\boldsymbol{q})^2 + 2\,\boldsymbol{k}^2 (\boldsymbol{k}-\boldsymbol{q})^2 + 2\,\boldsymbol{q}^2 \boldsymbol{k}^2 - \left(\boldsymbol{k}^2\right)^2 - \left(\boldsymbol{q}^2\right)^2 - \left[(\boldsymbol{k}-\boldsymbol{q})^2\right]^2} \Bigg\}
\end{aligned}
$$

note that it is complex!

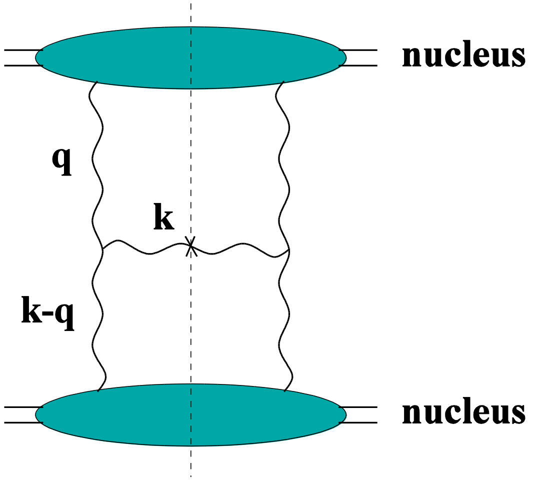
Simplified form

In an AA collision, with saturation effects included (as a cutoff), the formula can be simplified to:

$$\frac{d\sigma}{d^2k_T\,dy} \approx \frac{4\,C_F}{\pi}\,\frac{\alpha_s\left(\Lambda^2_{\mathrm{coll}}\right)\;\alpha_s(\boldsymbol{k}^2)\,\alpha_s(Q_s^2)}{\left(\boldsymbol{k}^2\right)^2}\,\ln\frac{\boldsymbol{k}^2}{Q_s^2}$$

Note that the monster scale becomes simple in the collinear limit:

$$Q \;\approx\; k_\perp \quad \text{if} \quad k_\perp \gg q_\perp$$



Ansatz for the full expression

Inspired by the LO result we make the following ansatz for the gluon production in pA including running coupling corrections

$$\frac{d\sigma}{d^2 k_T\, dy} = \frac{2\, C_F}{\pi^2} \frac{1}{\boldsymbol{k}^2} \int d^2 q\, \overline{\phi}_p(\boldsymbol{q}, y)\, \overline{\phi}_A(\boldsymbol{k} - \boldsymbol{q}, Y - y)\, \frac{\alpha_s\left(\Lambda^2_{\text{coll}}\right)}{\alpha_s\left(Q^2\right)\, \alpha_s\left(Q^{*\,2}\right)}$$

with the new unintegrated gluon distributions

$$\overline{\phi}_{p,\,A}(\boldsymbol{k}, y) = \frac{C_F}{(2\pi)^3} \int d^2 b\, d^2 r\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\ \nabla^2_r\, N_G^{p,\,A}(\boldsymbol{r}, \boldsymbol{b}, y)$$

(note that $\overline{\phi}(\boldsymbol{k}, y) = \alpha_s(\boldsymbol{k}^2)\, \phi(\boldsymbol{k}, y)$)

Integrated multiplicity

- dN/dy depends on how one defines Q_s :
  - If $Q_s^2 = 4\pi\,\alpha_s\left(Q_s^2\right)\,\alpha_s\left(\Lambda^2\right)\,\dfrac{A}{S_\perp}\,\ln\dfrac{Q_s}{\Lambda}$ then

$$\frac{dN}{dy} \approx \frac{C_F}{4\pi^2}\,S_\perp\,\frac{Q_s^2}{\ln^2\frac{Q_s}{\Lambda}}\,\frac{\alpha_s\left(\Lambda_{\rm coll}^2\right)}{\alpha_s^2\left(\Lambda^2\right)} \;\propto\; \frac{A}{\ln^2 A}$$

  - If $Q_s^2 = 4\pi\,\alpha_s^2\left(Q_s^2\right)\,\dfrac{A}{S_\perp}$ then

$$\frac{dN}{dy} \approx \frac{C_F}{4\pi^2}\,S_\perp\,\frac{Q_s^2}{\alpha_s^2\left(Q_s^2\right)}\,\alpha_s\left(\Lambda_{\rm coll}^2\right) \;\propto\; A$$

Multiplicity vs centrality

- We conclude that rc is not a defining factor in the centrality dependence of mutiplicity per participant, contrary to KLN interpretation of their results.

- Perhaps more importantly one needs two different saturation scales Q_{s1} and Q_{s2} to get the right centrality dependence. (cf. Drescher et al ’06, Kuhlman el at ’06).

Conclusions

- kT-factorization ansatz survives inclusion of running coupling corrections, at least at LO, after a slight redefinition of gluon distributions

- We have obtained a formula for LO gluon production + rc corrections.

- We have constructed an ansatz for the rc corrections to the pA gluon production including multiple rescatterings and small-x evolution.

- Our results can be used to model AA collisions as well.

Backup slides

Our Prediction

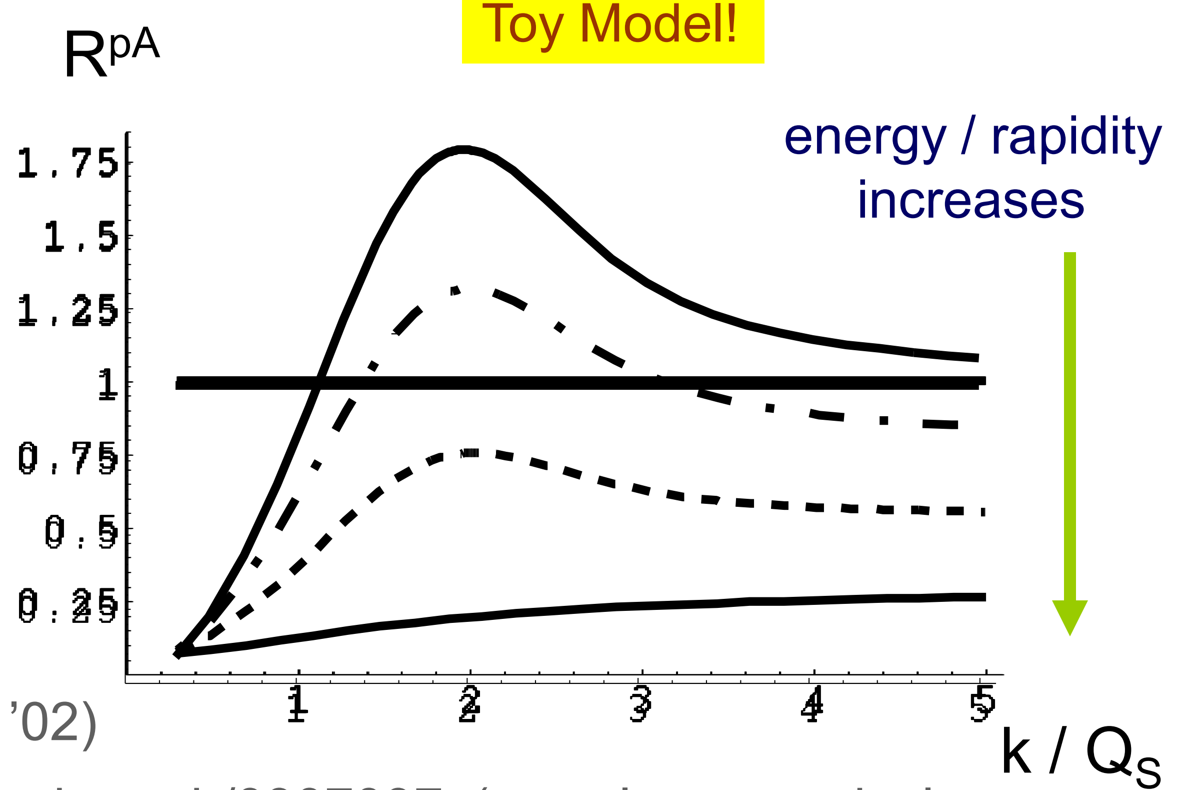
Our analysis shows that as energy/rapidity increases the height of the Cronin peak decreases. Cronin maximum gets progressively lower and eventually disappears.

- Corresponding R^{pA} levels off at roughly at

$$R^{pA} \sim A^{-1/6}$$

(Kharzeev, Levin, McLerran, '02)

Toy Model!

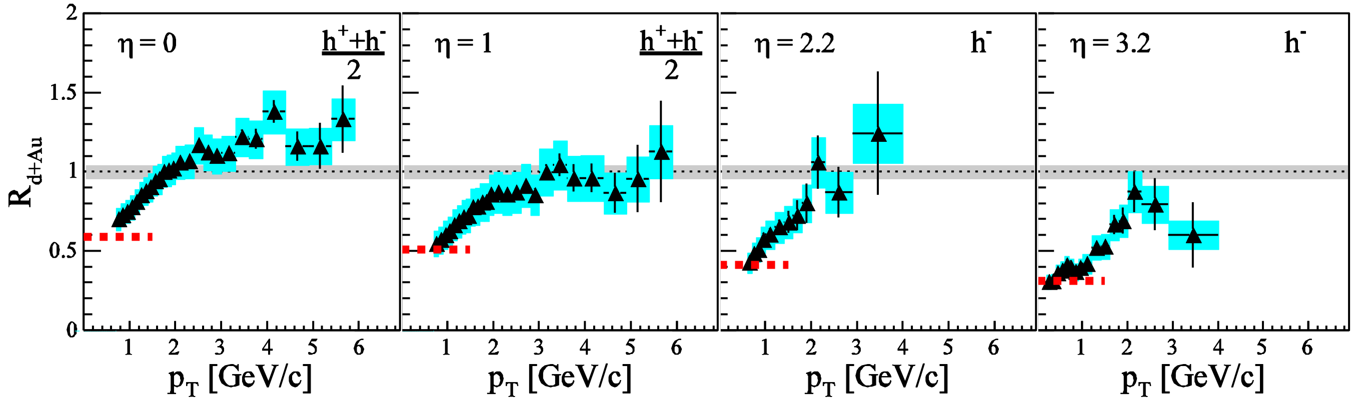


D. Kharzeev, Yu. K., K. Tuchin, hep-ph/0307037; (see also numerical simulations by Albacete, Armesto, Kovner, Salgado, Wiedemann, hep-ph/0307179 and Baier, Kovner, Wiedemann hep-ph/0305265 v2.)

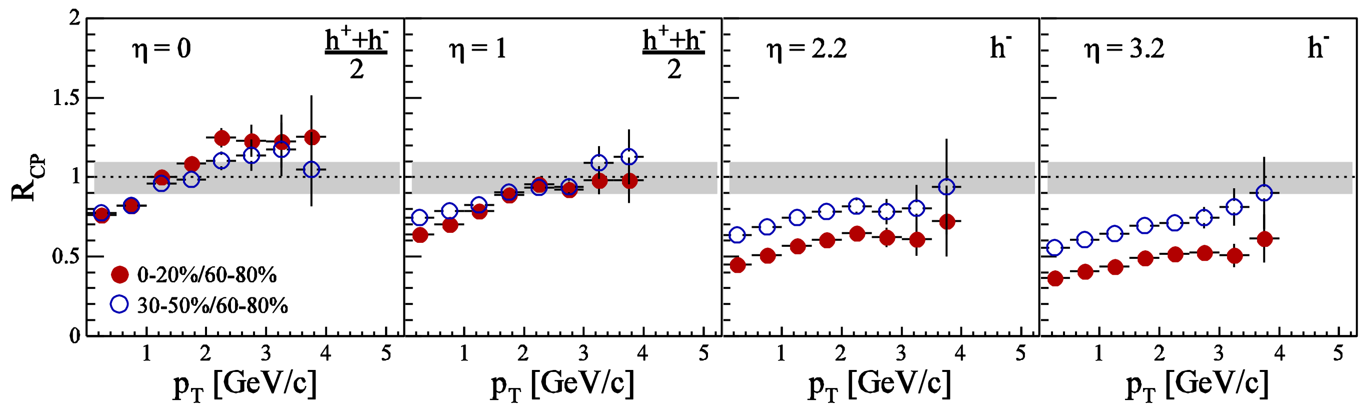
⇨At high energy / rapidity R^{pA} at the Cronin peak becomes a decreasing function of both energy and centrality.

R_{dAu} at different rapidities

R^{dAu}



R^{CP} – central to peripheral ratio



Most recent data from BRAHMS Collaboration nucl-ex/0403005

Our prediction of suppression was confirmed!