Heavy quarkonia in a quark-gluon plasma: coupled evolution and dynamics

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Abstract

We consider the possibility that heavy quarkonia admit different bound states in a QGP, between which they can transition dynamically. We show that the vacuum mass eigenstates are not the relevant eigenstates for the in-medium dynamics. This leads in particular to abundance ratios of the various states which deviate from the predictions of static models. Additionally, the quarkonium dynamics differ from that of states with a definite mass.

1. Introduction

Most studies of the suppression of heavy quarkonia in a quark-gluon plasma rely on the idea of sequential melting of quarkonium states with rising temperature. In this work, we adopt a more dynamical picture in which the effect of the plasma is not only to destroy the various states, but also to induce transitions between them, with temperature-dependent rates. Our purpose is to identify the impact of these transitions on the dynamical evolution of quarkonia in the medium.

For that, we first assume that the plasma is static, with a fixed temperature T. A $Q\bar{Q}$ pair in a given state is inserted at rest in this plasma at time $t=0$, and we follow the change of the various quarkonium state populations as well as the dynamical evolution of the $Q\bar{Q}$ pair under the influence of the momentum kicks it receives. In a second step, we consider a plasma with an evolving temperature.

2. Modelling of in-medium quarkonia

Consider a heavy quarkonium embedded in a thermal medium. The overall hamiltonian of the system is:

$$H = H_{Q\bar{Q}} + H_{\text{med.}} + V_{\text{int.}}.$$

$Q\bar{Q}$ pairs

Bound states

We model the heavy quarkonia as the successive bound states of a real-valued non-relativistic in-medium quark-antiquark potential.

For simplicity we consider a Coulomb-like binding potential:

$$U(r) \propto -\frac{\alpha_s}{r}.$$

Gluon-induced transitions from color-neutral $Q\bar{Q}$ states lead to colored states. For the latter, we assume the existence of an unspecified effect (soft color interaction, color evaporation...) which instantaneously turns a color octet into a color singlet state.

Unbound states

Such states should constitute a continuum and be modelled by scattering states. We adopt a simpler description:

- bound states of the potential U above a given "dissociation threshold" (fixed by the binding energy of stable quarkonia) describe unbound pairs;
- we forbid transitions between such unbound states and the bound ones.

Plasma

The plasma is modelled as a thermal bath:

- whose proper frequencies ω_λ span a large continuum;
- which is in a stationary state (the corresponding density operator is thus constant in interaction representation);
- which is non polarized and isotropic.

These properties and the bath temperature remain unmodified in the presence of the $Q\bar{Q}$ pair

Quarkonium-plasma coupling

For the interaction term, we assume:

- that the gluons interact with the $Q\bar{Q}$ pair through their chromoelectric field;
- dipolar coupling.

The interaction potential of the overall hamiltonian is thus given by:

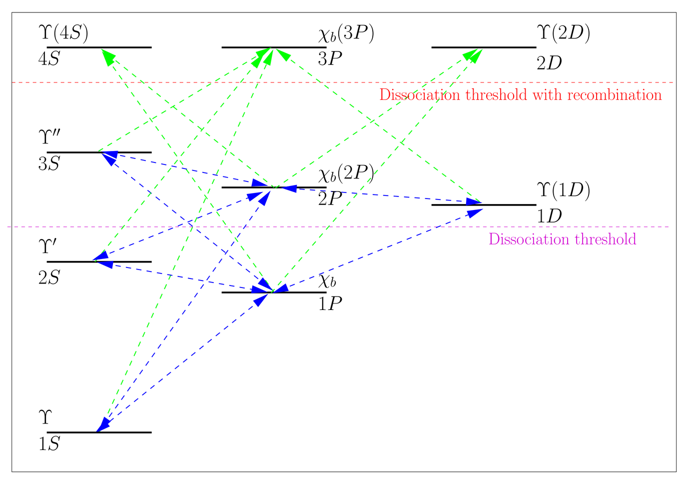
$$V_{\text{int.}} = -\mathbf{d}\cdot\mathbf{E}$$

$$= -\mathrm{i}\sqrt{C_F}\,\mathbf{r}\cdot\sum_\lambda\sqrt{\frac{4\pi\alpha_S\omega_\lambda}{2L^3}}\,\epsilon_\lambda\left(a_\lambda - a_\lambda^\dagger\right).$$

The chromoelectric dipolar operator $\mathbf{d}$ has real matrix elements between the different quarkonium states $|i\rangle$ and $|k\rangle$ directed along the angular momentum quantification axis.

3. Quarkonium spectroscopy

From now on, we focus on the bottomonium ($b\bar{b}$) system, which is expected to keep a much richer spectroscopy in medium than the charmonium. The states and transitions we consider are shown here:



For $b\bar{b}$ states bound in vacuum, we depart from the spectroscopy following from the potential U:

- the energies with respect to the ground ($1S$) state are the same as in vacuum;
- yet the wave functions are those given by the potential.

The $b\bar{b}$ dissociation threshold is indicated by a dashed line.

To mimic the possible recombination of unbound b and $\bar{b}$ quarks into bound bottomonia, we allow the transition from the lowest dissociated states back into bound states. Thus, the states that may recombine—sitting below a second "dissociation line"—are those which are below the threshold for heavy meson production in vacuum.

4. On the evolution equation

To describe the in-medium dynamics of the quarkonia, we use the master equation for the density matrix. In this phenomenological approach, the dynamics of a small quantum system interacting with a reservoir $\mathcal{R}$ follow from performing a partial trace over the reservoir degrees of freedom. If ρ denotes the density matrix for the full system, the state of the small system is also described by a density operator σ, called reduced density matrix, given by $\sigma = \mathrm{Tr}_{\mathcal{R}}(\rho)$.

The master equation governs the time evolution of the reduced density matrix, whose diagonal elements describe, in our model, the populations of the different energy levels.

Dynamical evolution within the quantum master equation

Solving the quantum master equation amounts to solving a set of coupled Einstein equations for the quarkonium state populations:

$$\frac{\mathrm{d}\sigma_{ii}}{\mathrm{d}t} = -\sum_{k\neq i}\Gamma_{i\to k}\sigma_{ii} + \sum_{k\neq i}\Gamma_{k\to i}\sigma_{kk}, \qquad (1)$$

where the transition rates ($\Gamma_{k\to i}$) are given by Fermi's golden rule.

- If one is only after the evolution of the internal degrees of freedom [1], the transition rates are given by:

$$\Gamma_{k\to i} = 3\times 2\pi\sum_\lambda\left(\langle n_\lambda\rangle + 1\right)\left|\langle i;1_\lambda|\tilde{V}|0_\lambda;k\rangle\right|^2\delta(\omega_\lambda - \omega_{ik}),$$

$$\Gamma_{i\to k} = 3\times 2\pi\sum_\lambda\langle n_\lambda\rangle\left|\langle k;0_\lambda|\tilde{V}|1_\lambda;i\rangle\right|^2\delta(\omega_\lambda - \omega_{ik}).$$

The term $+1$ in the first equation accounts for spontaneous emission. The plasma temperature enters the master equation through the number $\langle n_\lambda\rangle$ of gluons in mode λ, while the factor 3 arises from the different polarizations.

- If one is interested in both the internal and external degrees of freedom [2], the quantum master equation gives the dynamical evolution of the quarkonium momentum distribution. The equation can be expanded for low momentum transfers with the medium. *In the absence of dissociation transitions and neglecting mass variations*, this expansion up to the second order allows one to recover the usual Fokker–Planck equation

$$\frac{\partial\pi}{\partial t}(t,\mathbf{p}) = \gamma\nabla_{\mathbf{p}}\big(\mathbf{p}\pi(t,\mathbf{p})\big) + D\,\Delta_{\mathbf{p}}\pi(t,\mathbf{p}) \qquad (2)$$

for the momentum distribution function $\pi(t,\mathbf{p})$, while the coefficients γ and D together with the quarkonium mass $2m_Q$ and the temperature obey the fluctuation-dissipation relation

$$\frac{D}{2m_Q} = \gamma k_B T. \qquad (3)$$

5. Underlying assumptions

This description relies on several assumptions:

- The bath characteristic time is much smaller that the characteristic time of a transition.
- The field seen by both quark and antiquark is the same (long wavelength assumption).
- The interaction term is linear in a_λ and $a_\lambda^\dagger$, which corresponds to a weak-coupling regime (we only consider single-gluon transitions).
- We assume that the bath and $Q\bar{Q}$-pair degrees of freedom factorize at any time.

6. Results: In-medium evolution of quarkonia

Evolution of the internal degrees of freedom

We first consider a constant plasma temperature T, and we follow the evolution of a $b\bar{b}$ pair initially created at rest in the fundamental state ($1S$).

The time evolution of the different populations shows that, after some transient regime, all states evolve with the same characteristic time (left). The different populations are coupled to each other through the medium: the vacuum mass eigenstates are not eigenstates of the in-medium evolution operator. It is possible to compute the quasi-stationary ratios of the different populations as function of the plasma temperature (right): the resulting ratios differ widely from those predicted in statistical models.

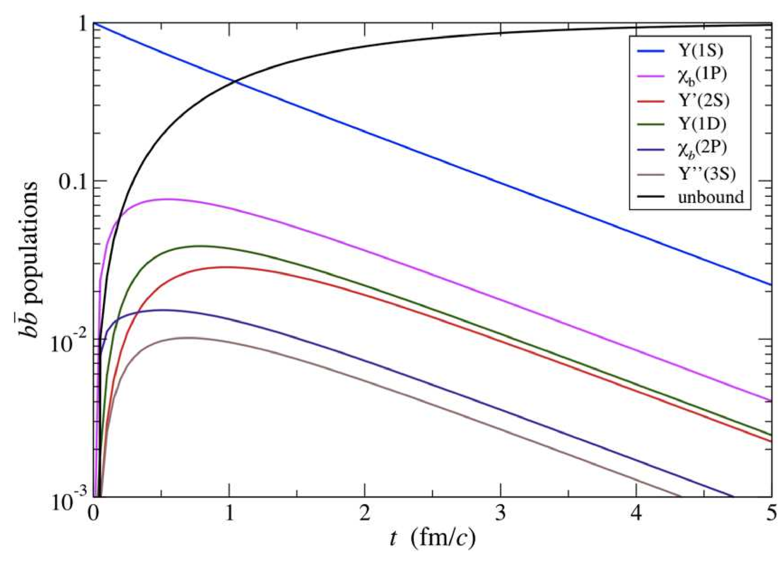


Figure: Time evolution of the populations of bottomonia for a plasma at $T=5T_c$.

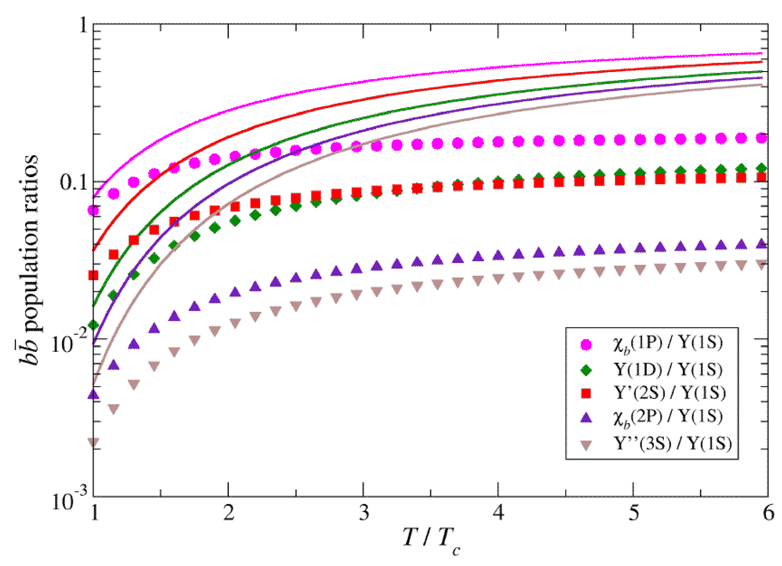


Figure: Temperature dependence of the "equilibrium" ratios of bottomonium populations; lines denote the ratios in statistical models.

Next, we consider an evolving plasma temperature, using the time dependence at the central point of the plasma given by a dissipative fluid dynamics computation for Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [3], with $b = 2.33$ fm, $\eta/s = 0.2$ and using the equation of state s95p-PCE. Such a time-dependent temperature (left) modifies our previous results: the change in temperature results in evolving transition rates, which prevents the $b\bar{b}$-state populations from reaching their stationary ratios (right).

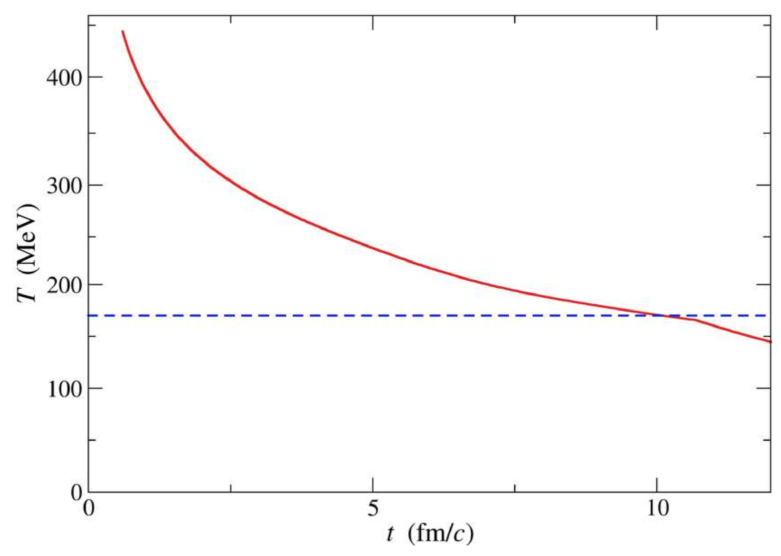


Figure: Time evolution of the plasma temperature in viscous hydrodynamics [3].

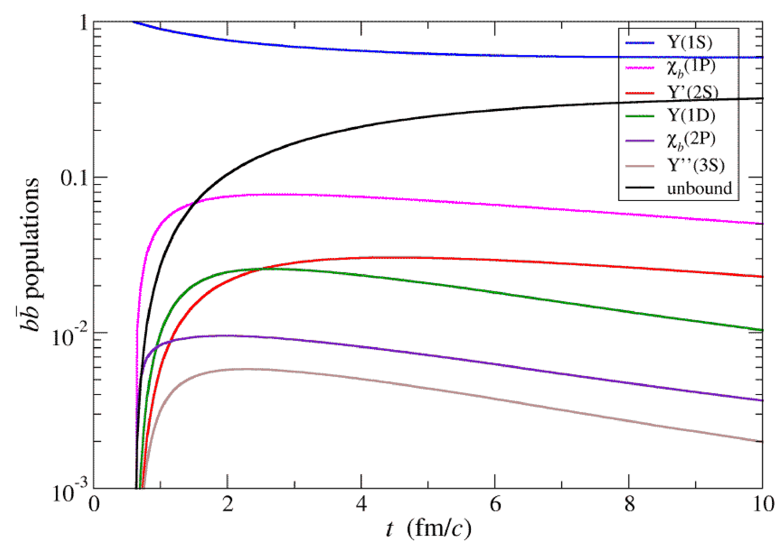


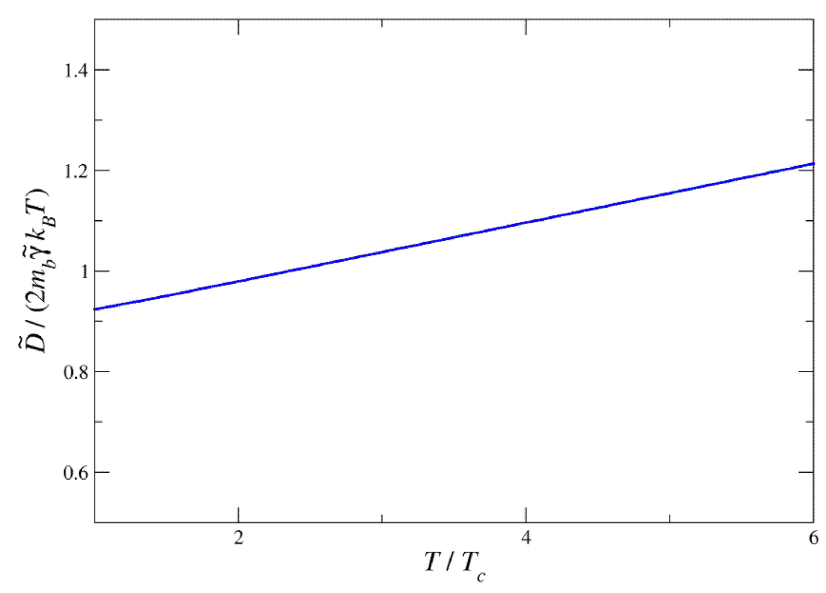
Figure: Time dependence of the populations of bottomonia for a plasma with evolving temperature.

Evolution of both internal and external degrees of freedom

When dissociation transitions are allowed and the different masses of the various bound states are taken into account, the quarkonium master equation for the momentum distribution function no longer leads to the Fokker–Planck equation (2), which is replaced by

$$\frac{\partial\pi}{\partial t}(t,\mathbf{p}) = \tilde{\gamma}\nabla_{\mathbf{p}}\big(\mathbf{p}\pi(t,\mathbf{p})\big) + \tilde{D}\,\Delta_{\mathbf{p}}\pi(t,\mathbf{p}) - \left(A + B\mathbf{p}^2\right)\pi(t,\mathbf{p}),$$

with A and B two positive numbers. Additionally, although $\tilde{\gamma}$ and $\tilde{D}$ are still constant, they no longer satisfy the fluctuation-dissipation relation (3), as shown in the figure.



7. Conclusion

In this work we show, in the framework of a microscopic description of the quarkonium-plasma interactions, that the possibility of internal transitions between the different quarkonium states strongly impacts the quarkonium evolution and dynamics:

- A given state is never totally suppressed, as it is constantly recreated from other states.
- Progressive losses to the continuum of dissociated states lead to a deviation of the population ratios from those of statistical models:

  the internal degrees of freedom are not equilibrated.
- In medium, the momentum distribution of heavy quarkonia obeys a modified Fokker–Planck-like equation, whose coefficients do not satisfy the usual fluctuation-dissipation relation:

  the external degrees of freedom are not thermalized.

These qualitative features are independent from the choice of microscopic model for the quarkonium.

Acknowledgments

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Bibliography

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