Medium Induced Collinear Radiation via Soft Collinear Effective Theory (SCET)

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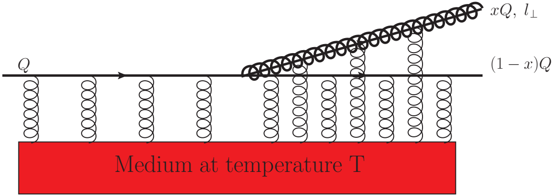
Jets propagation in dense media

Hard Probes for the QGP

Jet quenching observables make it possible to study
how parton fragmentation is affected by the medium.

The medium has two main effects on the propagating hard parton:

- changing direction of its momentum (transverse momentum broadening);
- inducing parton energy loss (dominated by QCD analogue of bremsstrahlung).



Hard partons constantly kicked by the medium: all subject to transverse momentum broadening.

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L}$$

Effective Field Theory description

Separation of scales

Particle with energy Q propagating through
a dense medium with characteristic scale T

$$\lambda \equiv \frac{T}{Q} \ll 1$$

We can ultimately hope for an Effective Field Theory description:

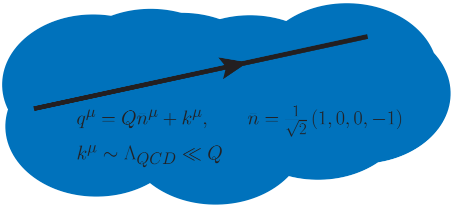
- physics at each scale cleanly separated at leading power;
- corrections systematically calculable, order by order in λ.

Our language: Soft Collinear Effective Theory (SCET)

In the $Q \gg T$ limit natural organization of the modes into kinematic regimes

Other works on SCET applied to parton propagation in dense media:
Idilbi, Majumder, PRD80(2009) [arXiv:0808.1087]; Ovanesyan, Vitev, [arXiv:1103.1074].

Soft Collinear Effective Theory (SCET)



SCET

Effective theory of highly energetic, approximately massless particles interacting with a soft background.

C. Bauer et al., PRD63(2001) [hep-ph/0005275]; PRD63(2001) [hep-ph/0011336]; PLB516(2001) [hep-ph/0107001]; PRD65(2002) [hep-ph/0109045].

SCET degrees of freedom

Introduce fields for infrared degrees of freedom (in operators)

Offshell modes with $q^2 \gg \lambda^2 Q^2$ are integrated out (in coefficients)

modes	$q^\mu = (q^+, q^-, q_\perp)$	fields
collinear	$Q\left(\lambda^2, 1, \lambda\right)$	$\xi_{\bar{n}},\ A^\mu_{\bar{n}}$
soft	$Q\left(\lambda, \lambda, \lambda\right)$	$\xi_s,\ A^\mu_s$
ultra-soft	$Q\left(\lambda^2, \lambda^2, \lambda^2\right)$	$\xi_{us},\ A^\mu_{us}$

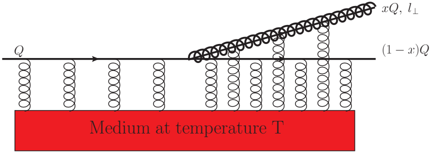
Hard particle propagating through the medium:
collinear mode of SCET

Relevance of Glauber modes

Incoming parton, outgoing parton and radiated gluon considered to be collinear.

Radiation vertex already present in SCET.

What about momentum broadening?
Are the SCET d.o.f. enough?



Momentum broadening in the high energy limit dominated by interactions between the hard collinear parton and Glauber modes from the medium.

- Glauber modes: $p = (\lambda^2, \lambda^2, \lambda)Q$.

Idilbi, Majumder, Phys.Rev.D80(2009) [arXiv:0808.1087];

Glaubers not d.o.f. of SCET, need to extend SCET to include them

SCET + Glauber Effective Lagrangian

Goal

Derive an effective Lagrangian to describe a theory of collinear partons (quarks or gluons) and Glauber gluons.

Top-down EFT: start from QCD Lagrangian, keep only relevant d.o.f.

EFT fields

Light-cone unit vectors: $\bar{n} \equiv \frac{1}{\sqrt{2}}(1,0,0,-1)\,, \qquad n \equiv \frac{1}{\sqrt{2}}(1,0,0,1)$.

Quark field decomposition:

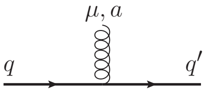
$$\xi(x) = \xi_{\bar{n}}(x) + \xi_n(x), \qquad \xi_{\bar{n}}(x) \equiv \frac{\not{\bar{n}}\not{n}}{2}\xi(x), \qquad \xi_n(x) \equiv \frac{\not{n}\not{\bar{n}}}{2}\xi(x).$$

Collinear quark field: "large" component $\xi_{\bar{n}}(x)$, the "small" component $\xi_n(x)$ is integrated out.

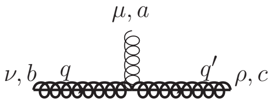
Collinear gluon field: $A^{\mu}_{\bar{n}}(x)$.

Glauber gluon field: $A^{\mu}_{G}(x)$ (background field).

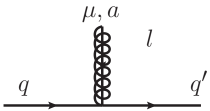
SCET + Glauber Feynman Rules



$$= i g t_F^a \overline{n}_\mu \not{n}$$



$$= -2\, i g \, (t_G^a)_{bc} \, \overline{n}^\mu \times [g^{\nu\rho} Q + n^\nu (q'_\perp - q_\perp)^\rho - n^\rho (q'_\perp - q_\perp)^\nu - \tfrac{\alpha-1}{2\alpha} \left(n^\rho q^\nu + n^\nu q'^\rho\right)]$$



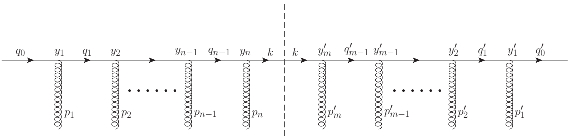
$$= i\, g\, t_F^a \left[\bar{n}_\mu - \frac{1}{2} \frac{\not{q}_\perp + \not{l}_\perp}{n\cdot(q+l)} \frac{\not{q}_\perp}{n\cdot q} n_\mu + \gamma_\mu^\perp \frac{\not{q}_\perp}{2\, n\cdot q} + \frac{\not{q}_\perp + \not{l}_\perp}{2\, n\cdot(q+l)} \gamma_\mu^\perp \right] \not{n}$$

We are ready to compute Feynman diagrams!

Transverse Momentum Broadening

Momentum broadening described by $P(k_\perp)$, probability to acquire transverse momentum $k_\perp$ after traversing the medium

FDE, Liu, Rajagopal, [arxiv:1006.1367]



Result for $P(k_\perp)$

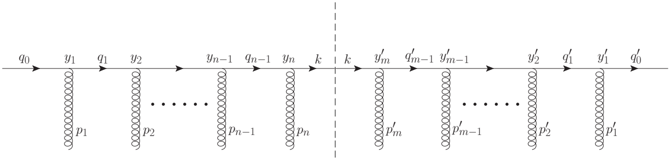
$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp), \qquad \mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr} \left[W^\dagger_\mathcal{R}[x_\perp] \, W_\mathcal{R}[0] \right] \right\rangle$$

for a collinear particle in the $SU(N)$ representation $\mathcal{R}$, with dimension $d(\mathcal{R})$.

- $P(k_\perp)$ is a soft function, it depends only on the medium property
- Transverse momentum broadening without radiation:
  field theoretically well-defined property of the medium
- $\hat{q} = \frac{1}{L} \int \frac{d^2k_\perp}{(2\pi)^2} \, k_\perp^2 \, P(k_\perp)$

$P(k_\perp)$ derived with other techniques: Casalderrey-Solana and Salgado, APPB38(2007) [arXiv:0712.3443 [hep-ph]], Liang, Wang and Zhou, PRD77(2008) [arXiv:0801.0434 [hep-ph]].

Glaubers from the medium and the $Q \to \infty$ limit



A_μ as a background field

(i) hard parton propagating in a specific field configuration $A_\mu(p)$;

(ii) average over field configurations: $\left\langle \mathrm{Tr}\left[W^\dagger_{\mathcal{R}}[x_\perp]\, W_{\mathcal{R}}[0] \right] \right\rangle$.

Nature of the medium (strongly coupled? weakly coupled?) only affects step (ii).

$Q \to \infty$ limit

Result for $P(k_\perp)$ valid if $Q \gg k_\perp^2 L \sim \hat{q} L^2$.

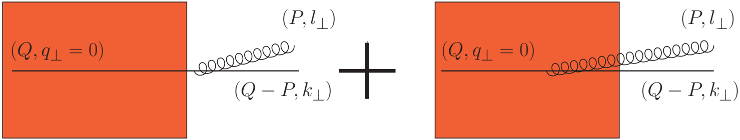
Same framework for the gluon radiation calculation

Gluon radiation calculation

Medium induced collinear gluon radiation

Incoming collinear quark radiates a gluon collinear in the same direction.

Effective theory valid for any collinear gluon energy.

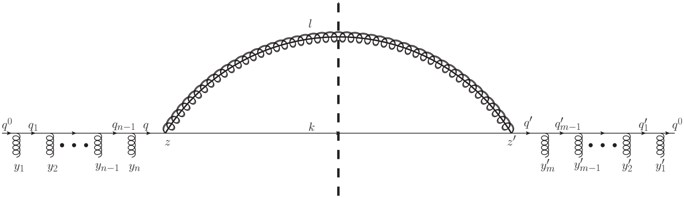


Total amplitude: vacuum + medium emission

$$\mathcal{M} = \mathcal{M}_v + \mathcal{M}_m \quad \Rightarrow \quad |\mathcal{M}|^2 = |\mathcal{M}_v|^2 + |\mathcal{M}_m|^2 + 2\,\mathrm{Re}\,(\mathcal{M}_m \mathcal{M}_v^*)$$

We integrate over the final quark kinematical variables, matrix element squared will only depend on $l_\perp$ and P

Vacuum emission



$$|\mathcal{M}_v|^2 = 4\,g^2\,C_F \int d^2x_\perp \exp\left[i\,l_\perp \cdot x_\perp\right]\,\mathcal{Y}[x_\perp, l_\perp, P]$$

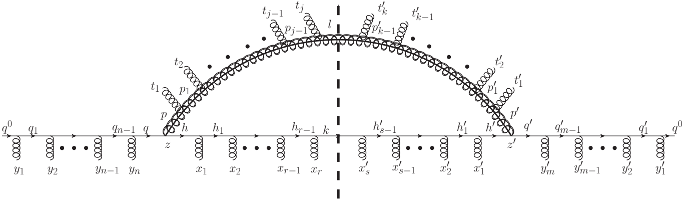
where the have the soft function

$$\mathcal{Y}[x_\perp, l_\perp, P] \equiv \mathcal{W}_F(x_\perp) \int \frac{d^2k_\perp}{(2\pi)^2}\,\frac{\exp[i\,k_\perp \cdot x_\perp]}{\left(\frac{k_\perp^2}{Q-P} + \frac{l_\perp^2}{P}\right)^2}\left[\frac{1}{2}\,\frac{k_\perp^2}{(Q-P)^2} + \frac{1}{2}\,\frac{q_\perp^2}{Q^2} + \frac{l_\perp^2}{P^2} - \frac{l_\perp \cdot k_\perp}{P(Q-P)} - \frac{l_\perp \cdot q_\perp}{QP}\right]$$

Check the $P \ll Q$ limit

$$\mathcal{Y}[x_\perp, l_\perp, P] \simeq \delta^2(x_\perp)\,\frac{1}{l_\perp^2} \qquad \Rightarrow \qquad |\mathcal{M}_v|^2 = \frac{4\,g^2\,C_F}{l_\perp^2}$$

Wiedemann, NPB588(2000) hep-ph/0005129; NPA690(2001) [hep-ph/0008241]



Calculation in progress....

So far: fundamental Wilson lines and adjoint path integrals in the expression, with transverse derivative acting on all of them

$P \ll Q$ limit correctly reproduced

$$|\mathcal{M}_m|^2 = \frac{1}{P^2} g^2 \int dz'^{-} dz^{-} \int d^2t'_\perp d^2t_\perp \exp\left[-i\, l_\perp \cdot (t_\perp - t'_\perp)\right] \; W_A^{ab}(z'^{-}, z^{-})\,[0_\perp]$$
$$\left[\frac{\partial}{\partial\, \bar{y}_\perp} G\left(\bar{y}_\perp = 0_\perp, z'^{-}; t'_\perp, L^{-};\right) \frac{\partial}{\partial\, y_\perp} G\left(t_\perp, L^{-}; y_\perp = 0_\perp, z^{-}\right)\right]_{ab}$$

Wiedemann, NPB588(2000) hep-ph/0005129; NPA690(2001) [hep-ph/0008241]

Summary and future directions

Medium induced gluon radiation within SCET

Kinematics:
radiated gluon collinear to the incoming quark, with any fraction of its energy

Future directions

- complete the collinear gluon spectrum
- evaluate the thermal average
- include λ power corrections

and also...

- introduce soft modes in the EFT ($p \sim Q(\lambda, \lambda, \lambda)$)
  and allow for large angle radiation
- allow for emission in any collinear direction