

QM2011



# The second act of hydro: the sound circles

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**“While throwing stones into the pond, watch carefully the circles they make, or else this occupation is meaningless” K.Prutkov**

# Outline:

1. Hydro1: sQGP remains a good liquid at LHC
2. Hydro2: perturbations. Initial “hot spots” => “circles” => are observed in correlations
3. Sounds from the “Tiny Bangs” are solved analytically (on top of “Gubser flow), even with viscosity
4. Mach cones separate (slightly) hotter matter from the unperturbed one: the “edge” should be observable in events with large  $O(100 \text{ GeV})$  deposition, and is perhaps already seen at LHC!)

# Viewpoint

## A “Little Bang” arrives at the LHC

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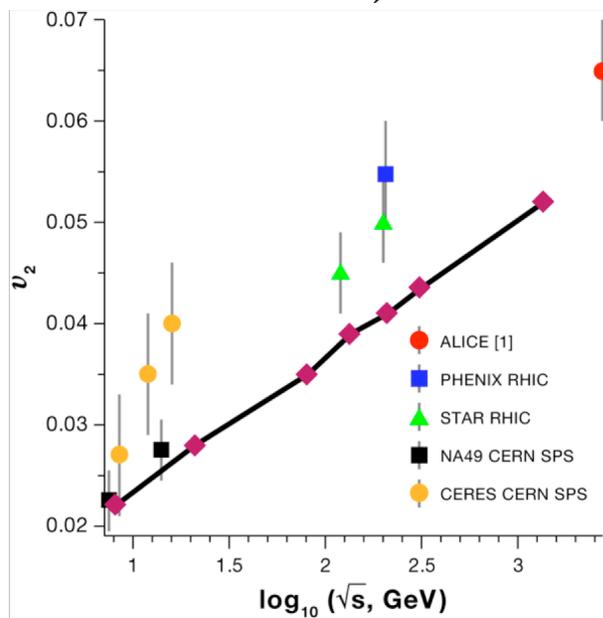


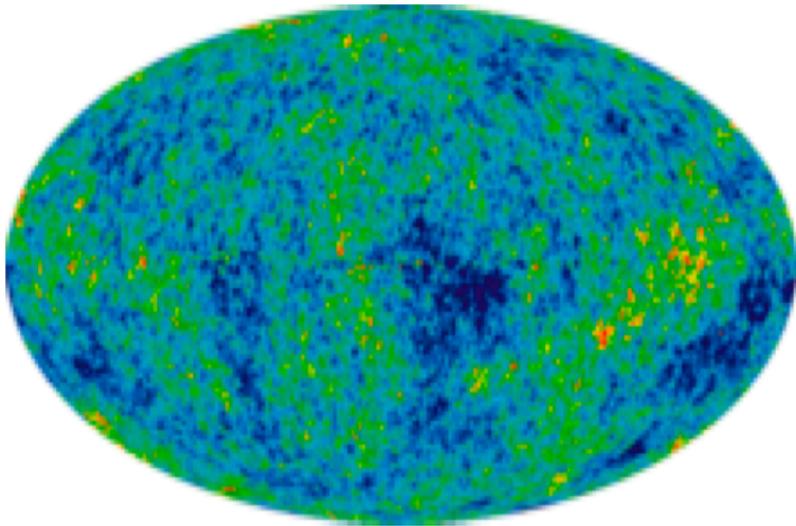
FIG. 1: The ALICE experiment suggests that the quark-gluon plasma remains a strongly coupled liquid, even at temperatures that are 30% greater than what was available at RHIC. The plot shows the “elliptic flow parameter”  $v_2$  (a measure of the coupling in the plasma) at different heavy-ion collision energies, based on several experiments (including the new data from ALICE [1]). (Note the energy scale is plotted on a logarithmic scale and spans three orders of magnitude.) The trend is consistent with theoretical predictions (pink diamonds) for an ideal liquid [4].

Increased elliptic and radial flows, as well as increased HBT radii/volume are all supporting “Hydro1”, the “Little Bang”

What do these results tell us about the quark-gluon plasma? The mean free path for particles in the plasma can be conveniently expressed via a dimensionless ratio ( $\eta/s\hbar$ ), where  $\eta$  is the shear viscosity,  $s$  is the entropy density and  $\hbar$  is Planck’s constant. In a weakly coupled quark-gluon plasma, the mean free path should be large ( $\eta/s\hbar \gg 1$ ), while it should be small in a strongly coupled plasma. RHIC data analysis has shown it to be extremely small, close to the theoretically conjectured lower limit  $\eta/s\hbar = 1/4\pi$  for infinitely strong coupling [5]. That this strong-coupling picture holds for the QGP seen at the LHC seems now likely. Naively, one might

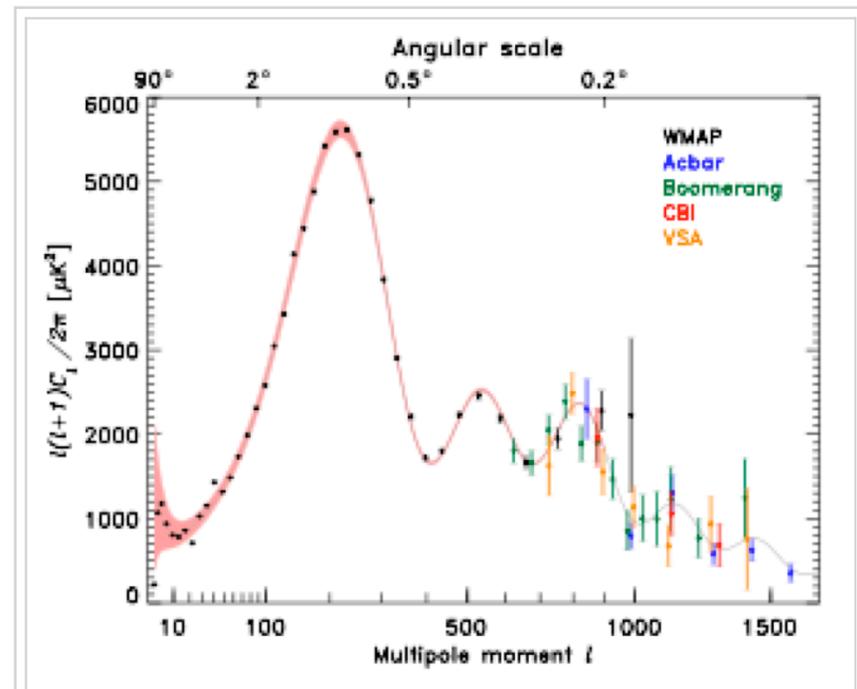
## More about CMB fluctuations

41. ^ Zeldovich, Y. B. (1972). "A hypothesis, unifying the structure and the entropy of the Universe". *Monthly Notices of the Royal Astronomical Society* **160**: 1P–4P. doi:10.1016/S0026-0576(07)80178-4 (<http://dx.doi.org/10.1016%2FS0026-0576%2807%2980178-4>) .
42. ^ Doroshkevich, A. G.; Zel'Dovich, Y. B.; Syunyaev, R. A. (12–16 September 1977). "Fluctuations of the microwave background radiation in the adiabatic and entropic theories of galaxy formation". In Longair, M. S. and Einasto, J.. *The large scale structure of the universe; Proceedings of the Symposium*. Tallinn, Estonian SSR: Dordrecht, D. Reidel Publishing Co.. pp. 393–404. Bibcode: 1978IAUS...79..393S (<http://adsabs.harvard.edu/abs/1978IAUS...79..393S>) . While this is the first paper to discuss the detailed observational imprint of density inhomogeneities as anisotropies in the cosmic microwave background, some of the groundwork was laid in Peebles and Yu, above.



The same scale is seen in CMB and correlations of the galaxies:

All correspond to Sound Horizon,  
The distance sound travel before neutralization



The power spectrum of the cosmic microwave background radiation temperature anisotropy in

# Perturbations of the Big and the Little Bangs

Frozen sound (from the era long gone) is seen on the sky, both in CMB and in distribution of Galaxies

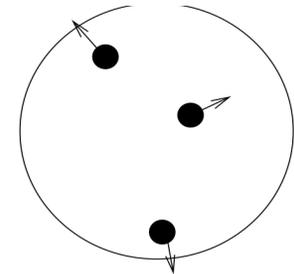
$$\frac{\Delta T}{T} \sim 10^{-5}$$

$$l_{\text{maximum}} \approx 210$$

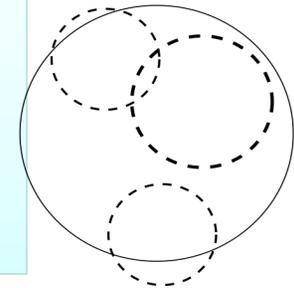
$$\delta\phi \sim 2\pi/l_{\text{maximum}} \sim 1^\circ$$

**Initial state fluctuations in the positions of participant nucleons lead to perturbations of the Little Bang also**

$$\frac{\Delta T}{T} \sim 10^{-2}$$



**Cylindrical (extended in z) at FO surface  $\tau=2R$  and sound velocity is  $\frac{1}{2} \Rightarrow$  radius is about  $R \Rightarrow$  Azimutal harmonics  $m=1,3,5\dots$**



**They are literally circles on the sky, around primordial density perturbations**

*Perhaps shumerians had managed to see them somehow...., why else had they introduced  $1^\circ$ ?*

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**Fate of the initial state perturbations in heavy ion collisions**

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(Received 20 July 2009; revised manuscript received 14 October 2009; published 13 November 2009)*

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# Visible shape of the sound (at freezeout, boosted by radial flow)

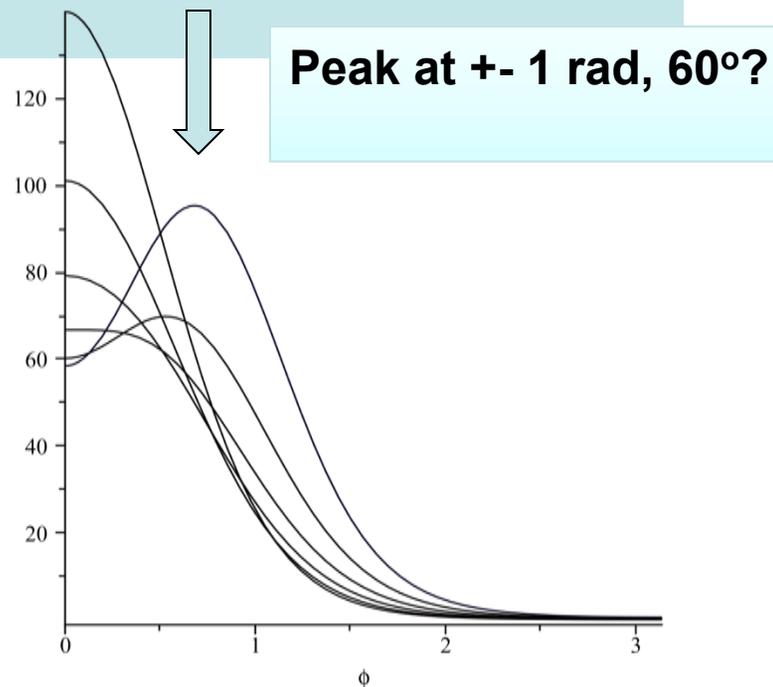


FIG. 5. (Color online) Dependence of the visible distribution in the azimuthal angle on the width of the (semi)circle at the time of freeze-out. Six curves, from the most narrow to the widest ones, correspond to the radius of the circle of 1, 2, 3, 4, 5, and 6 fm, respectively. The original spot position is selected to be at the edge of the nuclei. The distribution is calculated for a particle of  $p_t = 1$  GeV and fixed freeze-out  $T_f = 165$  MeV.

- The blue line is how asimuthal distribution would look like **for sound cylinders, double peak because of two points where the circle crosses the FO surface**
- **The circles were found and studied by Hama,Grassi et al in**
- **Event-by-event hydro**

# Two new fundamental scales, describing fluctuations **at freezeout**

(P.Staig,ES)

1.The sound horizon:  
radius

$$H_s = \int_0^{\tau_f} d\tau c_s(\tau)$$

**2.The viscous horizon:  
The width of the circle**

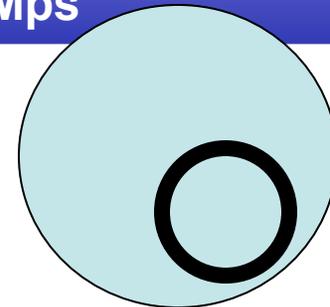
$$\delta T_{\mu\nu}(t) = \exp\left(-\frac{2}{3} \frac{\eta}{s} \frac{k^2 t}{3T}\right) \delta T_{\mu\nu}(0)$$

$$k_v = \frac{2\pi}{R_v} = \sqrt{\frac{3Ts}{2\tau_f\eta}} \sim 200 \text{ MeV}$$

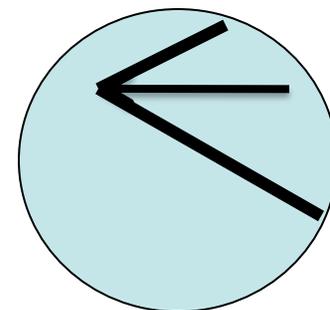
Let us finish this section by pointing out the hierarchy relation between all those four scales which we assume is true

$$R > H_s > R_v > l \quad (2.8)$$

For the Big Bang it was introduced by Sunyaev-Zeldovich about 40 years ago, was observed in CMB and galaxy correlations, it is about 150 Mps



**cylinders**



**cones**

# The Fate of the Initial State Fluctuations in Heavy Ion Collisions. III The Second Act of Hydrodynamics

Pilar Staig and Edward Shuryak

## Comoving coordinates with Gubser flow:

Gubser and Yarom, arXiv:1012.1314

$$\sinh \rho = -\frac{1 - q^2 \tau^2 + q^2 r^2}{2q\tau}$$

$$\tan \theta = \frac{2qr}{1 + q^2 \tau^2 - q^2 r^2}$$

$$\frac{\partial^2 \delta}{\partial \rho^2} - \frac{1}{3 \cosh^2 \rho} \left( \frac{\partial^2 \delta}{\partial \theta^2} + \frac{1}{\tan \theta} \frac{\partial \delta}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2 \delta}{\partial \phi^2} \right) + \frac{4}{3} \tanh \rho \frac{\partial \delta}{\partial \rho} = 0 \quad (3.16)$$

We have seen that in the short wavelength approximation we found a wave-like solution to equation 3.16, but now we would like to look for the exact solution, which can be found by using variable separation such that  $\delta(\rho, \theta, \phi) = R(\rho)\Theta(\theta)\Phi(\theta)$ , then

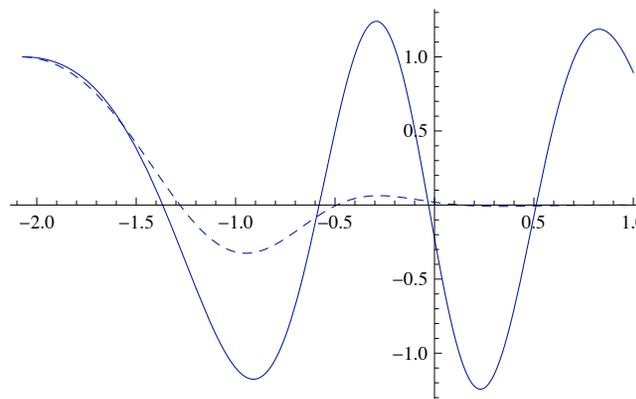
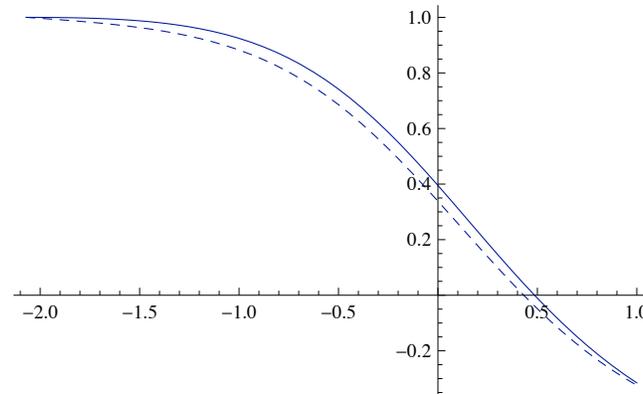
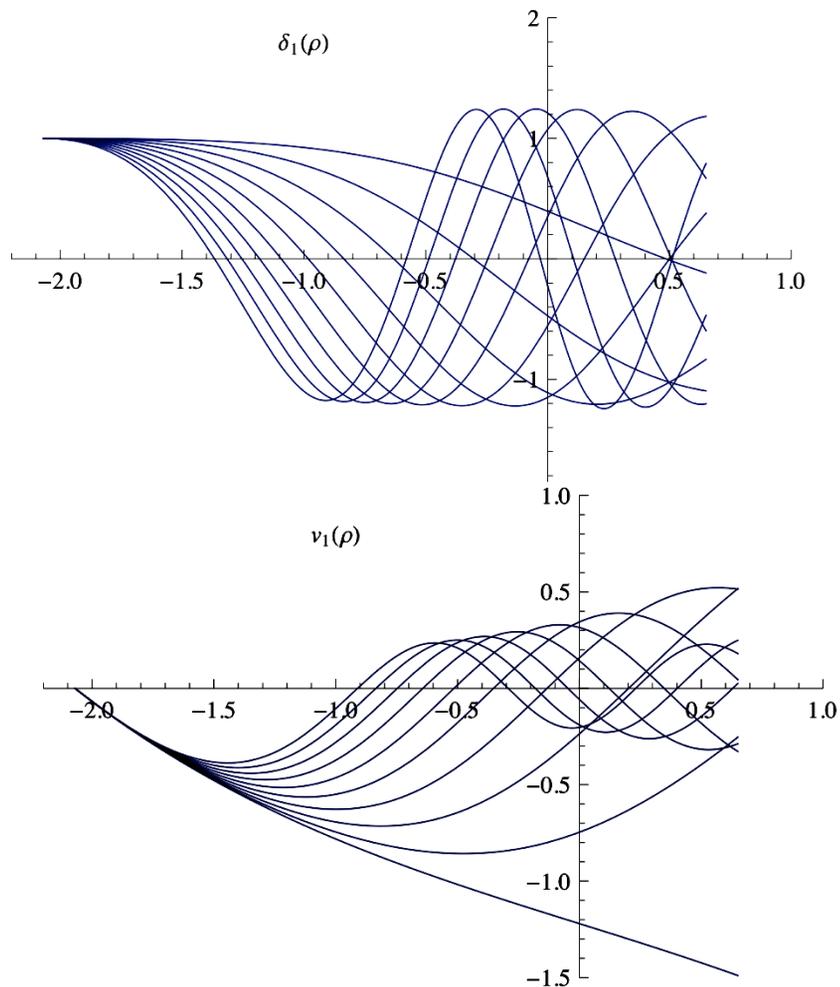
$$R(\rho) = \frac{C_1 P_{-\frac{1}{2} + \frac{1}{6} \sqrt{12\lambda + 1}}^{2/3}(\tanh \rho) + C_2 Q_{-\frac{1}{2} + \frac{1}{6} \sqrt{12\lambda + 1}}^{2/3}(\tanh \rho)}{(\cosh \rho)^{2/3}}$$

$$\Theta(\theta) = C_3 P_l^m(\cos \theta) + C_4 Q_l^m(\cos \theta)$$

$$\Phi(\phi) = C_5 e^{im\phi} + C_6 e^{-im\phi} \quad (3.26)$$

where  $\lambda = l(l + 1)$  and P and Q are associated Legendre polynomials. The part of the solution depending on  $\theta$  and  $\phi$  can be combined in order to form spherical harmonics  $Y_{lm}(\theta, \phi)$ , such that  $\delta(\rho, \theta, \phi) \propto R_l(\rho)Y_{lm}(\theta, \phi)$ .

# harmonics $l=1..10$ , $\delta T$ and velocity

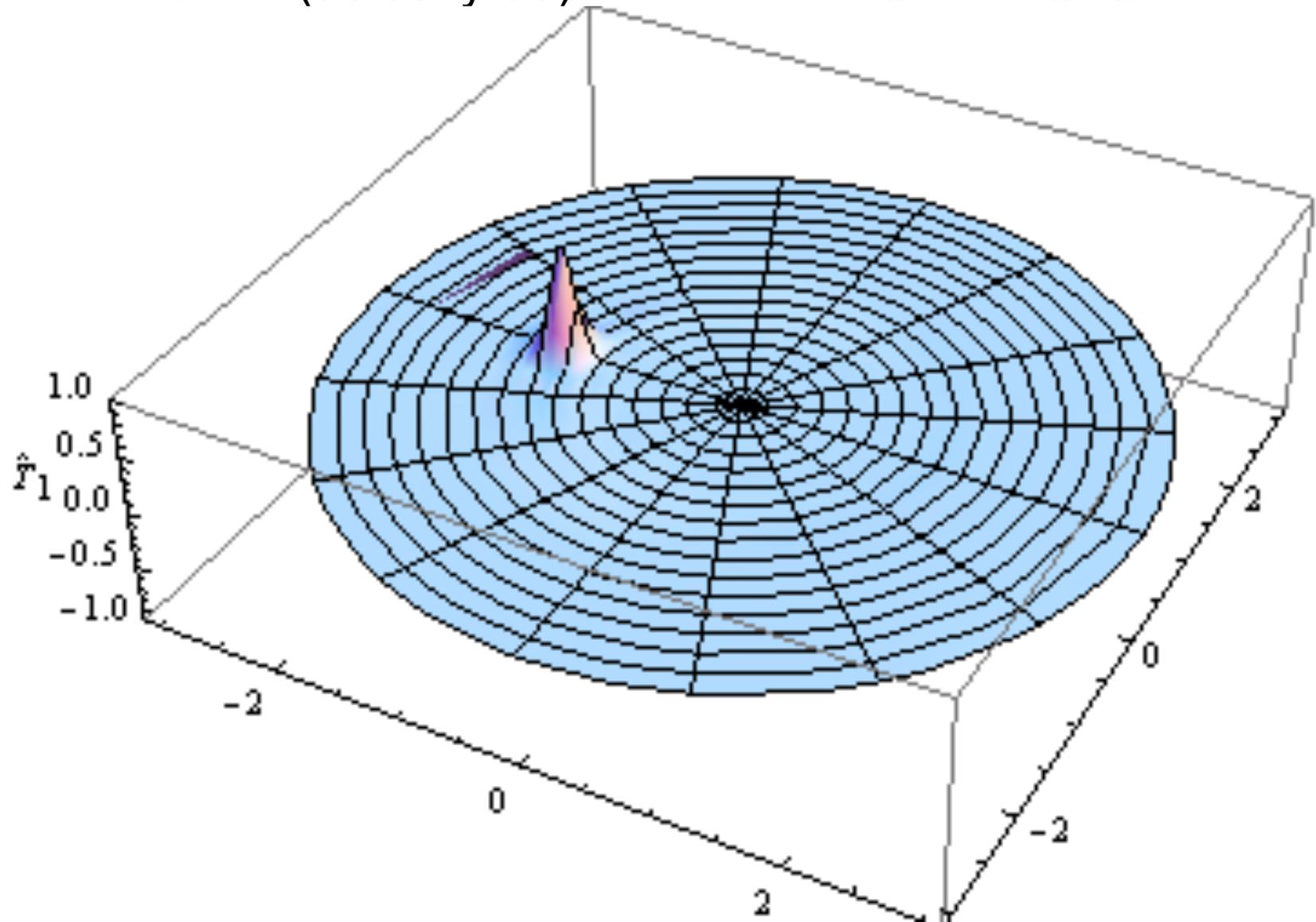


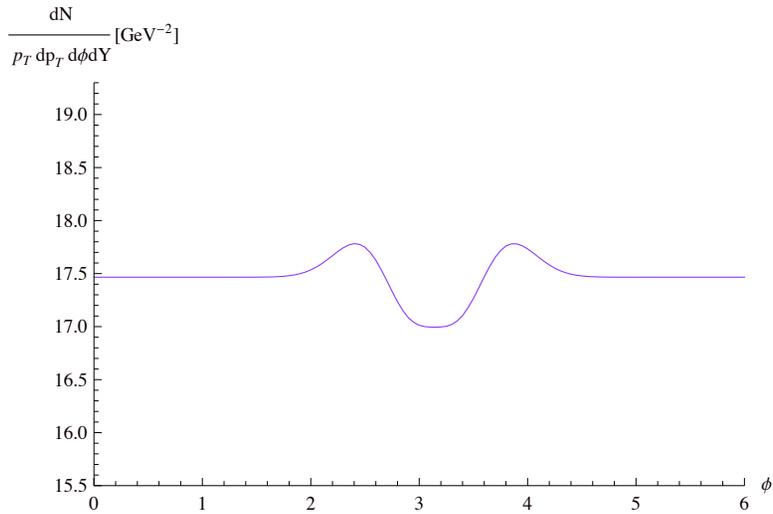
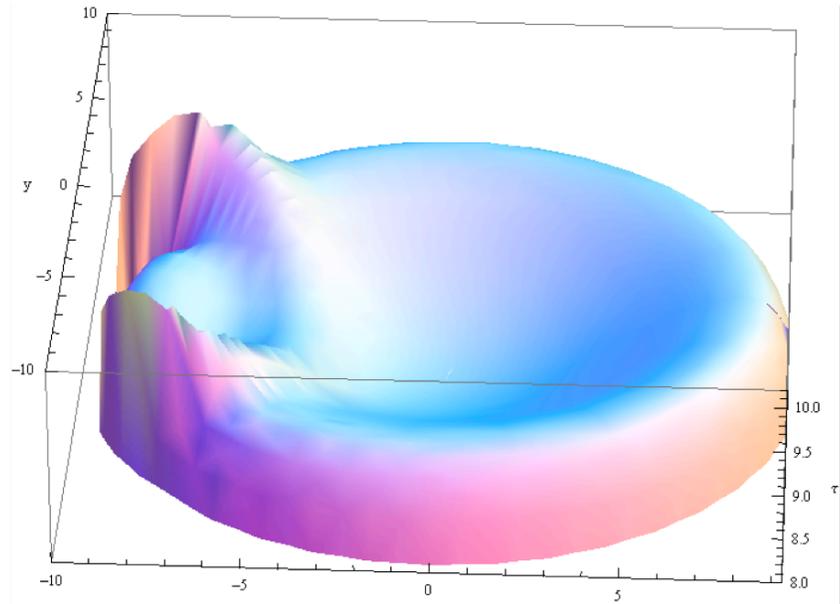
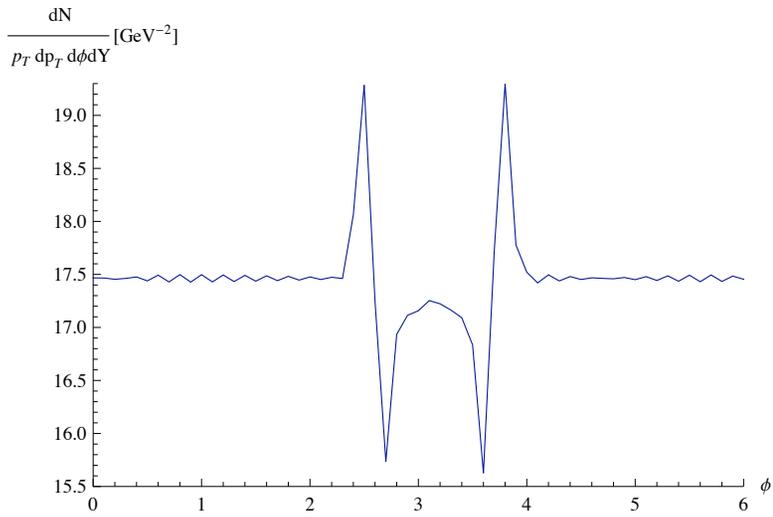
**Viscosity (dashed) hardly affect  
The 1<sup>st</sup> harmonic but kills the 10th**

lhs ( $\rho=-2$ ) is initiation time and FO time is around zero

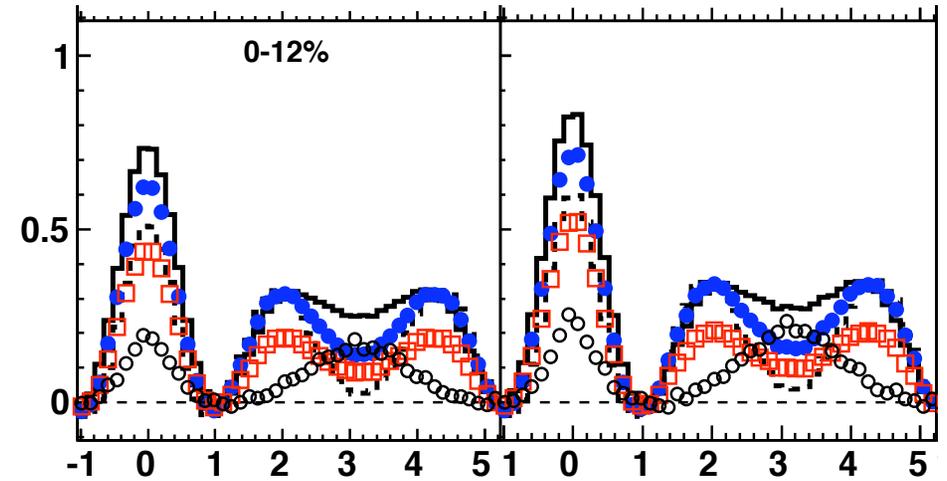
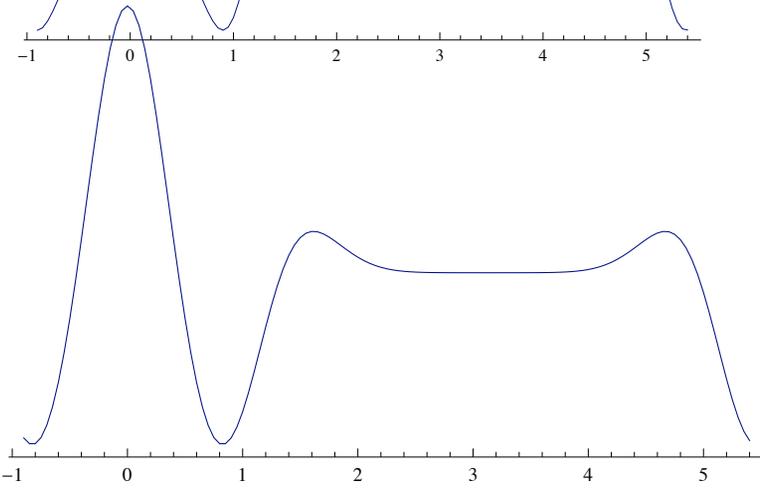
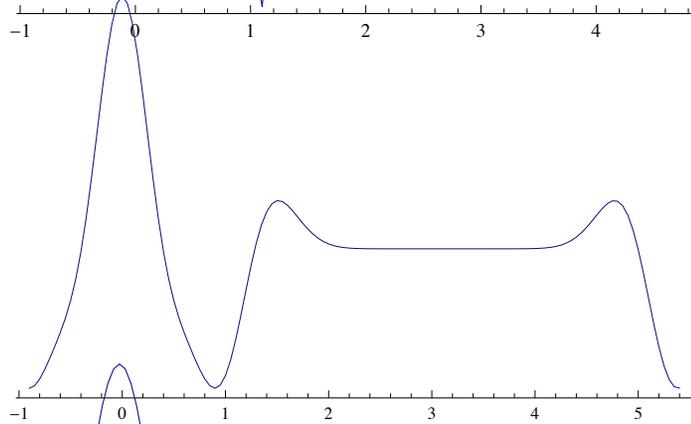
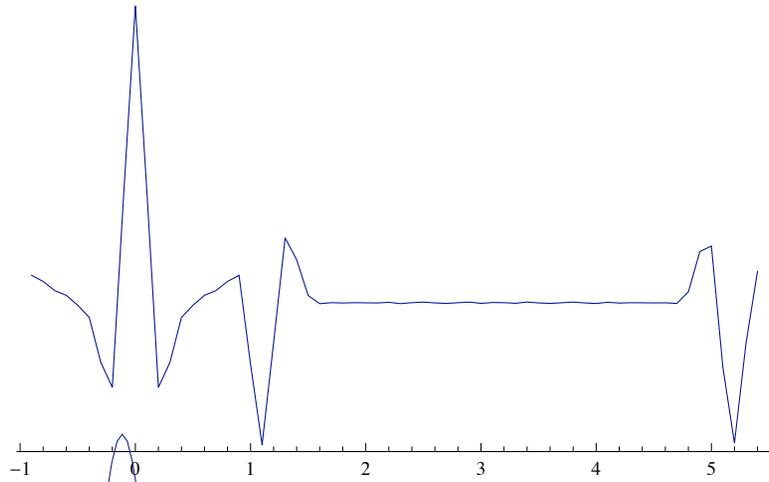
# HERE IS THE SUM OF

**all** (actually 30) **HARMONICS**





**The modified freezeout  
 Surface (right) leads to  
 A modified angular distribution  
 Of particles, with and without viscosity  
 (left)**



Left:  $4\pi \eta/s = 0, 1, 1.5$   
 Note shape change

Up: star central correlators  
 Note shape agreement  
 No parameters, just Green  
 Function from a delta function

# Jet/Fireball Edge should be observable!

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 (Dated: January 26, 2011)

Shock/sound propagation from the quenched jets have well-defined front, separating the fireball into regions which are and are not affected. While even for the most robust jet quenching observed this increases local temperature and flow of ambient matter by only few percent at most, strong radial flow increases the contrast between the two regions so that the difference should be well seen in particle spectra at some  $p_t$ , perhaps even on event-by-event basis. We further show that the effect comes mostly from certain ellipse-shaped 1-d curve, the intercept of three 3-d surfaces, the Mach cone history, the timelike and spacelike freezeout surfaces. We further suggest that this “edge” is already seen in an event released by ATLAS collaboration.

arXiv:1101.4839v1 [hep-ph] 25 Jan 2011

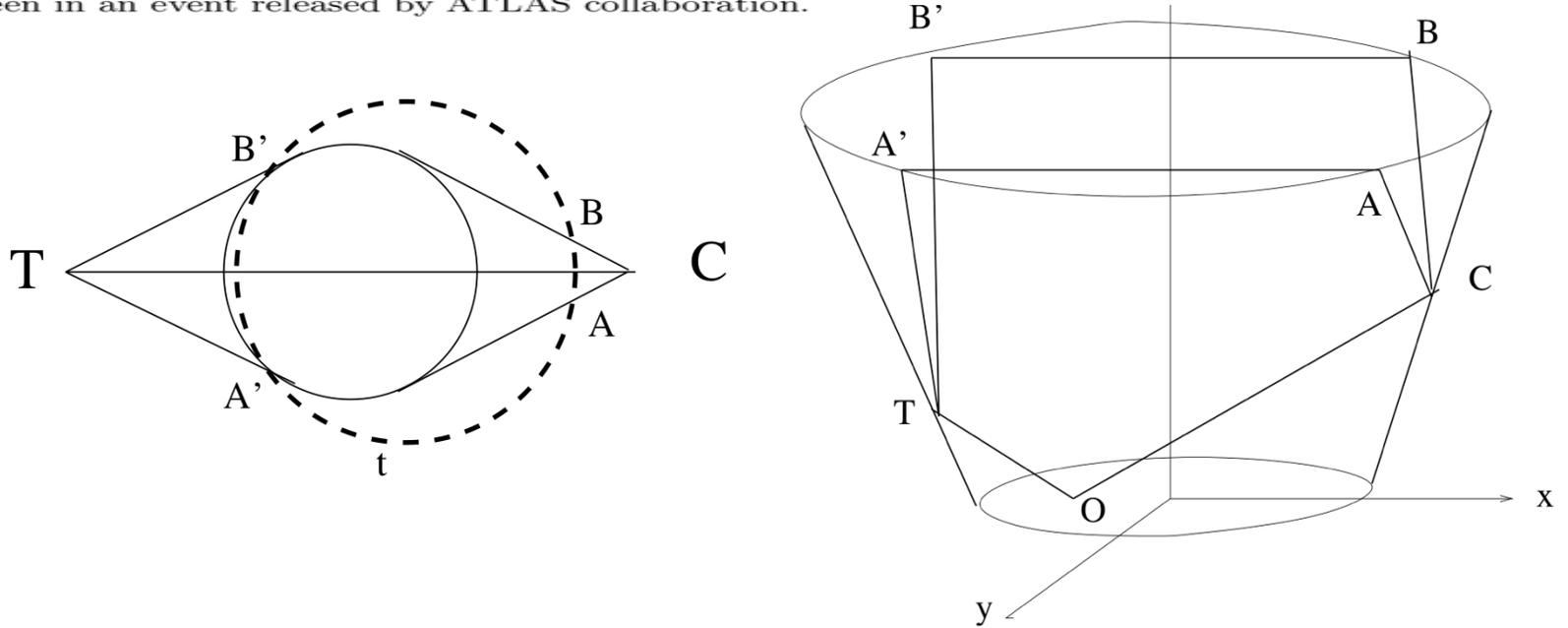
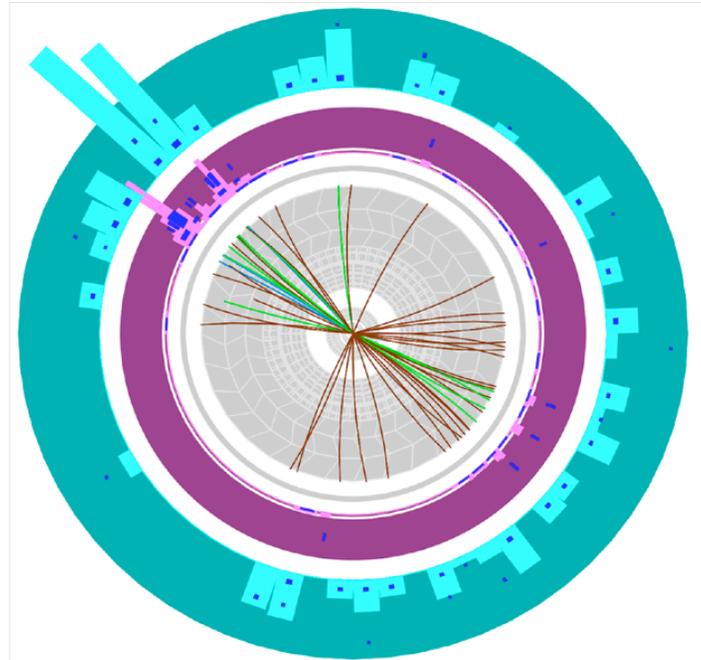


FIG. 1: Schematic shape of the Mach surface in the transverse  $x, y$  plane at  $z = 0$  and fixed time (upper plot), as well as its shape in 3d including the (proper longitudinal) time (lower plot). Mach surface  $\sigma_M$  is made of two parts,  $OCAA'T$  and  $OCBB'T$ . For more explanations see text.

The angular edge of the jets: matter inside is few % HOTTER => SHOULD BE SEEN at tuned pt

$$\Delta\phi = \pm \frac{Hs(t, t_f)}{R}$$



- ATLAS event, in which there is no identifiable jet
- Tracks  $p_t > 2.6$  GeV, cal.  $E > 1$  GeV/cell
- Note the sharp edge of the away-side perturbation! **Is it a “frozen sound“?**

# Summary

• LHC/ALICE sees large (30% larger) elliptic (and radial) flows, exactly as **Hydro 1** predicted already 10 years ago! =>

**QGP remains a very good liquid!**

• **Hydro 2**: Quantitative analytic theory in the linear approximation  
=> Green function from a point perturbation (for Gubser flow)  
Reproduces the correlators beautifully, best with viscosity

**So, we see the sound circle!**

$$4\pi\eta/s \approx 1.5$$

• “Tiny Bangs” on top of the “Little Bang”, from the initial state fluctuations are responsible for odd (1,3,5) harmonics, as seen in standard Glauber studies. Phases of higher harmonics can be measured e.g. in 3-particle correlators!

• Large energy deposition to matter from LHC jets make the inside of the Mach cone a bit hotter. Jet/fireball edge should be visible,

**• Even perhaps on event-by-event basis**

# Fluctuations from Glauber

B. Alver, G. Roland, *Phys.Rev.C*81:054905,2010.

e-Print: [arXiv:1003.0194](https://arxiv.org/abs/1003.0194) [nucl-th] => triangular flow paper

- $\varepsilon_{mn} = \langle r^m \cos(n(\phi - \psi_{mn})) \rangle$
- The dipole  $\varepsilon_{11} = 0$  by definition but not  $\varepsilon_{31}$
- Are angles of different harmonics correlated? Odd ones are quite correlated  $\psi_{31}, \psi_{33}, \psi_{55}$  pointing in y direction, to the tips of the almond
- (Pilar Staig, ES, [archive:1008.3139](https://arxiv.org/abs/1008.3139), aug.18<sup>th</sup>, 2010 )

Later in analytic  
Part  $m \Rightarrow |n|$

•The odds are **all** correlated!

There are “tips” and ”waist” peaks geometry tells us that peripheral events should be both 2- and 3-peaks

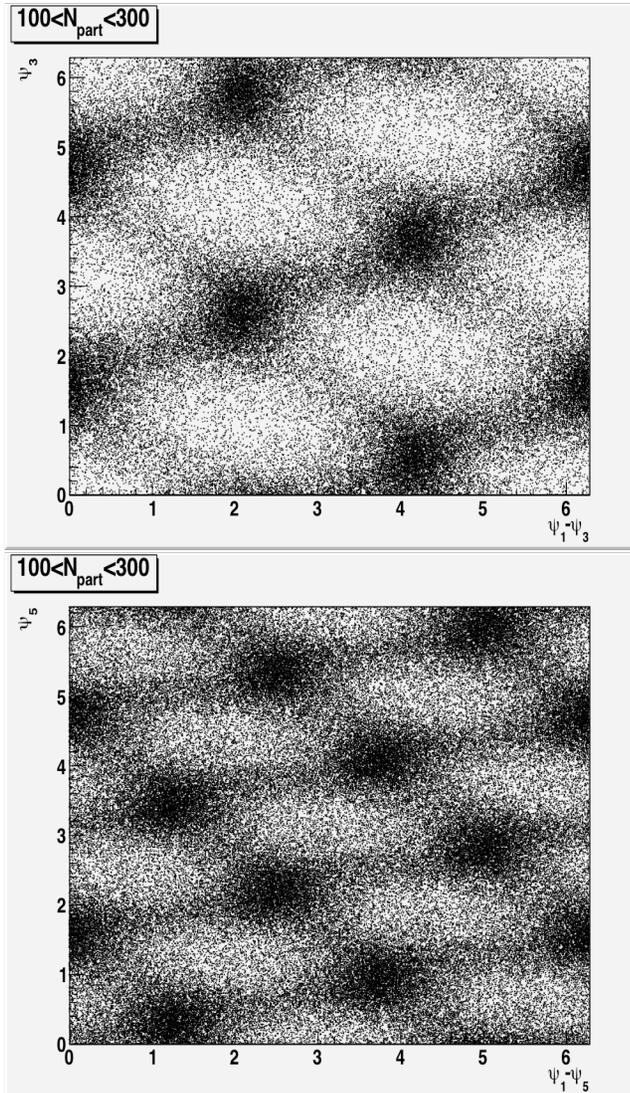


FIG. 8: Scatter plot of the  $\psi_3$  vs  $\psi_3 - \psi_1$  (above), and of the  $\psi_5$  vs  $\psi_5 - \psi_1$  (below), the same centrality

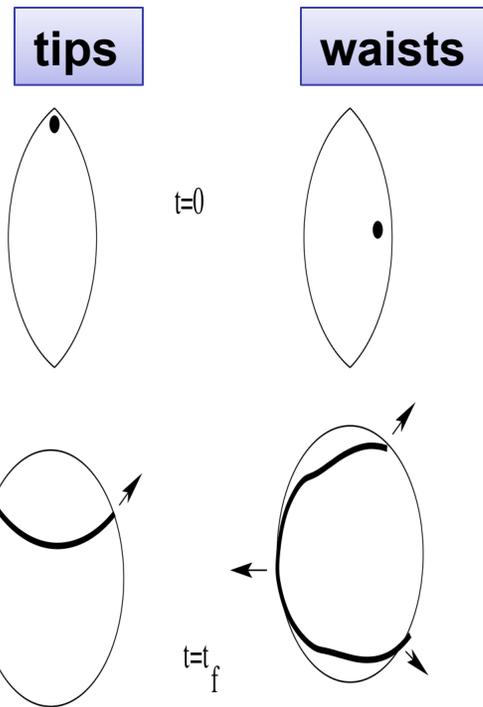


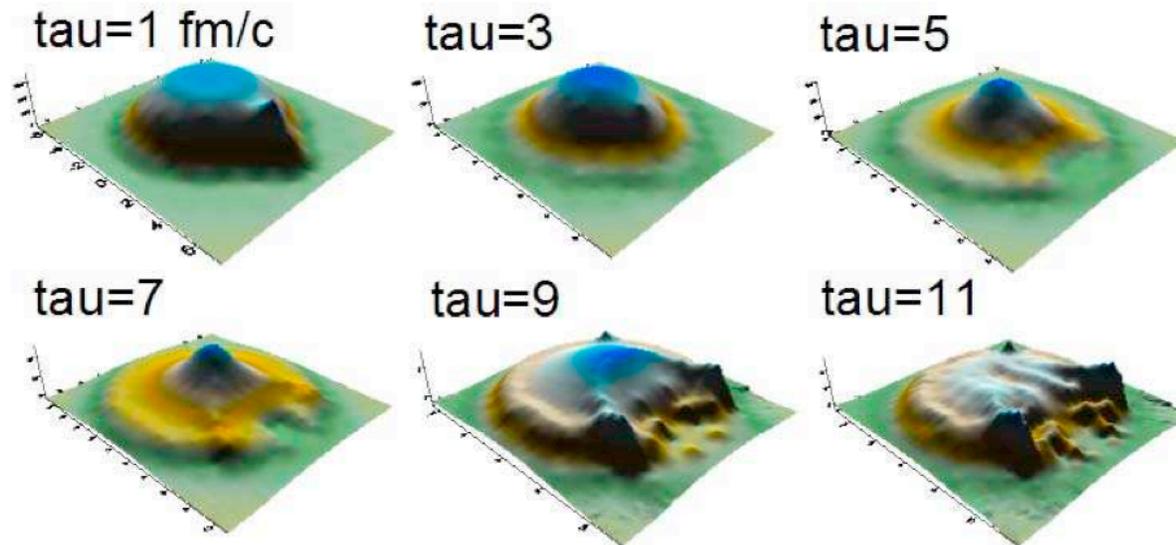
FIG. 4: Two upper picture correspond to initial time  $t = 0$ : the system has almond shape and contains perturbations (black spots). Two lower pictures show schematically location and diffuseness of the sound fronts at the freezeout time  $t_f$ . The arrows indicate the angular direction of the maxima in the angular distributions, 2 and 3 respectively.

**2- and 3-peak events can be Experimentally Separated by the relative phase of the 3ed and 1<sup>st</sup> harmonics**

The sound cylinders and two peaks are also seen by Brazilian group (Andrade, Grassi et al)

Origin of the two peaks

Tube “sinks” and matter around “rises” forming a hole+two horns

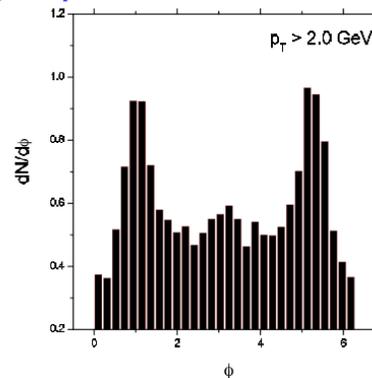


Temporal evolution of energy density for the one tube model.

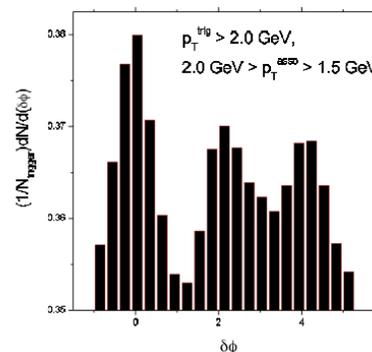
The peaks are at the same angles  
+/- 1 rad (as I got) from perturbation  
but **+/-2 rad in correlations**

One tube model

**MAIN RESULT:** single particle angular distribution has TWO  
PEAKS separated by  $\Delta\phi \sim 2$



**CONSEQUENCE:** two particle angular distribution has three  
peaks



# Geometric acoustics can describe modification of shapes by flow

$$\frac{d\vec{r}}{dt} = \frac{\partial\omega(\vec{k}, \vec{r})}{\partial\vec{k}},$$

$$\frac{d\vec{k}}{dt} = -\frac{\partial\omega(\vec{k}, \vec{r})}{\partial\vec{r}},$$

In this case the dispersion relation is obtained from that in the fluid at rest by a local Galilean transformation, so that

$$\omega(\vec{k}, \vec{r}) = c_s k + \vec{k}\vec{u}. \quad (4.3)$$

In the simplest case of constant flow vector  $\vec{u} = \text{const}(r)$  the first of these eqn just obtains an additive correction by flow

$$\frac{d\vec{r}}{dt} = c_s \vec{n}_{\vec{k}} + \vec{u}, \quad (4.4)$$

where  $\vec{n}_{\vec{k}} = \vec{k}/k$  is the unit vector in the direction of the momentum. The second eqn gives  $\frac{d\vec{k}}{dt} = 0$  as there is no

a (generalized) Hubble-like flow

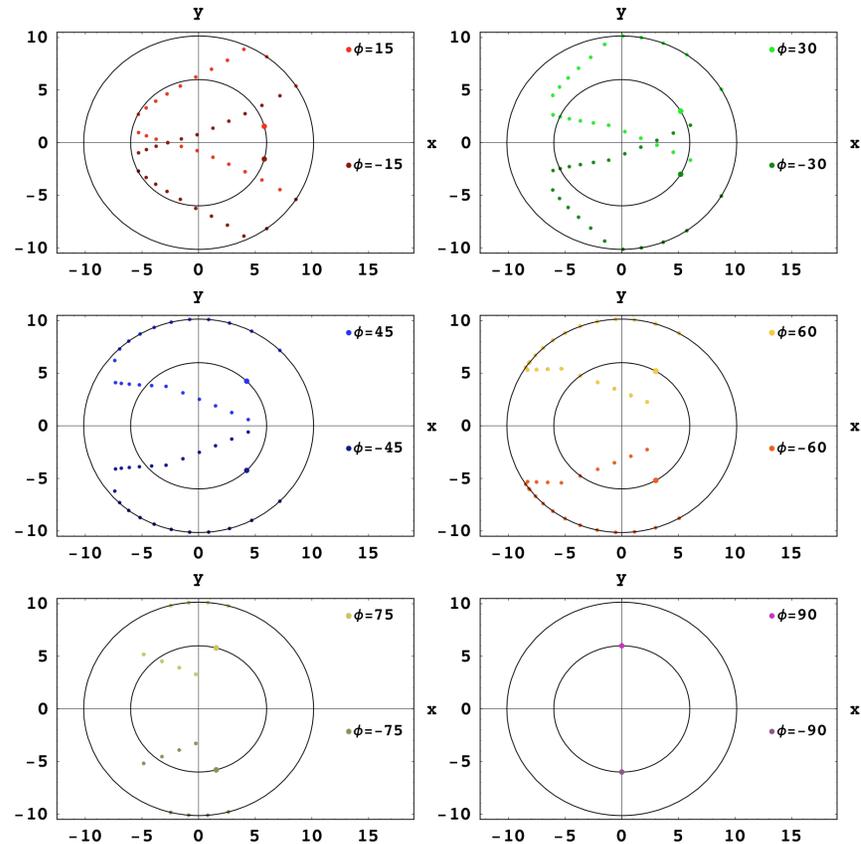
$$u_i(r) = H_{ij}r_j, \quad (4.5)$$

with some time and coordinate independent Hubble tensor. The eqn (4.2) now reads

$$\frac{dk_i}{dt} = -H_{ij}k_j, \quad (4.6)$$

$$k_i(t) = \exp(-H_i t)k_i(0). \quad \vec{r}(t) = tc_s \vec{n}_{\vec{k}} + \vec{r}(0)\exp(+Ht).$$

**Relativistic flow brings in Lorentz factor, easily solvable numerically: e.g.**



# Glauber fluctuations up to 6<sup>th</sup> are all comparable

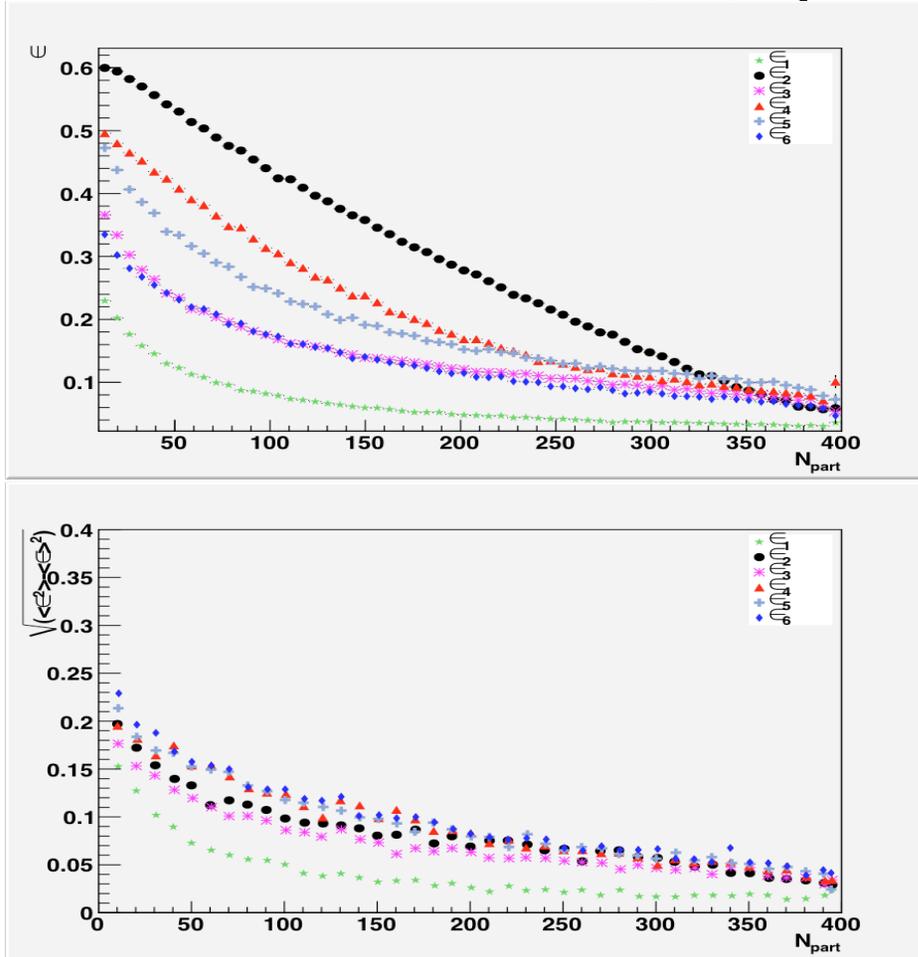


FIG. 5: Average anisotropies (upper plot) and their variations (lower), as a function of centrality expressed via the number of participants  $N_{part}$

$$\epsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2}}{\langle r^n \rangle}$$

$$\epsilon_1 = \frac{\sqrt{\langle r^3 \cos(\phi) \rangle^2 + \langle r^3 \sin(\phi) \rangle^2}}{\langle r^3 \rangle}$$

The angles  $\psi_n$  are defined by:

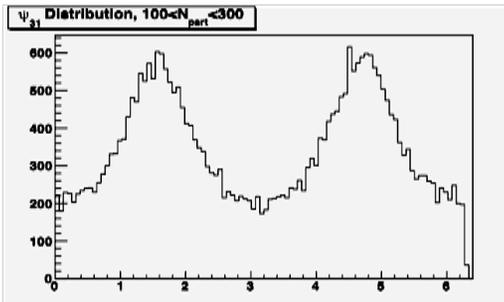
$$\tan(n\psi_n) = \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle}$$

and to calculate  $\psi_1$  we use:

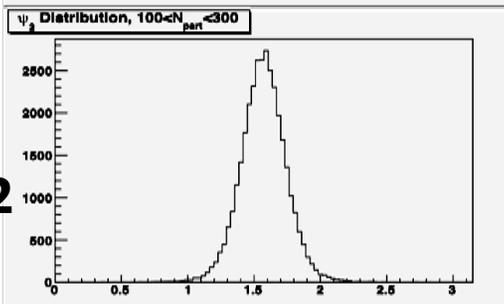
$$\tan(\psi_1) = \frac{\langle r^3 \sin(\phi) \rangle}{\langle r^3 \cos(\phi) \rangle}$$

# Distribution of the angles

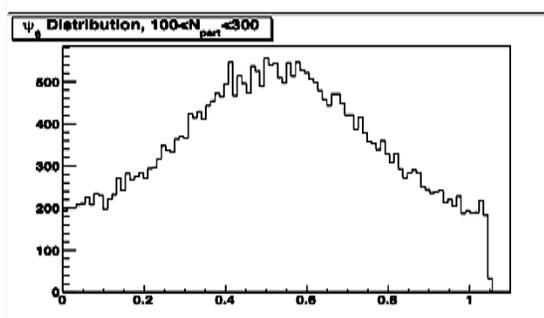
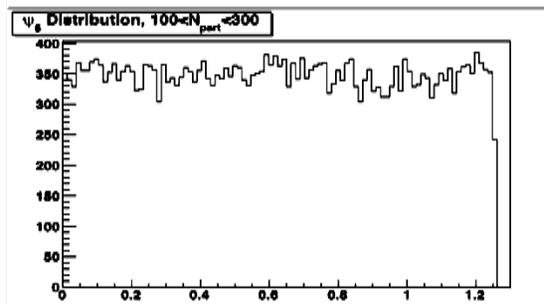
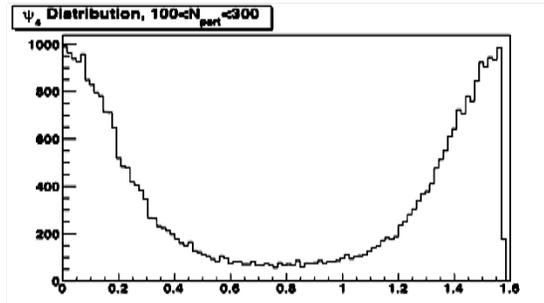
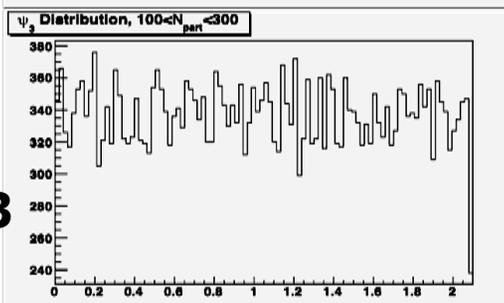
1



2

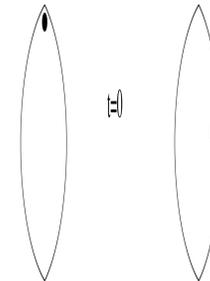


3



1<sup>st</sup> : tips and waist

4



5

3<sup>ed</sup> and 5<sup>th</sup>  
Uncorrelated?

6

# How to do phase-sensitive measurements?

- Central collisions: 2 vs 3 particles

This is of course all well known, and usually written as the 2-body correlator

$$C_2(\Delta\phi) = \left\langle \frac{d^2 N}{d\phi_1 d\phi_2} \right\rangle_{|\psi_p} \quad (4.4)$$

decomposed into harmonics of its argument, which can be easily computed

$$c_{n\Delta} = \frac{\int d(\Delta\phi) C_2(\Delta\phi) \cos(n\Delta\phi)}{\int d(\Delta\phi) C_2} = \langle v_n^2 \rangle \quad (4.5)$$

Note that this correlation function provides the *squared* amplitudes of the original harmonics, *averaged* over the events. (As we assumed the exactly central collisions, none of the harmonics have average values,  $\langle \epsilon_n \rangle = \langle v_n \rangle = 0$ : thus all effects actually come from the root-mean-square fluctuations of  $\epsilon_n$ .) This is e.g. how Alver and Roland [12] and others have obtained their estimates for the “triangular” flow. Note again, that the phases of the harmonics *disappear* in this function, and thus remain undetermined.

However, the situation is different for *three* (or more) body correlation functions: the phases survive and thus can be found. Indeed, now the single-body distribution (4.2) is cubed (or raised into higher power), so one finds a *triple* sum in which the random perturbation direction appears as  $\exp[i(n_1 + n_2 + n_3)\psi_p]$ . Averaging over it, one finds the condition

$$n_1 + n_2 + n_3 = 0 \quad (4.6)$$

One then can e.g. eliminate  $n_3$  and find the double sum

$$\sum_{n_1, n_2} \epsilon_{n_1} \epsilon_{n_2} \epsilon_{n_1+n_2} \exp\{i[n_1(\phi_1 - \phi_3) + n_2(\phi_2 - \phi_3) - n_1(\tilde{\psi}_{n_1} - \tilde{\psi}_{n_1+n_2}) - n_2(\tilde{\psi}_{n_1} - \tilde{\psi}_{n_1+n_2})]\}$$

# Non-central collisions, no integral $\Rightarrow n_1+2=n_2$ , such as $1+2=3, 3+2=5$

Let us present some details about this case, which will illustrate a general case. Let us make a simplification, writing only the second harmonics in the weight and ignoring small fluctuations in the magnitude and the angle  $\psi_2$  around  $\pi/2$  (see Fig.6b)

$$W(\psi_p) = 1 + 2W_2 \cos(2(\psi_p - \pi/2)) + \dots \quad (4.10)$$

where  $W_2 \approx 0.95$ . One can then calculate any moments of the 2-body distribution, for example the one corresponding to  $1+2=3$  term

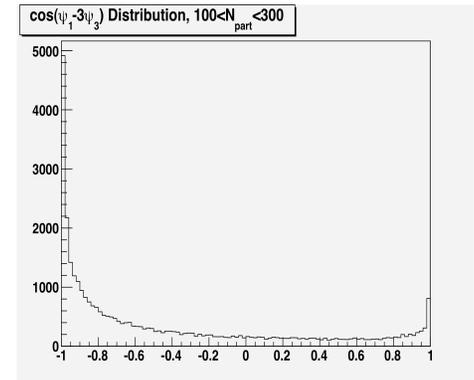
**experiment**  $\rightarrow$

$$\int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \cos(\phi_1 - 3\phi_2) \left\langle \frac{d^2N}{d\phi_1 d\phi_2} \right\rangle |w$$

$$\approx -W_2 \left( \frac{v_1}{\epsilon_1} \right) \left( \frac{v_3}{\epsilon_3} \right) \left\langle \epsilon_1 \epsilon_3 \cos(3\psi_3 - \psi_1) \right\rangle$$

**hydro**  $\rightarrow$

we have separated the ratios  $v_1/\epsilon_1, v_3/\epsilon_3$  (which are calculable by hydrodynamics) from the subsequent angular bracket containing the initial state deformations and their phases: those are to be averaged over the ensemble of initial conditions. For example, calculated in the Glauber model as explained at the beginning of the paper we obtain



“tips”  $\Rightarrow$  all angles are about  $90^\circ$

**Glauber**  
(or other initial state model)

