Soft-quark bremsstrahlung and energy losses

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Introduction

The purpose of this work is to develop unified approach to the description of the processes of nonlinear interactions of soft Fermi- and Bose-excitations of a quark-gluon plasma with hard thermal or external color-charged partons with integer (gluon) and half-odd-integer (quark and antiquark) spins. For the sake of simplification, we consider the QGP in the weak coupling limit, confined in unbounded volume and all hard quark excitations will be thought massless.

Our approach is based on the complete system of dynamical equations derived by J.-P. Blaizot and E. Iancu. The equations entirely describe the dynamics of soft bose- and fermi-excitations of hot QCD-medium at the soft momentum scale and contain in the right-hand sides either color currents or color Grassmann-valued sources induced by both the medium and hard test color particles. We have supplemented the Blaizot-Iancu equations by the generalized Wong equation describing a change of the classical color charge $Q = (Q^a)$, $a = 1, \dots, N_c^2 - 1$ of a hard particle and also by the generalized dynamical equations for the Grassmann color charges $\theta = (\theta^i)$ and $\theta^\dagger = (\theta^{\dagger i})$.

Soft-quark – hard-particle scattering.

First of all we will consider the scattering processes of soft (anti)quark excitations off hard thermal particles in a hot QCD-medium.

Equations of motion for soft gluon A^a_μ and soft quark $\psi^i_\beta(q), \bar\psi^i_\beta(q)$ excitations are generalized to a case of presence in the system of a color current and source generated by a spin-1/2 hard test particle propagating through the hot QCD-medium:

$$
{}^*\mathcal{D}^{-1}_{\mu\nu}(k)A^{a\nu}(k) = -j^{A(2)a}_\mu(A,A)(k) - j^{A(3)a}_\mu(A,A,A)(k) - j^{\Psi(0,2)a}_\mu(\bar\psi,\psi)(k)
$$
$$
- j^{\Psi(1,2)a}_\mu(A,\bar\psi,\psi)(k) - j^{(0)a}_{Q\mu}(k) - j^{(1)a}_{Q\mu}(A)(k) - j^{(2)a}_{Q\mu}(A,A)(k), \qquad (1)
$$

$$
{}^*S^{-1}_{\alpha\beta}(q)\psi^i_\beta(q) = -\eta^{(1,1)\,i}_\alpha(A,\psi)(q) - \eta^{(2,1)\,i}_\alpha(A,A,\psi)(q) -
$$
$$
-\eta^{(0)i}_{\theta\,\alpha}(q) - \eta^{(1)i}_{\theta\,\alpha}(A)(q) - \eta^{(2)i}_{\theta\,\alpha}(A,A)(q). \qquad (2)
$$

$j^{a\mu}_Q(x) = gv^\mu Q^a(t)\delta^{(3)}(\mathbf{x} - \mathbf{v}t)$ is the usual color current of a point particle and $\eta^i_{\theta\,\alpha}(x) = g\{\chi_\alpha\theta^i(t) - (C_F/2T_F)\chi_\alpha Q^a(t)(t^a)^{ij}\theta^j(t)\}\,\delta^{(3)}(\mathbf{x} - \mathbf{v}t)$ is the Grassmann-valued color source, where $v^\mu = (1, \mathbf{v})$ and χ_α is some constant spinor.

The Grassmann color charge $\theta^i(t)$ obeys the following equation, linearly depending on the background fermion field:

$$
\frac{d\theta^i(t)}{dt} + igv^\mu A^a_\mu(t,\mathbf{v}t)(t^a)^{ij}\theta^j(t) + ig\big(\bar\chi_\alpha\psi^i_\alpha(t,\mathbf{v}t)\big) \qquad (3)
$$
$$
- ig\left(\frac{C_F}{2T_F}\right)Q^a(t)(t^a)^{ij}\big(\bar\chi_\alpha\psi^j_\alpha(t,\mathbf{v}t)\big)
$$
$$
- ig\left(\frac{C_F}{2T_F}\right)(t^a)^{ij}\theta^j(t)\left\{\theta^{\dagger l}(t)(t^a)^{lk}\big(\bar\chi_\alpha\psi^k_\alpha(t,\mathbf{v}t)\big) + \big(\bar\psi^k_\alpha(t,\mathbf{v}t)\chi_\alpha\big)(t^a)^{kl}\theta^l(t)\right\} = 0
$$

with the initial condition $\theta^i(t)\big|_{t=t_0} = \theta^i_0$. A similar equation is valid for the usual color charge $Q^a(t)$ (the *generalized Wong equation*).

The medium-induced currents $j^{A(2)a}_\mu$, $j^{\Psi(0,2)a}_\mu$, $j^{\Psi(1,2)a}_\mu$ and sources $\eta^{(1,1)\,i}_\alpha$, $\eta^{(2,1)\,i}_\alpha$ are chosen in the *hard thermal loop approximation*. For example,

$$
j^{\Psi(0,2)a}_\mu(\bar\psi,\psi)(k) = g\,(t^a)^{ij}\!\int {}^*\Gamma^{(G)}_{\mu,\,\alpha\beta}(k;q_1,-q_2)\bar\psi^i_\alpha(-q_1)\psi^j_\beta(q_2)\times
$$
$$
\times\,\delta(k + q_1 - q_2)dq_1dq_2,
$$

where ${}^*\Gamma^{(G)}_{\mu,\,\alpha\beta}$ is the HTL-resummed vertex between quark pair and gluon.

A system (1), (2), (3) can be solved by the approximation scheme method:

$$
A^a_\mu(k) = A^{(0)a}_\mu(k) + \sum_{s=1}^{\infty} A^{(s)\,a}_\mu(k), \quad \psi^i_\alpha(q) = \psi^{(0)i}_\alpha(q) + \sum_{s=1}^{\infty}\psi^{(s)i}_\alpha(q), \qquad (4)
$$

where $A^{(0)a}_\mu(k)$ and $\psi^{(0)i}_\alpha(q)$ are free-field solutions, and $A^{(s)\,a}_\mu(k)$ and $\psi^{(s)i}_\alpha(q)$ are contributions of g^s order. We have suggested an approach allowing practically completely to automate procedure of the calculation of effective amplitudes for any values $s = 1,\,2,\,3,\,\dots$.

A solution of the system can be also presented in the following form:

$$
A^a_\mu(k) = A^{(0)a}_\mu(k) - {}^*\mathcal{D}_{\mu\nu}(k)j^{(0)a\nu}_Q(k) - {}^*\mathcal{D}_{\mu\nu}(k)\tilde{j}^{\,a\nu}[A^{(0)},\bar\psi^{(0)},\psi^{(0)},Q_0,\theta^\dagger_0,\theta_0](k),
$$
$$
\psi^i_\alpha(q) = \psi^{(0)i}_\alpha(q) - {}^*S_{\alpha\beta}(q)\,\eta^{(0)i}_{\theta\,\beta}(q) - {}^*S_{\alpha\beta}(q)\,\tilde\eta^i_\beta[A^{(0)},\psi^{(0)},Q_0,\theta_0](q),
$$

where $\tilde{j}^{\,a\nu}[A^{(0)},\bar\psi^{(0)},\psi^{(0)},Q_0,\theta^\dagger_0,\theta_0](k)$ and $\tilde\eta^i_\beta[A^{(0)},\psi^{(0)},Q_0,\theta_0](q)$ are some *effective current and source*, which in turn are functionals of the free fields $A^{(0)a}_\mu$, $\psi^{(0)i}_\alpha$ and $\bar\psi^{(0)i}_\alpha$ (and initial color charges Q^a_0, $\theta^{\dagger i}_0$ и θ^i_0).

The lowest order effective sources

There exist two effective sources of the lowest order in the coupling constant:

$$
\tilde\eta^{(1)i}_\alpha[\psi^{(0)},Q_0](q) = \frac{g^2}{(2\pi)^3}\,(t^a)^{ii_1}Q^a_0\!\int K^{(Q)}_{\alpha\alpha_1}(\chi,\bar\chi|\,q,-q_1)\psi^{(0)i_1}_{\alpha_1}(q_1)\delta(v\cdot(q-q_1))dq_1,
$$

where the integrand is

$$
K^{(Q)}_{\alpha\alpha_1}(\chi,\bar\chi|\,q,-q_1) \equiv \alpha\,\frac{\chi_\alpha\bar\chi_{\alpha_1}}{v\cdot q_1} - {}^*\Gamma^{(Q)\mu}_{\alpha\alpha_1}(q-q_1;q_1,-q)\,{}^*\mathcal{D}_{\mu\nu}(q-q_1)v^\nu,
$$

and

$$
\tilde\eta^{(1)i}_\alpha[A^{(0)},\theta_0](q) = \frac{g^2}{(2\pi)^3}\,(t^a)^{ij}\theta^j_0\!\int K^{(Q)\mu}_\alpha(\mathbf{v},\chi|k,-q)A^{(0)a}_\mu(k)\delta(v\cdot(k-q))dk, \qquad (5)
$$

where, in turn

$$
K^{(Q)\mu}_\alpha(\mathbf{v},\chi|\,k,-q) \equiv \frac{v^\mu\chi_\alpha}{v\cdot q} - {}^*\Gamma^{(Q)\mu}_{\alpha\beta}(k;q-k,-q)\,{}^*S_{\beta\beta'}(q-k)\chi_{\beta'}.
$$

Effective source (5) generates the most simple process of inelastic scattering of soft quark excitation off hard test particle bringing into change of statistics of hard and soft modes, as it is depicted in Fig. 1.

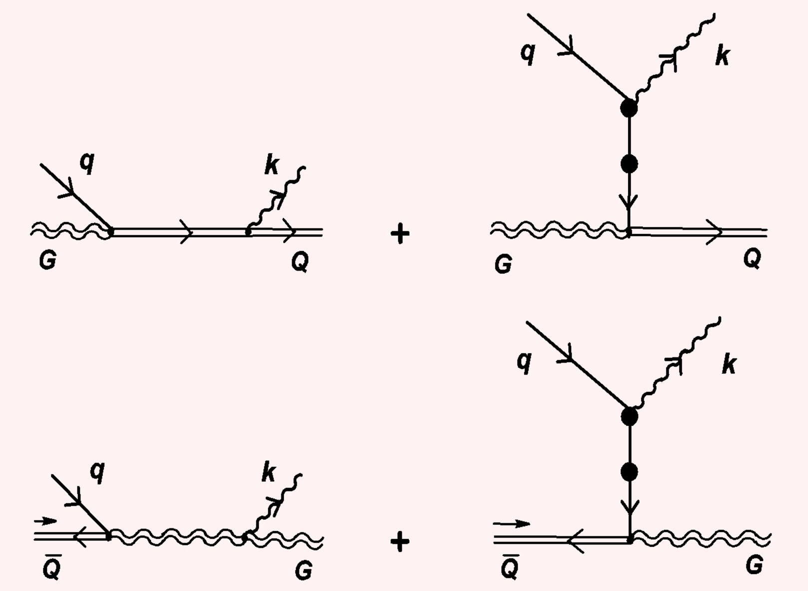

Fig. 1. The lowest order scattering process of soft fermion excitations off the hard test particles with a change of statistics of hard and soft excitation. The blob stands for the HTL resummation, and the double lines denote hard particles.

Inelastic polarization losses

Then we have suggested the general formula defining energy losses of high-energy parton (quark or gluon) traversing the hot QCD medium induced by the scattering off soft-quark excitations. As a basic formula for parton energy losses per unit length generated by the effective current $\tilde{j}^{\,a}_\mu[A^{(0)},\bar\psi^{(0)},\psi^{(0)},Q_0,\theta^\dagger_0,\theta_0](k)$ and effective source $\tilde\eta^i_\alpha[A^{(0)},\psi^{(0)},Q_0,\theta_0](q)$ we accepted the following expression:

$$
-\frac{dE}{dx} = \left(-\frac{dE}{dx}\right)_{\mathcal{B}} + \left(-\frac{dE}{dx}\right)_{\mathcal{F}},
$$

where pure 'fermionic' part of energy loss is

$$
\left(-\frac{dE}{dx}\right)_{\mathcal{F}} \equiv \frac{1}{|\mathbf{v}|}\lim_{\tau\to\infty}\frac{(2\pi)^4}{\tau}\sum_{\lambda=\pm}\int dQ_0\!\int d\theta^\dagger_0 d\theta_0\!\int q^0dq^0d\mathbf{q}\,\times
$$
$$
\times\Big\{\mathrm{Im}({}^*\Delta_+(q))\,\langle|\,\bar u(\hat{\mathbf{q}},\lambda)\tilde\eta^{\,i}[A^{(0)},\psi^{(0)},Q_0,\theta_0](q)|^2\rangle +
$$
$$
+\mathrm{Im}({}^*\Delta_-(q))\,\langle|\,\bar v(\hat{\mathbf{q}},\lambda)\tilde\eta^{\,i}[A^{(0)},\psi^{(0)},Q_0,\theta_0](q)|^2\rangle\Big\}.
$$

Energy losses associated with initial source $\eta^{(0)\,i}_{\theta\,\alpha}(q) = g/(2\pi)^3\chi_\alpha\theta^i_0\delta(v\cdot q)$ are defined by the following expression

$$
\left(-\frac{dE}{dx}\right)_{\mathcal{F}} = \frac{1}{2E}\,\frac{1}{|\mathbf{v}|}\left(\frac{C_\theta\alpha_s}{2\pi^2}\right)\times \qquad (6)
$$
$$
\times\int q^0dq^0d\mathbf{q}\Big\{(1 - \mathbf{v}\cdot\hat{\mathbf{q}})\mathrm{Im}({}^*\Delta_+(q)) + (1 + \mathbf{v}\cdot\hat{\mathbf{q}})\mathrm{Im}({}^*\Delta_-(q))\Big\}\delta(v\cdot q).
$$

It is *'inelastic' polarization losses* decreasing with parton energy E as $1/E$.

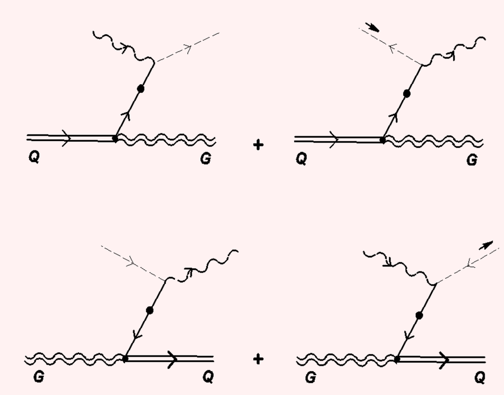

Fig. 2. Energy losses induced by long-distance collision processes wherein a change of a type of energetic parton takes place. The dotted lines denote thermal partons absorbing virtual soft-quark excitation.

Soft-quark bremsstrahlung

The general approach to calculation of effective currents and sources generating bremsstrahlung of an arbitrary number of soft quarks and soft gluons at collision of a high-energy color-charged particle with thermal partons in a hot quark-gluon plasma, is developed.

We take, as the starting point, a system of field equations (1) and (2), the right-hand side of which contains the color current or source of a hard test particle. Further, the color current or source of the second hard test particle must be added to the right-hand side of these equations. So, for example, the Yang-Mills equation (1) takes the following form

$$
{}^*\mathcal{D}^{-1}_{\mu\nu}(k)A^{a\nu}(k) = -j^{A(2)a}_\mu(A,A) - j^{\Psi(0,2)a}_\mu(\bar\psi,\psi) - j^{\Psi(1,2)a}_\mu(A,\bar\psi,\psi)
$$
$$
-\Big\{j^{(0)a}_{Q_1\mu}(k) + j^{(1)a}_{Q_1\mu}(A)(k) + j^{(2)a}_{Q_1\mu}(A,A)(k) + j^{(1)a}_{\theta_1\mu}(\bar\psi,\psi)(k) +
$$
$$
+ j^{(2)a}_{\theta_1\mu}(A,\bar\psi,\psi)(k) + j^{(2)a}_{\Xi_1\mu}(Q_{01},\bar\psi,\psi)(k) + (1\to 2)\Big\}.
$$

The lowest order effective source is defined by derivation of the right-hand side of the Dirac field equation with respect to initial values of usual and Grassmann color charges Q^a_{01} and θ^i_{02} (or Q^a_{02} and θ^i_{01}):

$$
\tilde\eta^i_\alpha(\mathbf{v}_1,\mathbf{v}_2;\dots;\theta_{01},\theta_{02};Q_{01},Q_{02};\dots|\,q) \qquad (7)
$$
$$
= K^{a,\,ij}_\alpha(\mathbf{v}_1,\mathbf{v}_2;\dots|\,q)\,Q^a_{01}\theta^j_{02} + K^{a,\,ij}_\alpha(\mathbf{v}_2,\mathbf{v}_1;\dots|\,q)\,Q^a_{02}\theta^j_{01},
$$

where the coefficient function on the right-hand side is

$$
K^{a,\,ij}_\alpha(\mathbf{v}_1,\mathbf{v}_2;\dots|\,q) = \frac{g^3}{(2\pi)^6}\,(t^a)^{ij}\times \qquad (8)
$$
$$
\times\int\mathcal{K}_\alpha(\mathbf{v}_1,\mathbf{v}_2;\chi_1,\chi_2|q,-q_1)\mathrm{e}^{-i(\mathbf{q}-\mathbf{q}_1)\cdot\mathbf{x}_{01}}\mathrm{e}^{-i\mathbf{q}_1\cdot\mathbf{x}_{02}}\delta(v_1\cdot(q-q_1))\delta(v_2\cdot q_1)dq_1,
$$

$$
\mathcal{K}_\alpha(\mathbf{v}_1,\mathbf{v}_2;\chi_1,\chi_2|\,q,-q_1) = \qquad (9)
$$
$$
= -\,\alpha\,\frac{\chi_{1\alpha}}{v_1\cdot q_1}\left[\bar\chi_1\,{}^*S(q_1)\chi_2\right] - \frac{\chi_{2\alpha}}{v_2\cdot(q-q_1)}\left(v_{2\mu}{}^*\mathcal{D}^{\mu\nu}_C(q-q_1)v_{1\nu}\right) +
$$
$$
+\,v_{1\mu}{}^*\mathcal{D}^{\mu\nu}_C(q-q_1){}^*\Gamma^{(Q)}_{\nu,\,\alpha\beta}(q-q_1;q_1,-q)\,{}^*S_{\beta\beta'}(q_1)\chi_{2\beta'}.
$$

In Fig. 3 diagrammatic interpretation of the first term on the right-hand side of the lowest order effective source (7) is presented.

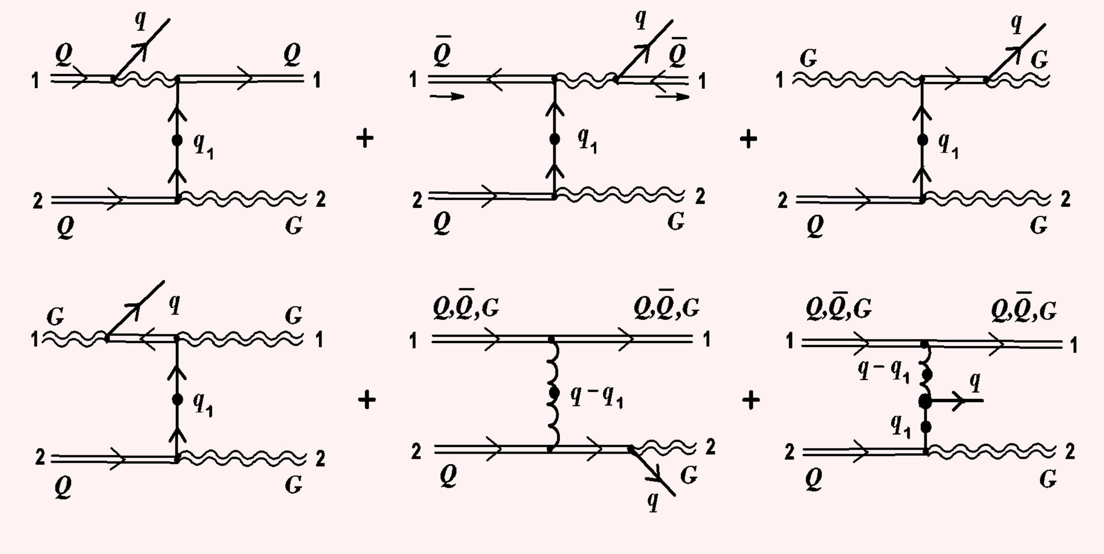

Fig. 3. The simplest process of bremsstrahlung of soft quark generated by effective Grassmann source (7). Here the first four diagrams are associated with the first term on the right-hand side of Eq. (8).

Because this contribution is proportional to usual color charge Q^a_{01}, the type of a hard parton 1 is the same as at the beginning of interaction so at the end (similar statement holds for contribution with charge Q^a_{02}).

Radiation intensity of bremsstrahlung of soft quark

As a formula for the radiation intensity of bremsstrahlung of soft quark we use the following expression:

$$
\mathcal{I} = \sum_{\zeta=\mathrm{Q},\bar{\mathrm{Q}}}\int\frac{d\mathbf{p}_2}{(2\pi)^3}\Big[f^{(\zeta)}_{\mathbf{p}_2} + f^{(G)}_{\mathbf{p}_2}\Big]\left(\int d\mathbf{b}\,W(\mathbf{b};\zeta)|\,\mathbf{v}_1 - \mathbf{v}_2|\right) \equiv \left\langle\frac{dW(\mathbf{b})}{dt}\right\rangle_{\mathbf{b}}, \qquad (10)
$$

where $\mathbf{b}$ is impact parameter and $W(\mathbf{b};\zeta)$ is the energy of soft-quark radiation field

$$
W(\mathbf{b};\zeta) = -\frac{i(2\pi)^4}{2}\int d\mathbf{q}\,dq^0q^0\!\int dQ_{01}dQ_{02}\!\int d\theta^\dagger_{01}d\theta_{01}\!\int d\theta^\dagger_{02}d\theta_{02}
$$
$$
\times\,\langle\bar{\tilde\eta}^i(-q;\mathbf{b})\{{}^*S(-q) + {}^*S(q)\}\tilde\eta^i(q;\mathbf{b})\rangle\,.
$$

In a static limit $\mathbf{v}_2 = 0$ the formula (10) coincides with an expression for energy loss: $\mathcal{I}|_{\mathbf{v}_2=0} = -dE_1/dt$, where E_1 is energy of a fast parton 1.

Approximation of static color center

With allowance made for some approximations from the radiation intensity (10) and an explicit form of effective source (7)–(9), the expression for energy losses of high-energy parton 1 induced by bremsstrahlung of soft quark normal mode $(+)$, is derived

$$
\left(-\frac{dE_1}{dx}\right)^{+} = -\frac{\alpha_s^3}{\pi^2}\left(\frac{C_FC^{(1)}_2}{d_A}\right)\left(\sum_{\zeta=\mathrm{Q},\bar{\mathrm{Q}}}C^{(\zeta)}_\theta\!\int|\mathbf{p}_2|\left[f^{(\zeta)}_{|\mathbf{p}_2|} + f^{(G)}_{|\mathbf{p}_2|}\right]\frac{d|\mathbf{p}_2|}{2\pi^2}\right)
$$
$$
\times\int\frac{d\omega}{\omega}\int d\mathbf{q}_\perp\!\int d\mathbf{q}_{1\perp}\frac{1}{\left(\mathbf{q}^2_{1\perp}+\omega^2_0\right)^2+\left(\frac{\omega^2_0\pi}{2}\right)^2}\,\frac{(\mathbf{q}_{1\perp}\cdot\mathbf{l}_\perp)^2 + 2\,(\mathbf{q}_{1\perp}\times\mathbf{l}_\perp)^2}{\left[(\mathbf{q}-\mathbf{q}_1)^2_\perp + m^2_g\right]^2}.
$$

The distinguishing features of the expression obtained are its logarithmic divergence as $\omega\to 0$ and also the absence of suppression factor $1/E_1$, as is the case for the inelastic polarization losses Eq. (6).

Application to processes of jet conversions

We would like to mention a connection of our study with a research of the interaction processes of a jet with the surrounding medium at which the flavor of the jet, i.e. the flavor of the leading parton can change. Thus in the papers by W. Liu, R.J. Fries et al. (Phys. Rev. C, 2007, 2008) (see also S. Sapeta, U.A. Wiedemann (Eur. Phys. J. C, 2008)) it was shown that taking into account the effects of conversions between quark and gluon jets in traversing through the quark-gluon plasma is important along with the energy losses to explain some experimental observations. In our approach the processes of jet conversions in the QGP is already 'built into' the formalism a priory and they are its fundamental part.

In addition as opposed to the approach developed by W. Liu et al., in which the flavor charging processes were considered only via two-body scattering of the type $gq \to \gamma g$, $q\bar q \to gg$, … and were served as addition to the processes of energy losses, our formalism enables to consider more complicated processes, where conversions of the jets are indissolubly related to radiative energy losses which are induced by soft quark bremsstrahlung. Perhaps such a type of interactions of a jet with the hot QCD medium can give appreciable contribution to the flavor dependent measurements of jet quenching observables and finally to the definitive jet hadron chemistry.

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