Lattice study of the second order transport coefficients

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I. Background

①Success of ideal hydrodynamic model for QGP @ the RHIC → **strongly coupled QGP.**

②The LHC can reach higher temperature than the RHIC → QCD coupling becomes small
→ **viscous effects & transport coefficients must be more important.**

③The evolution equation of shear current in Israel-Stewart theory (aka causal viscous hydrodynamics)

$$\frac{d\pi_{ij}}{dt} = -\frac{1}{\tau_\pi}[\pi_{ij} + \ldots]$$

π_{ij}: shear current, τ_π: relaxation time

④Our purpose is **to constrain the transport coefficients** in IS theory by lattice simulation.

II. Formalism

Kubo formula for the shear viscosity

$$\eta = i\int d^3x \int_0^\infty dt\, t\, \langle[\pi_{12}(t,\vec{x}), \pi_{12}(0,\vec{0})]\rangle$$

$$\langle[A,B]\rangle = i\int_0^\beta d\lambda \langle \dot{A}(-i\lambda)B\rangle$$

$$\langle \pi_{12}(t-i\lambda)\pi_{12}(0)\rangle \simeq e^{-t/\tau_\pi}\langle \pi_{12}(-i\lambda)\pi_{12}(0)\rangle$$

λ: imaginary-time

The ratio of shear viscosity to relaxation time

$$\frac{\eta}{\tau_\pi} \simeq \int d^3x \int_0^\beta d\lambda \langle \pi_{12}(-i\lambda,\vec{x})\pi_{12}(0,\vec{0})\rangle$$

Remarks on η/τ_π:
①No contribution from the origin
②UV divergence → T=0 subtraction
③Euclidean correlator → Lattice observable

According to operator-product expansion, the behavior of correlation function in the limit x→0

$$\pi_{12}(x)\pi_{12}(0) \simeq \frac{C}{x^8} + \left[\frac{2}{3}T_{00} + \frac{1}{6}F^2\right]\delta^4(x)$$

T=0 subtraction — contact term

C: c-number constant, F: gluon field strength tensor

1st term vanishes by T=0 subtraction, but 2nd & 3rd terms (contact terms) remain.

S. Caron-Huot, Phys. Rev. D79, 125009(2009).
P. Romatschke and D. T. Son, Phys. Rev. D80, 065021(2009).
H. B. Meyer, Phys. Rev. D82, 054504(2010).

In the local rest frame of a matter, $\pi_{12}=T_{12}$.
What we calculate is

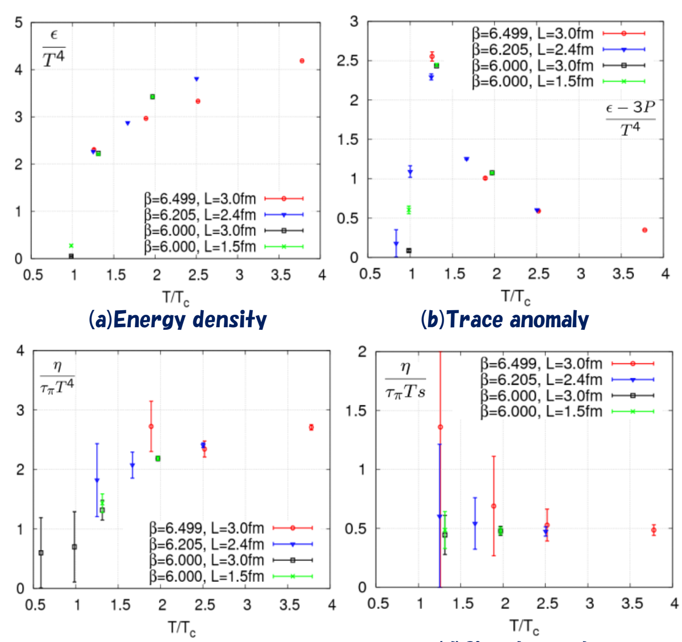
$$\frac{\eta}{\tau_\pi} = \int_{x\neq 0} d^4x \langle T_{12}(x)T_{12}(0)\rangle_{T-0}$$

$$\simeq \left[\int d^4x \langle T_{12}(x)T_{12}(0)\rangle - \frac{2}{3}\langle T_{00}\rangle - \frac{1}{6}\langle F^2\rangle\right]_{T-0}$$

III. Results

Lattice setup
- **SU(3) pure gauge theory** with standard action
- Heat bath*1 + Over relaxation*4 for update
- **Clover plaquette for energy-momentum tensor**
- JackKnife method for error estimation

β	a[fm]	Nt	Ns	Ls[fm]
6.499	0.049	4,6,8,12,32	32	1.6
6.205	0.075	4,6,8,32	32	2.4
6.000	0.094	4,6,8,16	32	3.0
6.000	0.094	4,6,8,16	16	1.5



(a)Energy density (b)Trace anomaly (c) η/τ_π (d)Signal speed

IV. Summary & Future plan

①We evaluated **a ratio of the shear viscosity to relaxation time** with SU(3) lattice gauge theory
→**constrained transport coefficients.**

②**Signal speed ~ 0.5** for T>1.5Tc (causality OK).

③Future plan : bulk channel.