Charmonium spectral functions at finite momenta in the quark-gluon plasma from lattice QCD

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LQGP collaboration
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**J/Ψ Suppression**

- QGP signature in heavy ion collisions
  - Current situation
    - Experiments: SPS, RHIC, LHC
    - Lattice QCD: $\eta_c$, $J/Ψ$ survive at $T \sim 1.7T_c$, Asakawa, Hatsuda, Umeda,....
  - Relativistic heavy ion collisions
    - space-time expansion
      - temperature $\sim 200$ MeV @ RHIC
      - charmonia $\sim 3.0$ GeV

Charmonium spectral functions at finite momenta
Lattice QCD

- First principle calculation
- Useful tool for analyses of non-perturbative aspects: sQGP
- Monte Carlo simulations, numerical experiments

\[ \langle A \rangle = \frac{1}{Z} \int dU d\bar{\psi} d\psi e^{-S_G - S_F} A(\bar{\psi}, \psi, U) \]

ex. A: correlators

\[ S_G : \text{gauge action} \quad S_F = \bar{\psi} W \psi : \text{fermion action} \]

\[ \langle A \rangle = \frac{1}{Z} \int \det W dU e^{-S_G} A(\bar{\psi}, \psi, U) \]

\( \det W = 1 \) : quenched approximation
Charmonia in Heavy Ion Collisions

- Spectral functions with finite momenta
- Ill-posed problems

\[ C(t, \vec{p}) = \int d\omega \rho(\omega, \vec{p}) K(t) \]

\[ C(T, \vec{p}) = \sum \exp(i\vec{p} \cdot \vec{x}) \langle O_\Gamma(\vec{x}, t) \Gamma O^\dagger_\Gamma(0, 0) \rangle \]

\[ O_\Gamma(\vec{x}, t) = \bar{\Psi}(\vec{x}, t) \Gamma \psi(\vec{x}, t) \]

\[ \sim O(10) \]

Maximum Entropy Method
### Maximum Entropy Method

Asakawa, Hatsuda, Nakahara

\[ C(t, \vec{p}) = \int d\omega \rho(\omega, \vec{p}) K(t) \]

- **Bayes’ theorem**

\[ P[\rho|CH] = \frac{P[C|\rho H \cdot P[\rho|H]}{P[C|H]} \]

- **\(\chi^2\)-likelihood function**

\[ P[C|\rho H] = \exp(-L)/Z_L \]

- **Shannon-Jaynes entropy**

\[ P[\rho|H] = \exp(\alpha S)/Z_S \]

\[ S = \int \left[ A(\omega) - m(\omega) - A(\omega) \log \left( \frac{A(\omega)}{m(\omega)} \right) \right] \]

- **MEM solution**: maximum of \(\alpha S - L\)

- **Error analysis**: essential in MEM analysis
### Parameters

#### Actions
- standard plaquette action, Wilson fermion
- quenched approximation \( \leftrightarrow \) heavy flavor

#### Lattice sizes
- anisotropic lattice: \( \xi = a_\sigma / a_\tau = 4 \)
- \( \beta = 7.0, \ a_\tau = 9.75 \times 10^{-3} \) fm
- large spatial volume: \( N_\sigma \times N_\tau = 64^3 \times N_\tau, \ P_{\text{min}} \sim 0.5 \) GeV

<table>
<thead>
<tr>
<th>( N_t ) (T/Tc)</th>
<th>96 (0.78)</th>
<th>54 (1.38)</th>
<th>46 (1.62)</th>
<th>44 (1.70)</th>
<th>42 (1.78)</th>
<th>40 (1.87)</th>
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</tr>
</thead>
<tbody>
<tr>
<td># of conf.</td>
<td>260</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
</tbody>
</table>

heat bath: overrelaxation=1:4 1000 sweeps between measurements

Asakawa and Hatsuda, PRL

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φ@Nagoya

C.NONAKA  QM2011
$\eta_c$ at $T=0.78T_c$

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- $\kappa = 0.08285$ $\eta_c$
  - The first peak $\sim 2.9(2)$GeV Consistent with experimental value
  - The second peak resonance?
  - Other structure: lattice artifact
$\eta_c$ at $T=1.38T_c$

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Even above the $T_c$ the qualitative feature of $\rho$ is the same as that at $T=0.78 \, T_c$.

- The first peak
- The second peak resonance?
- Other structure: lattice artifact
\( \eta_c \text{ at } T=1.62T_c \)

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Even above the \( T_c \)

The first peak exists clearly.

- The first peak
- The second peak
- Lattice artifact?
- Other structure: lattice artifact
**η_c at T=1.70T_c**

| N_t (T/T_c) | 96 (0.78) | 54 (1.38) | 46 (1.62) | 44 (1.70) | 42 (1.78) | 40 (1.87) | 32 (2.33) |

At T=1.70 T_c
the spectral function dramatically changes.
From comparison with error bars, at T=1.70T_c η_c melts.
Melting Temperature

\[ N_t \left( \frac{T}{T_c} \right) \]

| \( N_t \) | 96 (0.78) | 54 (1.38) | 46 (1.62) | 44 (1.70) | 42 (1.78) | 40 (1.87) | 32 (2.33) |

\[ \eta_c \text{ melts between } T=1.62 \, T_c \text{ and } T=1.70 \, T_c. \]
The mass of $\eta_c$ increases with temperature.
Dispersion Relations at $T<T_c$

\[
\omega^2 = m^2 \left| p=0 \right| + \hat{p}^2_{\text{lattice}}
\]

free bosons:

\[
\hat{p}^2_{\text{lattice}} = \frac{2}{a_\sigma} \sin(pa_\sigma/2)
\]

The dispersion relation at $T=0.78T_c$ on the lattice is consistent with that of vacuum.

$P_{\min} \approx 0.5 \text{ GeV}$
Spectral functions at $P \neq 0$ ($T = 1.62T_c$)

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$P_{\text{min}} \sim 0.5$ GeV

Qualitatively, the shape of spectra functions at $p \neq 0$ is almost the same.
1st peak at finite momenta

$T = 1.62T_c$

- $\eta_c$ is stable even at higher momentum.
- The strength of the peak becomes smaller at higher momentum.
- The peak shifts to larger $\omega$ at high momentum.
Dispersion Relation

\[ \omega^2 = m_{p=0}^2 + \hat{p}^2_{\text{lattice}} \]

free bosons:  
\[ \hat{p}^2_{\text{lattice}} = \frac{2}{a_\sigma} \sin(pa_\sigma/2) \]

- The deviation from dispersion relation at vacuum starts to appear around \( p \sim 3.0 \text{ GeV} \).
- \( \Rightarrow \) medium effect
- different from dispersion relation at \( T < T_c \)
Summary

- Charmonium in Relativistic Heavy Ion Collisions
  - $\eta_c$ (PS channel) at $P=0$
    - $\eta_c$ melts between $T=1.62~T_c$ and $T=1.70~T_c$
    - Mass of $\eta_c$ increases with temperature ($T < 1.7~T_c$)
  - $\eta_c$ at finite momenta
    - At $T=1.62~T_c$ medium effects appears in dispersion relations

- Work in progress
  - $J/\Psi$ (vector channel)

- Future
  - Dilepton productions
    - Experimental data (NA60@SPS, PHENIX, STAR@RHIC, LHC)
    - Ex. light mesons ($\rho$, $\omega$...)