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Interests

- **Large-scale structure**: how can we use it to test the theory of gravity? Motivation: understand if the acceleration of the universe is due to **dark energy** or to a break down of General Relativity.

- **Primordial magnetic fields**: what impact do they leave on the Cosmic Microwave Background (CMB). Example: **B-modes** polarisation observed by BICEP2.
Large-scale structure

The distribution of galaxies is determined by:

- The initial conditions.
- The theory of gravity.
- The content of the universe.

The large-scale structure contains a lot of information about the global properties of our universe.
Large-scale structure

Two different and complementary regimes:

- The **large-scale** regime: we can test predictions from General Relativity.

- The **small-scale regime**: we can test gravity in the non-linear regime. Many modified gravity models undergo a non-trivial transition. To test these models we need **new observables**, well adapted to extract information in the non-linear regime (e.g. **phases** of the density field).
The large-scale regime

We usually assume that galaxies are a tracer of the underlying dark matter distribution:

$$\Delta = \frac{N - \bar{N}}{\bar{N}} = \frac{\delta \rho}{\bar{\rho}} = \delta$$

Incomplete picture: surveys do not provide the true positions of galaxies. They observe incoming photons’ directions and redshifts.

In a homogeneous universe:

- light propagates on straight line
- redshift is due to the expansion
Relativistic distortions

- Our universe is not homogeneous ➔ this **distorts** our coordinate system.

- Interest: the distortions can be used to **test gravity**.
  
  - **Doppler** effect: velocity shifts the photons’ frequency.
  
  - **Gravitational redshift**: the gravitational field shifts the photons’ frequency.

- Measure of the distortions ➔ measure of the gravitational potential ➔ **test** of Einstein’s equations: relation between the density and the gravitational field.
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Backup slides
Redshift

\[ ds^2 = -a^2 \left[ (1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right] \]

Effect of inhomogeneities on the redshift: \[ 1 + z = \frac{\nu_S}{\nu_O} = \frac{E_S}{E_O} \]

Photons travel on **null geodesics**.

\[ 1 + z = \frac{a_O}{a_S} \left[ 1 + V_S \cdot n - V_O \cdot n + \Psi_O - \Psi_S + \int_0^{r_S} dr (\dot{\Phi} + \dot{\Psi}) \right] \]

- **Doppler**
- **Gravitational redshift**
- **Integrated Sachs-Wolfe**

**Gravitational redshift:**

Observer
Result

\[ \Delta(z, n) = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (V \cdot n) \]

- \[ \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega (\Phi + \Psi) \]

\[ + \left( 1 - \frac{\mathcal{H}}{\mathcal{H}^2} - \frac{2}{r \mathcal{H}} \right) V \cdot n + \frac{1}{\mathcal{H}} \dot{V} \cdot n + \frac{1}{\mathcal{H}} \partial_r \Psi \]

\[ + \Psi - 2\Phi + \frac{1}{\mathcal{H}} \dot{\Phi} - 3\frac{\mathcal{H}}{k} V + \frac{2}{r} \int_0^r dr' (\Phi + \Psi) \]

\[ + \left( \frac{\mathcal{H}}{\mathcal{H}^2} + \frac{2}{r \mathcal{H}} \right) \left[ \Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \right] \]

Doppler  redshift space distortion  lensing  gravitational redshift  potential
Density

The density contribution $\Delta = b \cdot \delta$, generates an isotropic correlation function.

$$\xi(s) = \langle \Delta(x)\Delta(x') \rangle$$
depends only on the separation $s = |x - x'|$

$$\xi(s) = \frac{Ab^2 D_1^2}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s-1} T_\delta^2(k) j_0(k \cdot s)$$
Redshift distortions break the isotropy of the correlation function.

$$\Delta = b \cdot \delta - \frac{1}{H} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

Quadrupole Hamilton (1992)

$$\xi_2 = -D_1^2 \left( \frac{4f}{3} + \frac{4f^2}{7} \right) \mu_2(s) P_2(\cos \beta)$$

$$\mu_\ell(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s-1} T_\delta^2(k) j_\ell(k \cdot s)$$

$$P_2(\cos \beta) = \frac{3}{2} \cos^2 \beta - \frac{1}{2}$$
Relativistic effects

The relativistic effects break the symmetry of the correlation function.

The correlation function differs for galaxies behind or in front of the central one.

This differs from the breaking of isotropy, which is symmetric: redshift distortions have even powers of $\cos \beta$.

$\Rightarrow$ The amplitude is the same for $\beta = 0$ and $\beta = \pi$.

To measure the asymmetry, we need two populations of galaxies: faint and bright.
Cross-correlation

The following terms break the symmetry:

\[ \Delta_{\text{rel}} = \frac{1}{\mathcal{H}} \partial_r \Psi + \left( 1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r \mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} \]
Dipole in the correlation function

CB, Hui and Gaztanaga (2013)

\[ \xi(s, \beta) = D_1^2 f \frac{\dot{H}}{H_0} \left( \frac{\dot{H}}{H^2} + \frac{2}{rH} \right) (b_B - b_F) \nu_1(s) \cdot \cos(\beta) \]

\[ \nu_1(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left( \frac{k}{H_0} \right)^{n_s - 1} T_\delta(k) T_\Psi(k) j_1(k \cdot s) \]

By fitting for a dipole in the correlation function, we can measure relativistic effects, and separate them from the density and redshift space distortions.

\[ \xi_1(s) = \frac{3}{2} \int_{-1}^{1} d\mu \ \xi(s, \mu) \cdot \mu \quad \mu = \cos \beta \]
**Multipoles**

\[ z = 0.25 \]

**Monopole**

\[ s^2 \xi_0 \]

**Quadrupole**

\[ s^2 \xi_2 \]

**Hexadecapole**

\[ s^2 \xi_4 \]

**Dipole**

\[ s^2 \xi_1 \]
Interest

The dipole is sensitive to the **gravitational potential**.

\[
\Delta_{\text{rel}} = \frac{1}{\mathcal{H}} \partial_r \Psi + \left( 1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r \mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n}
\]

In general relativity, Euler equation: \( \dot{\mathbf{V}} \cdot \mathbf{n} + \mathcal{H} \mathbf{V} \cdot \mathbf{n} + \partial_r \Psi = 0 \)

\[
\Delta_{\text{rel}} = - \left( \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r \mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n}
\]

Combining the dipole with the quadrupole, we can test **Euler equation**.