

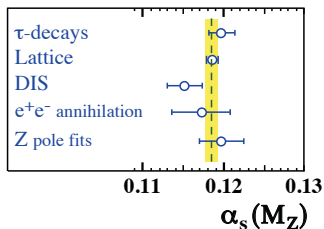


**Before I came to CERN ...**  
**(see Prezi)**

## Fundamental parameters of QCD from non-perturbative methods

$N_f$	$\Lambda_{\overline{MS}}$	Experiment	Theory
0	238(19)	$m_K, K \rightarrow \mu\nu\mu, ; K \rightarrow \pi\mu\nu\mu$	LGT, 
2	310(20)	$m_K, K \rightarrow \mu\nu\mu, ; K \rightarrow \pi\mu\nu\mu$	LGT, 
5	160(11)	DIS, HERA, ...	NNLO PT, fits to PDFs [Alekhin'12]
5	198(16)	DIS, HERA, ...	NNLO PT, fits to PDFs [Martin'09]
5	275(57)	$e^+e^- \rightarrow \text{hadrons}$ , LEP	4-loop PT at $M_Z$

- Error coming from the theory is dominating



- Lattice QCD: *ab initio* determination of QCD parameters
- full control over systematics
  - non-perturbative renormalization
  - chiral extrapolation
  - continuum extrapolation

- See Alberto's talk...

## RBC-UKQCD Collaboration members

### UKQCD

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Luigi Del Debbio (Edinburgh)  
Shane Drury (Southampton)  
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Together with: J.Hudspith, R. Lewis, K. Maltman (York University)

## $a_\mu$ as a stringent test of the SM

- $a_\mu^{exp} = 11659208.9(6.3) \times 10^{-10}$
- Current theoretical and experimental estimates: 3 – 4 $\sigma$  discrepancy
  - HVP (unceirt.  $O(10^{-10})$ )
  - HLbL (unceirt.  $O(10^{-10})$ )
  - other contributions (unceirt.  $O(10^{-11})$  or less)
- New experiments (J-PARC, Fermilab) expected to perform 4 $\times$  more precise measurement
- Improved precision of the theoretical estimates with dominating uncertainty required

## Hadronic vacuum polarisation from the lattice

Can be computed in Euclidean space-time [Blum '02, Lautrup-de Rafael 69]

$$a_{\mu}^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$$



- $\Pi_{\mu\nu} = a^4 \sum_x e^{iQx} \langle J_{\mu}^{em}(x) J_{\nu}^{em}(0) \rangle$
- $\Pi_{\mu\nu}(Q) = (Q^2 \delta_{\mu\nu} - Q_{\mu} Q_{\nu}) \Pi(Q^2)$
- $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$

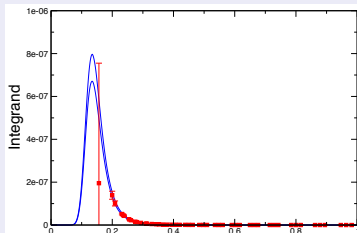
### Systematic uncertainties to be controlled - general

- 1 Simulations at physical  $m_{\pi}$
- 2 Controlled continuum limit, FV effects
- 3 Disconnected diagrams [Della Morte et al. '10]
- 4 Obtaining a real world result: charm quark, isospin effects ...

### Systematic uncertainties to be controlled - HVP related

- Conventional simulations do not allow access to sufficiently low Fourier momenta
- Integral is dominated in the region where relative errors are enhanced
- Structure of HVP tensor is such that  $\Pi(0)$  is not directly accessible
- Systematic uncertainty introduced by extrapolation

### Conventional procedure

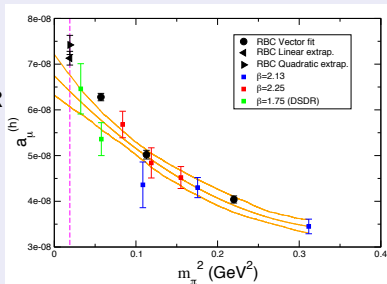


- $\Pi(Q^2) = \frac{\Pi_{\mu\nu}(Q^2)}{Q_\mu Q_\nu - \delta_{\mu\nu} Q^2}$
- Transverse projection:  $Q_\mu = 0$
- Take only diagonal components  $\Pi_{\mu\mu}$
- $a_\mu^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$

# Previous RBC-UKQCD computation of $a_\mu^{HLO}$ [Boyle et al'11]

Non physical  $m_\pi$ ,  $a^{-1} \approx 1.3, 1.7, 2.3$  GeV

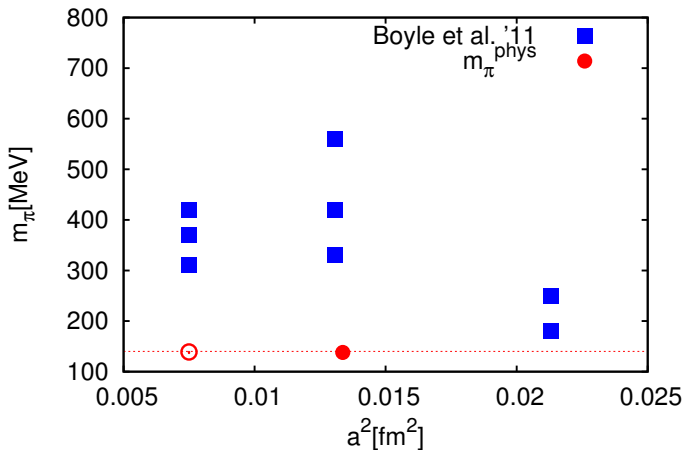
- Domain Wall Fermion, Iwasaki/DSDR gauge action
- Fitting  $Q^2$ -dependence of  $\Pi(Q^2)$  up to  $Q_C^2 \approx 2.5 - 9$  GeV<sup>2</sup>



- Strong  $m_\pi$  dependence
- Eliminate the systematics of chiral extrapolation: computing HVP at  $m_\pi^{phys}$



# RBC-UKQCD $N_f = 2 + 1$ Domain Wall ensembles



- $a_\mu^{HLO}$  from DWF for non-physical  $m_\pi$  [Boyle et al '11]
- physical point HVP ( $\bullet, \circ$ ) recently measured

## Summary and outlook

- $\Lambda_{N_f=2,4}, m_s$
  - Heavy quark physics (exploratory study - colab. with Southampton-KEK,  $f_B, f_{D_S}, \dots$ )
  - Algorithmic development
- 
- $a_\mu^{HLO}$  with full control over syst. and stat. uncertainties ( $< 1\%$ )
  - QCD+QED at finite volume
  - Precise determination of  $\alpha_s$  (gradient flow coupling, changing BC's ...)
  - ... (some other non-perturbative problem you are interested in? [4-2-040])