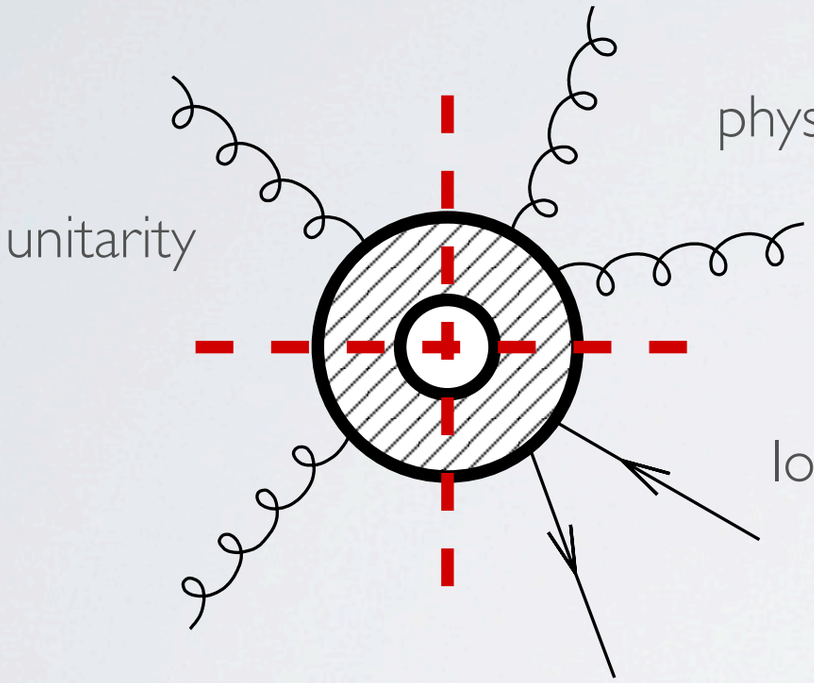
Simon Badger

on-shell methods



new methods for NNLO

integrand level reduction

infra-red subtraction

precision QCD

strong coupling measurements

backgrounds in BSM searches

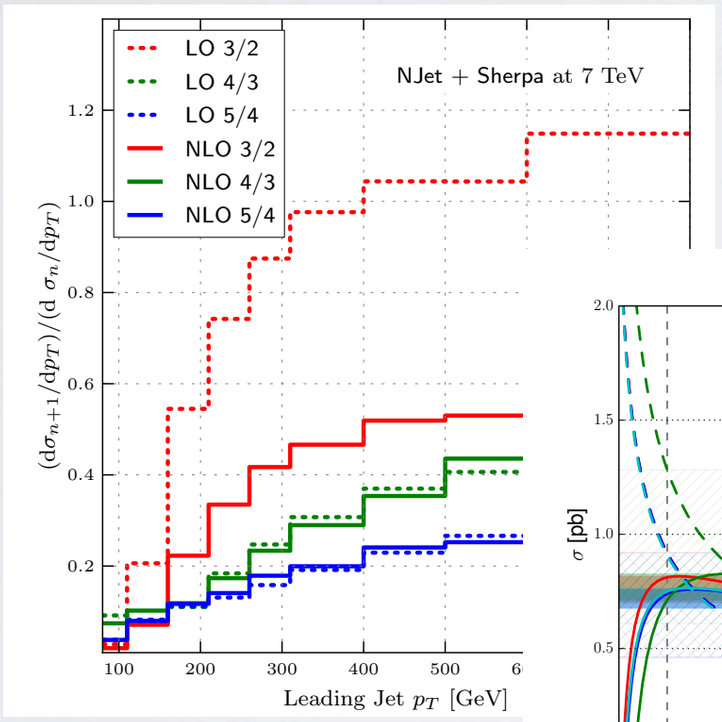
multi-leg NLO

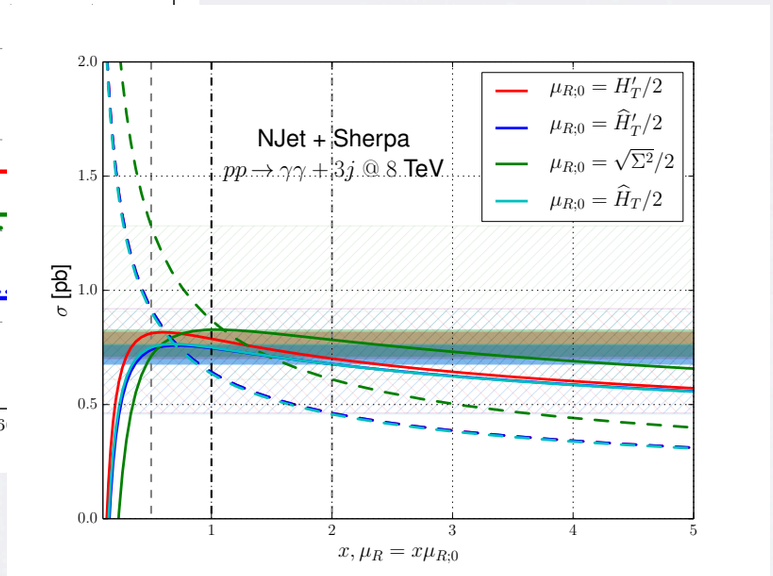
available and automated

matching and merging

Precision backgrounds

automated NLO corrections to high multiplicity jet production





MC interfaces with Herwig++/matchbox, Sherpa, Geneva

AppIgrid/FastNLO interfaces for PDF uncertainties

C++ library

NJET

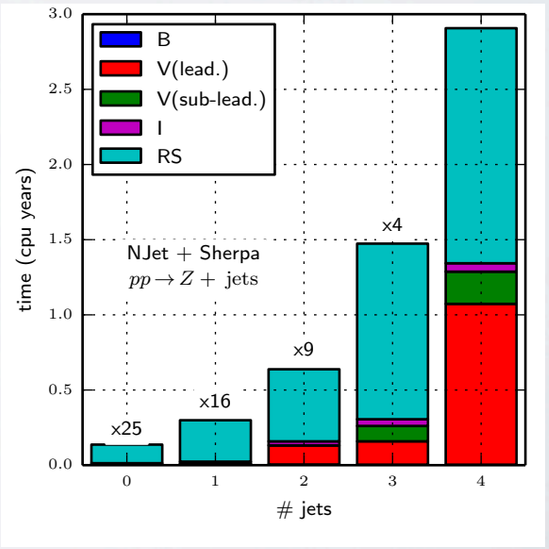
SB, Biedermann, Uwer, Yundin

$pp \longrightarrow \leq 5j$

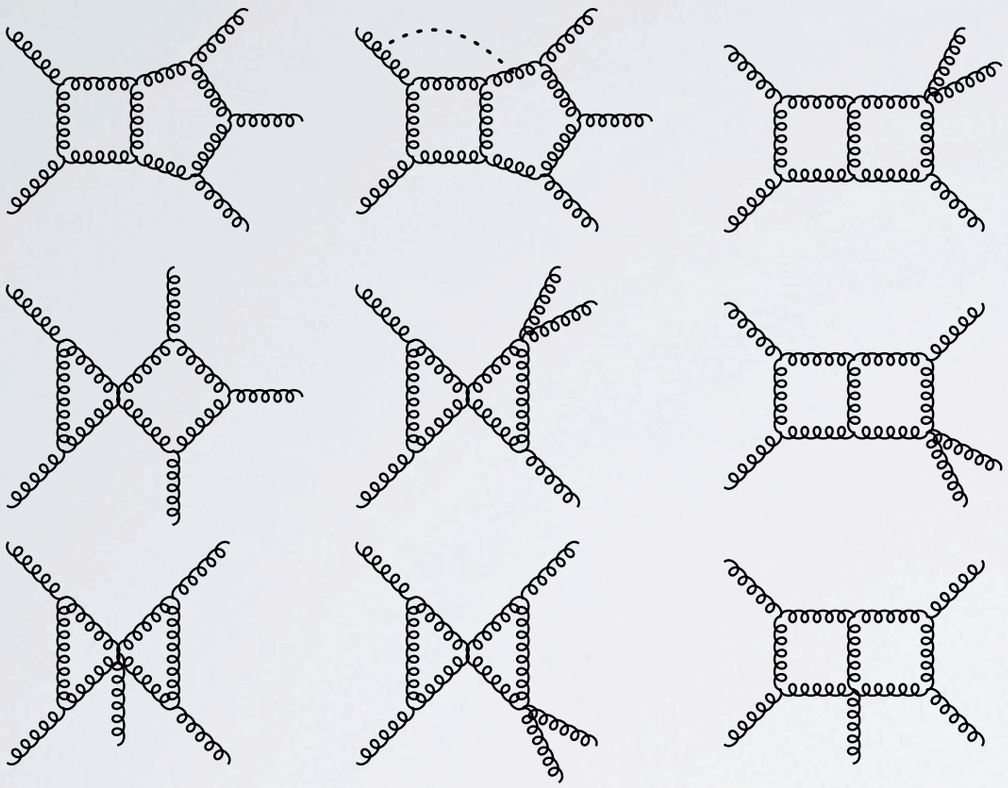
$pp \longrightarrow V(W/Z/\gamma) + \leq 5j$

$pp \longrightarrow \gamma\gamma + \leq 4j$

CPU bottleneck in real radiation



Integrand reduction methods



compact analytic form

[work with Yang Zhang, Hjalte Frellesvig, Donal O'Connell, Gustav Mogull]

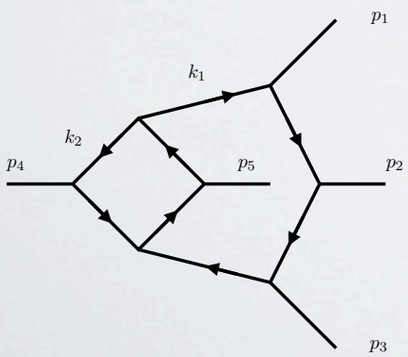
planar five-gluon all-plus helicity amplitude in QCD [SB, Frellesvig, Zhang (2013)]

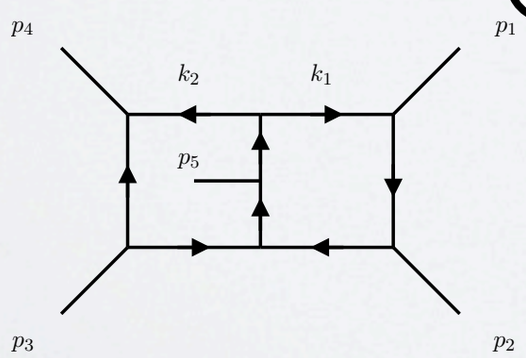
general D-dimensional reduction procedure

6D helicity trees

algebraic geometry

momentum twistors





non-planar amplitudes via BCJ colour-kinematic relations

Momentum twistors

usual spinor-helicity / invariants
representations extremely redundant

[Hodges (2009)]

spinor-helicity

$$p_i \cdot \sigma_{\alpha\dot{\alpha}} = \lambda_\alpha(p_i)\tilde{\lambda}_{\dot{\alpha}}(p_i)$$

$$Z_i = (\lambda_\alpha(p_i), \mu_\alpha(p_i))$$

$$W_{i,\dot{a}} = (\tilde{\mu}_{i,\dot{a}}, \tilde{\lambda}_{i,\dot{a}}) = \frac{\epsilon_{\dot{a},b,c,d} Z_{i-1,b} Z_{i,c} Z_{i+1,d}}{\langle i-1\, i\rangle\langle i\, i+1\rangle}$$

$$Z = \begin{pmatrix} 1 & 0 & f_1 & f_2 & f_3 & \dots & f_{n-3} & f_{n-2} \\ 0 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & 0 & \frac{x_{n-1}}{x_2} & x_n & \dots & x_{2n-6} & 1 \\ 0 & 0 & 1 & 1 & x_{2n-5} & \dots & x_{3n-11} & 1 - \frac{x_{3n-10}}{x_{n-1}} \end{pmatrix}$$

$$f_i = \sum_{k=1}^{i} \frac{1}{\prod_{l=1}^{k} x_l}$$

$$x_i = \begin{cases} s_{12} & i = 1 \\ -\frac{\langle i\, i{+}1\rangle\langle i{+}2\, 1\rangle}{\langle 1\, i\rangle\langle i{+}1\, i{+}2\rangle} & i = 2, \dots, n-2 \\ \delta_{n,4} + (1-\delta_{n,4})\frac{s_{23}}{s_{12}} & i = n-1 \\ -\frac{[2|P_{2,i-n+4}|i{-}n{+}5\rangle}{[21]\langle 1|\, i{-}n{+}5\rangle} & i = n, \dots, 2n-6 \\ \frac{\langle 1|P_{23}P_{2,i-2n+9}|i{-}2n{+}10\rangle}{s_{23}\langle 1|\, i{-}2n{+}10\rangle} & i = 2n-5, \dots, 3n-11 \\ \frac{s_{123}}{s_{12}} & i = 3n-10 \end{cases}$$

$$\delta^{(4)}\left(\sum_{i=1}^{n} p_i\right) \leftrightarrow \delta^{(4)}(0)$$

3n-10 independent variables

simplifying expressions
becomes multi-variable
polynomial factorisation
problem

all multiplicity parameterisation
suitable for H+jets, W+jets etc.

useful tool for analytic calculations

On-shell counter-terms

	local	ISR	FSR	analytic	$2 \to n$
Antenna	✘	✔	✔	✔	?
STRIPPER	✔	✔	✔	✘	?
q_T	✔	✔	✘	✔	?
Trócsányi et al.	✔	?	✔	?	?

[work in progress with Tom Melia and Franz Herzog]

$$\sigma_n^{NNLO} = \int_n (d\sigma^B + d\sigma^V + d\sigma^{VV}) + \int_{n+1} d\sigma^R + d\sigma^{RV} + \int_{n+2} d\sigma^{RR}$$

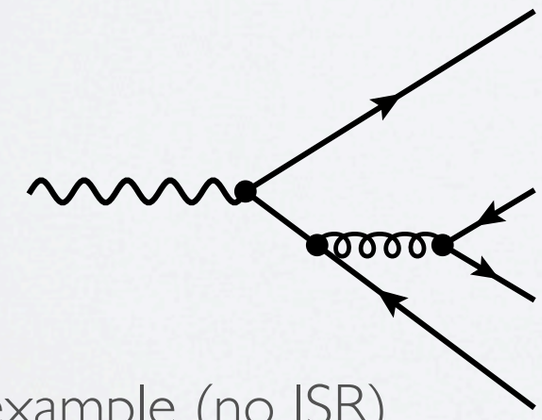
$$\int_{n+2} d\sigma^{RR} = \int (1-\Delta) \left(d\Phi_{1\ldots(n+2)} |M_{n+2}|^2 \right) + \int \Delta \left(d\Phi_{1\ldots(n+2)} |M_{n+2}|^2 \right)$$

$$\Delta = \Delta^S + \Delta^C - \Delta^S \Delta^C$$

soft and collinear limits

$$C_{123} d\Phi_{1234} = d\Phi_{\widetilde{1234}} d\Phi^C_{123} = \mathrm{Jac}(m^4_{123}) d\Phi_{1234}$$

simple integration
using IBP relations



test example (no ISR)

$$\begin{aligned}
\sigma^{RRSS} = & \int d\Phi_{1234} |M(P, 1_u, 2_{\bar{u}}, 3_d, 4_{\bar{d}})|^2 J_2(1,2,3,4) \\
& + m^3_{12}\left(\frac{s_{\widetilde{124}}}{s_{123} + s_{\widetilde{124}}} C_{12}\right) + m^4_{12}\left(\frac{s_{\widetilde{123}}}{s_{\widetilde{123}} + s_{124}} C_{12}\right) \\
& + m^1_{34}\left(\frac{s_{\widetilde{234}}}{s_{134} + s_{\widetilde{234}}} C_{34}\right) + m^2_{34}\left(\frac{s_{\widetilde{134}}}{s_{\widetilde{134}} + s_{234}} C_{34}\right) \\
& + m^4_{123}(\overline{C}_{123} - \widehat{S}^{(123)}_{12}) + m^3_{124}(\overline{C}_{124} - \widehat{S}^{(124)}_{12}) \\
& + m^2_{134}(\overline{C}_{134} - \widehat{S}^{(134)}_{34}) + m^1_{234}(\overline{C}_{134} - \widehat{S}^{(234)}_{34})
\end{aligned}$$