A study of microjets

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Outline

1. Introduction
   - Jet algorithms
   - Perturbative properties of jets
2. Generating functionals
   - Evolution equations
3. Observables
   - Inclusive microjet observables
   - Hardest microjet observables
   - Microjet vetoes
   - Filtering & Trimming
4. Conclusion

Overview

1 Introduction
- Jet algorithms
- Perturbative properties of jets

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Jets

Concept

Collimated bunches of particles produced by hadronization of a quark or gluon.

- Jets can emerge from a variety of processes
  - scattering of partons inside colliding protons,
  - hadronic decay of heavy particles,
  - radiative gluon emission from partons, ...
- We use jet algorithms to combine particles in order to retrieve information on what happened in the event.
- No unique or optimal definition of a jet, but good jet definitions are as close as we can get to observing single partons

Generalised k_t algorithms with incoming hadrons

Definition

1. For any pair of particles i, j find the minimum of

$$d_{ij} = \min\{k_{ti}^{2p}, k_{tj}^{2p}\}\frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

   where $\Delta R_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$.

2. If the minimum distance is d_{iB} or d_{jB}, then the corresponding particle is removed from the list and defined as a jet, otherwise i and j are merged.
3. Repeat until no particles are left.

The index p defines the specific algorithm

- $p = 1$ for the k_t algorithm,
- $p = 0$ for the Cambridge/Aachen algorithm,
- and $p = -1$ for the anti-k_t algorithm

Perturbative properties

Jet properties will be affected by gluon radiation and $g \to q\bar{q}$ splitting.

In particular, considering gluon emissions from an initial parton for a jet of radius R, then

- radiation at angles $> R$ reduces the jet energy,
- radiation at angles $< R$ generates a mass for the jet.

We will try to investigate the effects of perturbative radiation on a jet analytically, particularly in the small-R limit.

Example: Jet mass with emissions at angle $\theta < R$

To evaluate the effect of emissions within the reach of the jet definition, we study the mean squared invariant mass of a jet.

In the small-R limit we can write for an initial quark

$$\langle M^2 \rangle_q = \int \frac{d\theta^2}{\theta^2} \int dz \, \underbrace{p_t^2 z(1-z)\,\theta^2}_{\text{jet inv. mass}} \, \frac{\alpha_s}{2\pi} p_{qq}(z)\, \Theta(R-\theta)$$

$$= \frac{3}{8} C_F \frac{\alpha_s}{\pi} p_t^2 R^2$$

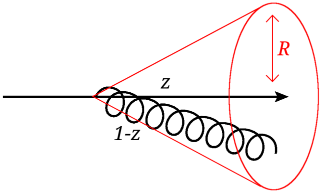

Figure: Gluon emission within the reach of the jet.

Example: Jet p_t with emissions at angle $\theta > R$

We can calculate the average energy difference between the hardest final state jet and the initial quark, considering emissions beyond the reach of the jet. In the small-R limit, we find

$$\langle \Delta z \rangle_q^{\text{hardest}} = \int^{\mathcal{O}(1)} \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) \frac{\alpha_s}{2\pi} p_{qq}(z) \Theta(\theta - R)$$

$$= \frac{\alpha_s}{\pi} C_F \left(2 \ln 2 - \frac{3}{8} \right) \ln R + \mathcal{O}(\alpha_s)$$

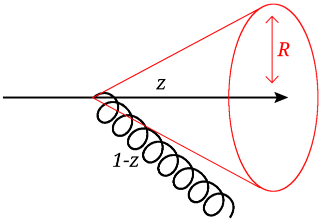

Figure: Gluon emission beyond the reach of the jet.

Example: Jet p_t with intial gluon

We can perform the same calculation in the case of an initial gluon.

$$\langle \Delta z \rangle_g^{\text{hardest}} = \int^{\mathcal{O}(1)} \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) \frac{\alpha_s}{2\pi}$$
$$\times \left[\frac{1}{2} p_{gg}(z) + n_f\, p_{qg}(z)\right] \Theta(\theta - R)$$
$$= \frac{\alpha_s}{\pi} \left[C_A \left(2 \ln 2 - \frac{43}{96} \right) + \frac{7}{48} n_f T_R \right] \ln R + \mathcal{O}(\alpha_s)$$

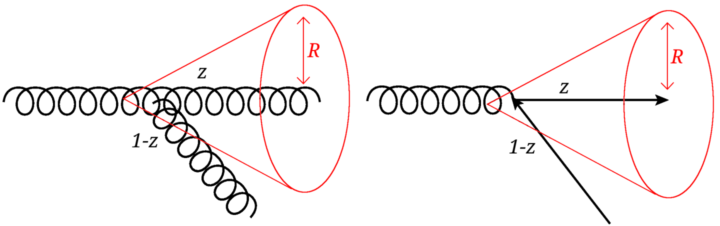

Figure: Gluon emission or $q\bar{q}$ splitting beyond the reach of the jet.

Microjets

Definition

Microjets are jets with small values for the jet radius

$$R \ll 1$$

Small-R limit relevant in a number of contexts, e.g.

- In Higgs physics, where complicated dependence on the jet radius appears due to clustering, in particular in the resummation of jet veto logarithms.
- Decay of heavy particles to boosted W, Z bosons and top quarks.
- Heavy-ion physics where small values for R are used due to the large background.
- In high pileup environments, where use of smaller R might help mitigate adverse effects of pileup.

Theoretically interesting because $\alpha_s \ln R \gg \alpha_s$, therefore calculations simplify and one can investigate all-order structure.

How relevant are small-R effects?

We can evaluate numerically how important the effect of perturbative $\ln R$ terms is on the microjet p_t.

Taking $R = 0.1$ we find that

- quark-induced jets have a hardest microjet $p_t \sim 10 - 15\%$ smaller than the original quark,
- gluon-induced jets have a hardest microjet $p_t \sim 20 - 30\%$ smaller than the original gluon.

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How important can contributions from higher orders be, e.g. $(\alpha_s \ln R)^n$, especially at smaller values of R ?
We will approach this question using generating functionals.

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Evolution variable t

Start with a parton and consider emissions at successively smaller angular scales.

We introduce an evolution variable t corresponding to the integral over the collinear divergence weighted with α_s

$$t = \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \frac{\alpha_s(p_t\theta)}{2\pi} = \frac{1}{b_0} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\alpha_s b_0}{2\pi} \ln \frac{1}{R^2} \right)^n$$

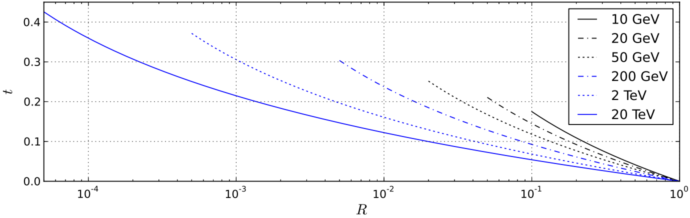

Figure: Plot of t as a function of R down to $Rp_t = 1\,\mathrm{GeV}$ for $p_t = 0.01 - 20\,\mathrm{TeV}$.

Generating functional

Definition

$Q(x, t_1, t_2)$ is the generating functional encoding the parton content one would observe when resolving a quark with momentum xp_t at scale t_1 on an angular scale $t_2 > t_1$ (ie. $R_1 \gg R_2$).

The mean number of quark microjets of momentum zp_t produced from a quark of momentum p_t are

$$\frac{dn_{q(z)}}{dz} = \frac{\delta Q(1, 0, t_2)}{\delta q(z)}\bigg|_{\forall q(z)=1, g(z)=1}$$

We can formulate an evolution equation for the generating functionals

$$Q(x, 0, t) = Q(x, \delta_t, t)\left(1 - \delta t \int dz\, p_{qq}(z)\right) + \delta_t \int dz\, p_{qq}(z)\Big[Q(zx, \delta_t, t) G((1-z)x, \delta_t, t)\Big] .$$

The gluon generating functional $G(x, t_1, t_2)$ is defined the same way.

Evolution equations

We can then easily rewrite the equation on slide [10] as a differential equation,

Quark

$$\frac{dQ(x,t)}{dt} = \int dz\, p_{qq}(z)\, [Q(zx,t)\, G((1-z)x,t) - Q(x,t)]\,.$$

The same procedure in the gluon case yields,

Gluon

$$\frac{dG(x,t)}{dt} = \int dz\, p_{gg}(z)\, [G(zx,t) G((1-z)x,t) - G(x,t)] \\ + \int dz\, n_f\, p_{qg}(z)\, [Q(zx,t) Q((1-z)x,t) - G(x,t)]\,.$$

Solving the evolution equations

We can solve these equations order by order as a power expansion in t, writing

$$Q(x,t) = \sum_n \frac{t^n}{n!} Q_n(x)\,,$$

$$G(x,t) = \sum_n \frac{t^n}{n!} G_n(x)\,.$$

Furthermore the evolution equations can be used to perform an all-order resummation of $(\alpha_s \ln R)^n$ terms.

These methods allow us to calculate observables in the small-R limit up to a fixed order in perturbation theory, or to resum them to all orders numerically.

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Inclusive microjet observables

Definition

Given a parton of flavour i, the inclusive microjet fragmentation function $f^{\text{incl}}_{j/i}(z,t)$ is the inclusive distribution of microjets of flavour j carrying a momentum fraction z.

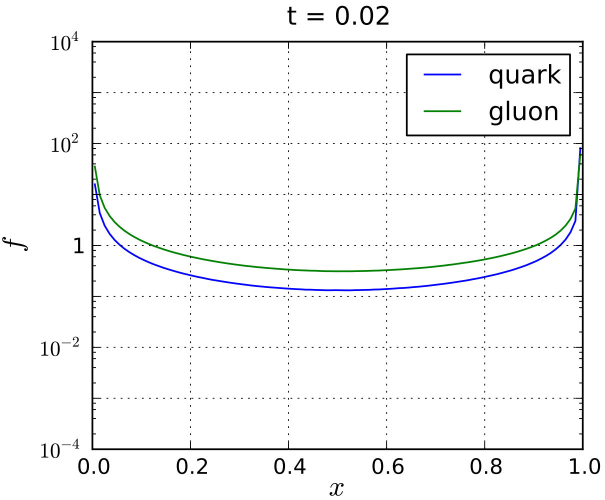
The inclusive microjet fragmentation function satisfies a DGLAP-like equation.

The inclusive microjet spectrum is given by

$$\frac{d\sigma_{\text{jet}}}{dp_t} = \sum_i \int_{p_t} \frac{dp_t'}{p_t'} \frac{d\sigma_i}{dp_t'} f^{\text{incl}}_{\text{jet}/i}(p_t/p_t', t)$$

Inclusive microjet fragmentation function

Peak at 1 is original parton, peak at 0 is soft gluon microjets.



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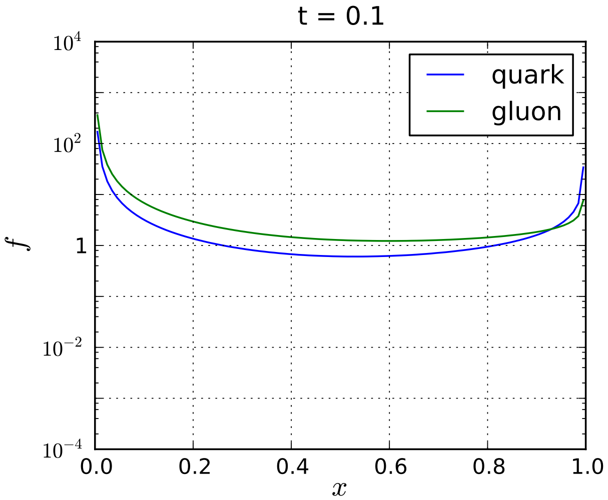

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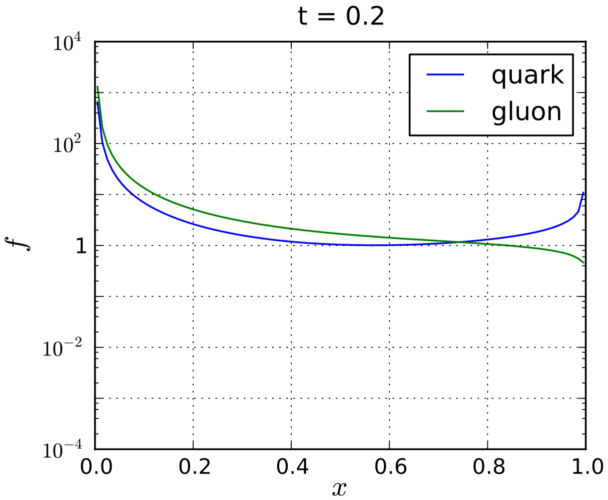

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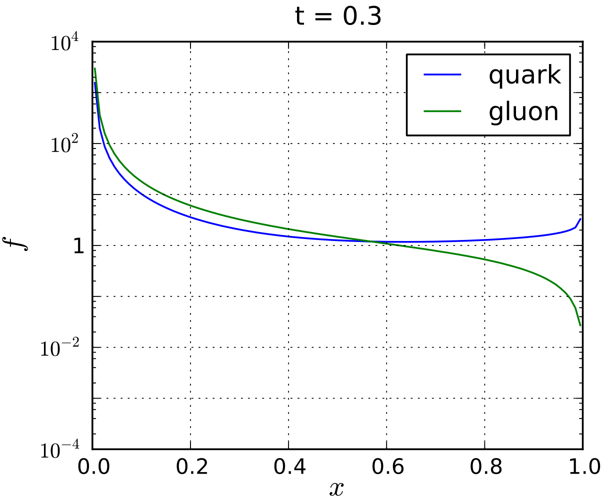

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Hardest microjet observables

Definition

$f^{\text{hardest}}(z)$ is the probability that the hardest microjet carries a momentum fraction z.

Probability conservation imposes

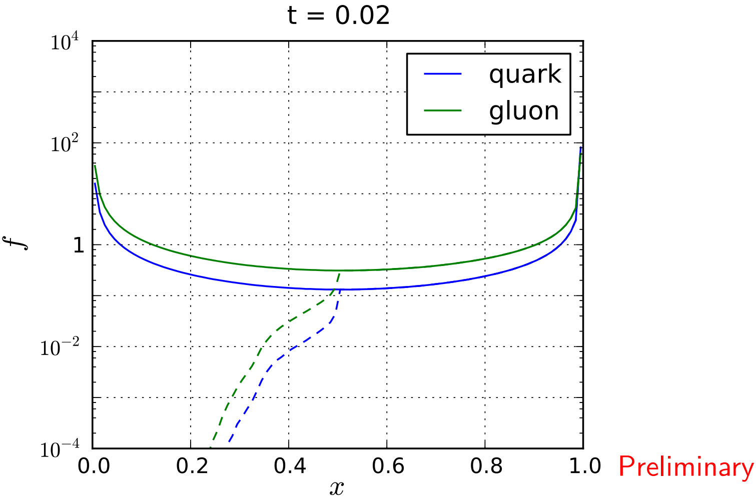
$$\int_0^1 dz\, f^{\text{hardest}}(z) = 1$$

No general DGLAP-like equation, but equal to the inclusive microjet fragmentation function for $z > 0.5$.

Hardest microjet fragmentation function

Solid line: inclusive microjet fragmentation function.

Dashed line: hardest microjet fragmentation function.



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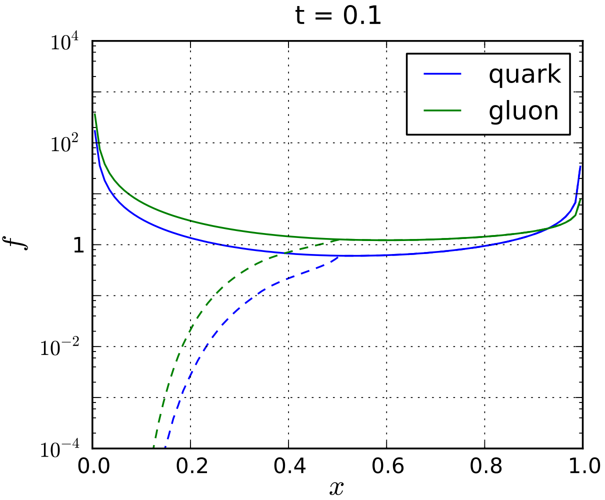

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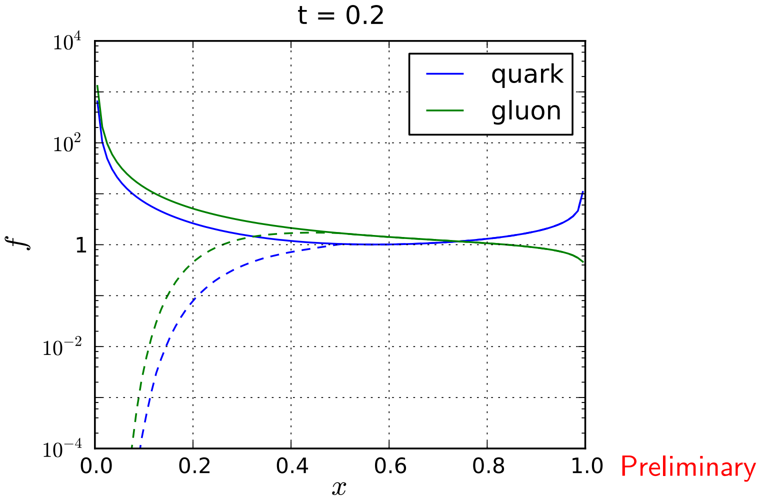

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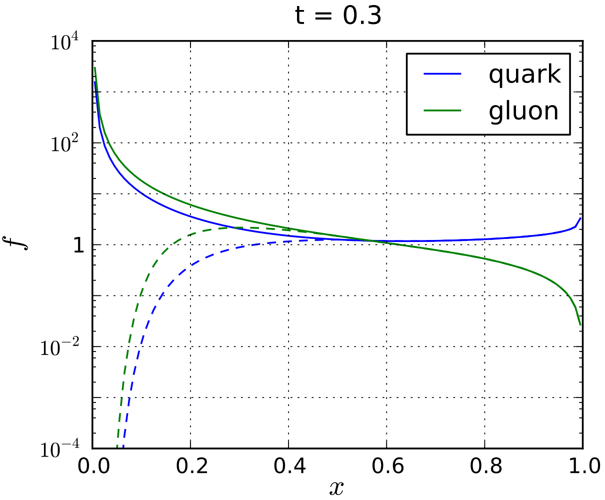

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Hardest microjet Δz revisited

The energy difference between the hardest microjet and the initial parton is given by

$$\langle \Delta z \rangle^{\text{hardest}} \equiv \int_0^1 dz\, f^{\text{hardest}}(z)(z-1)\,.$$

Legend

- Solid line: all-order result
- Dotted line: up to order t
- Dashed line: up to order t^2
- Dash-dotted line: up to order t^3

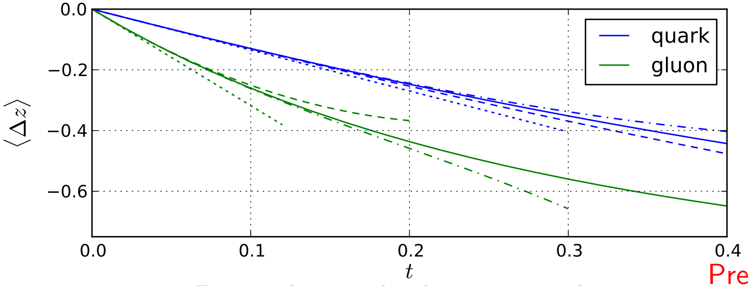

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Figure: Average hardest microjet Δz

Analytical results for Δz

It is now straightforward to calculate higher order contributions to the hardest microjet Δz from slide [6].

Orders t and t^2 are calculated analytically, higher orders are obtained numerically.

$$
\begin{aligned}
\langle \Delta z \rangle_g^{\text{hardest}} = t \left[-\frac{7}{48} n_f T_R + C_A \left(\frac{43}{96} - 2 \ln 2 \right) \right] \\
+ \frac{t^2}{2} \big(0.962984\, C_A^2 + 0.778515\, C_A n_f T_R \\
- 0.50674\, C_F n_f T_R + 0.0972222 n_f^2 T_R^2 \big) \\
+ \frac{t^3}{6} \big(-1.11718(2)\, C_A^3 - 1.557542(7)\, C_A^2 n_f T_R \\
+ 0.375492(7)\, C_A C_F n_f T_R + 0.75869(1)\, C_F^2 n_f T_R \\
- 0.635406(3)\, C_A n_f^2 T_R^2 + 0.305404(3)\, C_F n_f^2 T_R^2 \\
- 0.0648152(4) n_f^3 T_R^3 \big) + \mathcal{O}(t^4)
\end{aligned}
$$

Hardest microjet $\langle z^6 \rangle$

Compared with microjet spectrum

$$\frac{d\sigma}{dp_t} \propto p_t^{-6}$$

The convergence is quite slow : for R=0.2 and $p_t = 50\,\mathrm{GeV}$ the t^3 term contributes at 10% level in the initial gluon case.

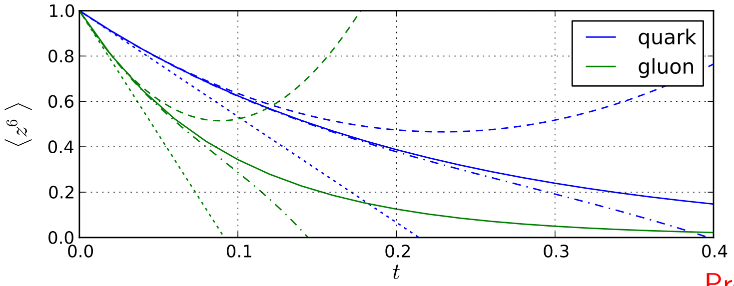

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Figure: Average hardest microjet $\langle z^6 \rangle$.

Hardest microjet $\ln z$

The logarithmic moment of f^{hardest} is relevant for jet vetoes

$$\langle \ln z \rangle^{\text{hardest}} \equiv \int_0^1 dz\, f^{\text{hardest}}(z) \ln z\,.$$

This seems to have a particularly stable perturbative expansion.

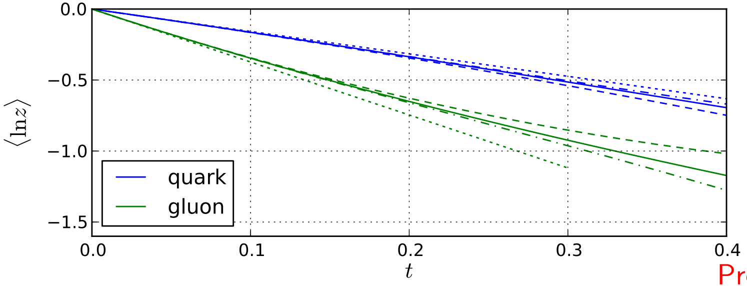

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Figure: Average hardest microjet $\ln z$

Filtering

Definition

Reclustering of a jet on a smaller angular scale $R_{\text{sub}} < R$, discarding all but the n_{filt} hardest subjets.

Here the convergence of the power series is slow.

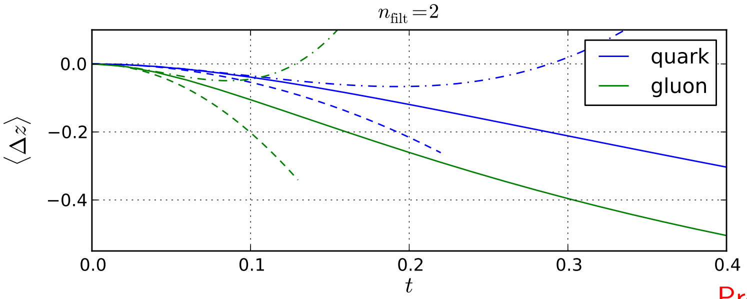

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Figure: Average jet energy loss Δz after filtering with $n_{\text{filt}} = 2$.

Trimming

Definition

- Recluster all particles within a jet into subjets with $R_{\text{sub}} < R$.
- Resulting microjets with $p_t \geq f_{\text{cut}} p_t^{\text{parton}}$ are merged and form the trimmed jet, others are discarded.

Caveat: this would need double resummation of $\ln R$ and $\ln f_{\text{cut}}$.

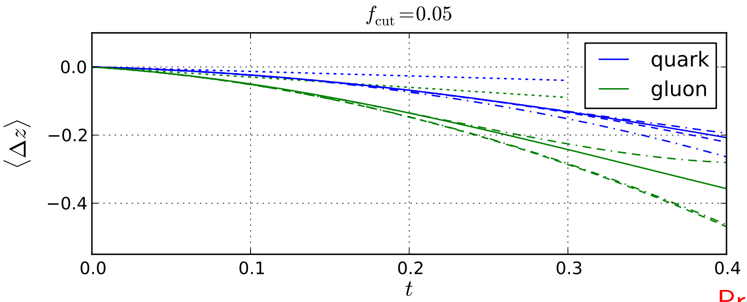

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Figure: Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.05$.

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Conclusion

- Small-R jets affected by radiation.
- Using a generating-functional approach, we carried out numerical LL resummation of $\ln R$ enhanced-terms in small-R jets.
- Resummation complemented by analytical calculations of the LL expansion for the first few orders in perturbation theory.
- Studied inclusive microjet spectrum and identified the spectrum of the hardest microjet emerging from parton fragmentation.
- Calculated a number of observables of interest, such as
  - energy losses of trimmed and filtered jets
  - logarithmic moment of hardest microjet spectrum, relevant in particular for jet vetoes in Higgs-boson production.
- Study of first phenomenological implications are in progress.

Backup slides

Jet flavour

Let us examine how quark and gluon jets evolve into each other.

Definition

The probability of finding a hardest subjet of flavour a is

$$\mathcal{P}(a) = \int_0^1 dz\, f_a^{\text{hardest}}(z)$$

where $f_a^{\text{hardest}}(z)$ is the probability distribution of hardest microjets of flavour a.

Tracking jet flavour can be of relevance for quark-gluon tagging.

Jet flavour

Given a parton flavour, we look at the probability that the hardest resulting microjet has the same flavour.

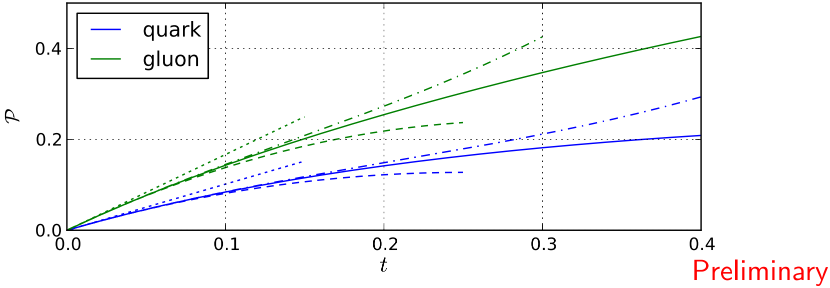

Figure: Flavour change probability.

Inclusive microjet fragmentation function

The inclusive microjet fragmentation functionally satisfies a DGLAP-style equation

$$\frac{df^{\mathrm{incl}}_{j/i}(z,t)}{dt} = \sum_k \int_z^1 \frac{dz'}{z'}\, P_{jk}(z')\, f^{\mathrm{incl}}_{k/i}(z/z',t)\,,$$

with an initial condition

$$f^{\mathrm{incl}}_{j/i}(z,0) = \delta(1-z)\delta_{ji}\,.$$

Filtering

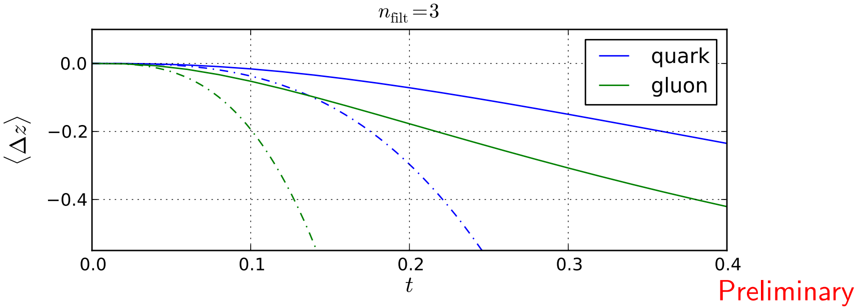

Figure: Average jet energy loss Δz after filtering with $n_{\rm filt} = 3$.

Trimming coefficients

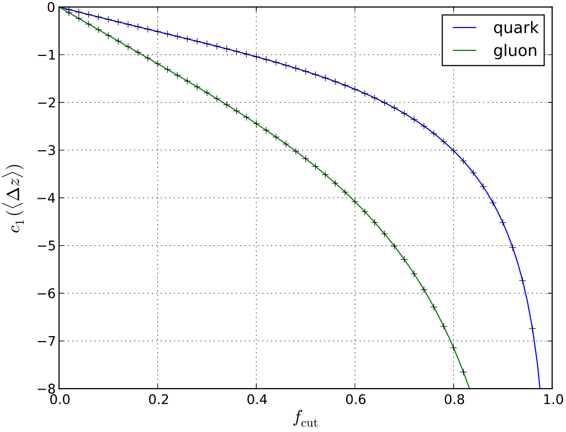

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Figure: First order coefficients $c_1(\Delta z)$ as a function of f_{cut}.

Trimming coefficients

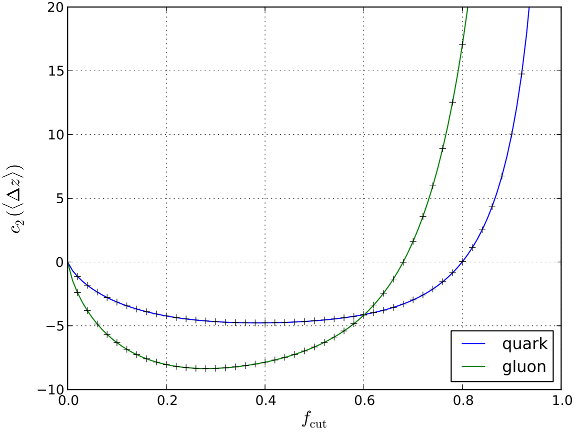

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Figure: Second order coefficients $c_2(\Delta z)$ as a function of f_{cut}.

Trimming coefficients

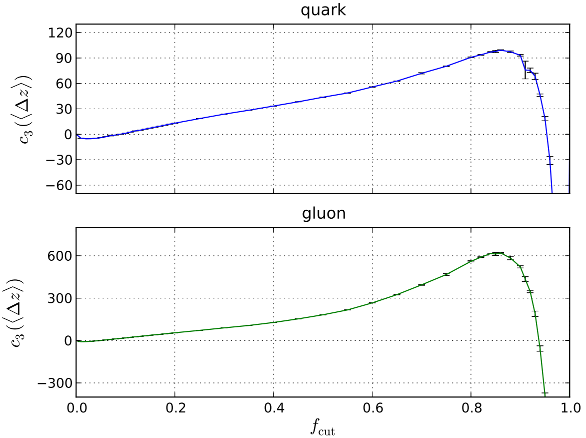

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Figure: Third order coefficients $c_3(\Delta z)$ as a function of f_{cut}.

Trimming coefficients

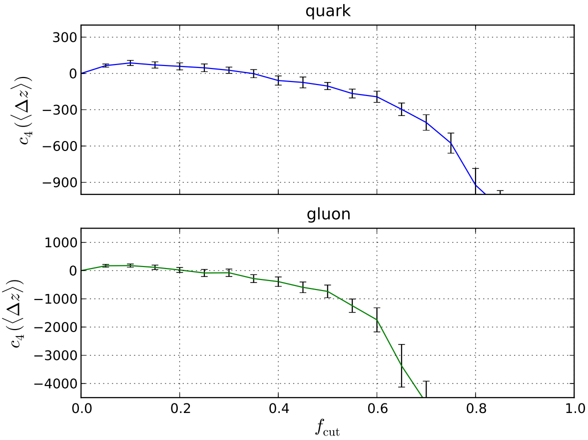

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Figure: Fourth order coefficients $c_4(\Delta z)$ as a function of f_{cut}.