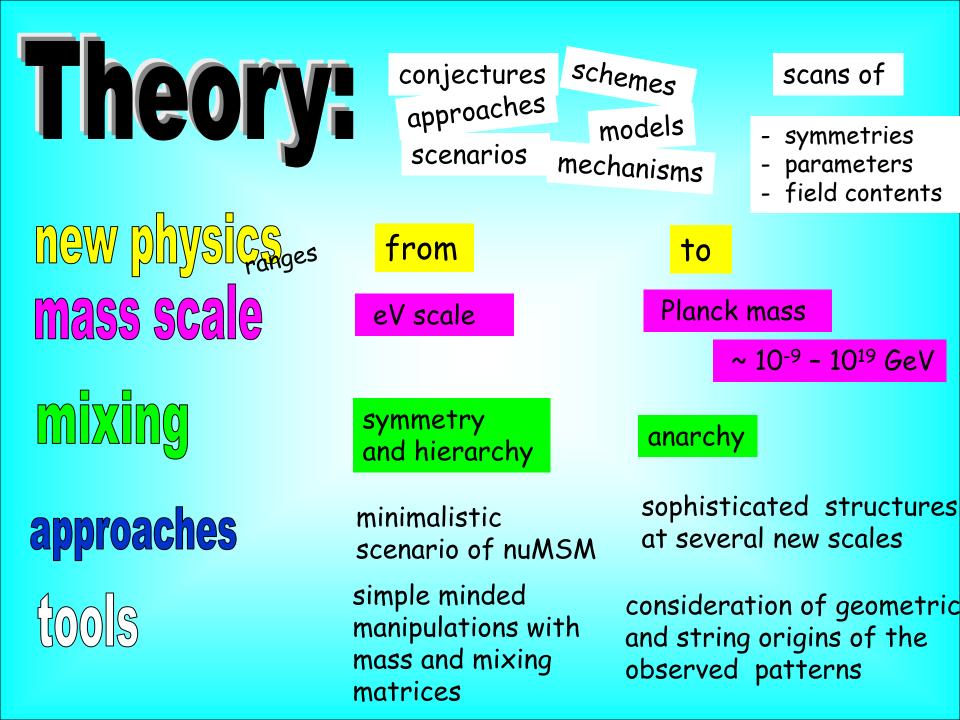
Theory of neutrino mass & mixing

A. Yu. Smirnov Max-Planck Institute for Nuclear Physics, Heidelberg, Germany

International Meeting for Large Neutrino Infrastructures June 23 - 24, 2014 , Paris



At the crossroads

Some of models and approaches may indeed reflect (correspond to) reality

And still some key elements can be missed

Recent developments:

> No new physics at LHC, MEG, searches for FCNC, Higgs properties are in agreement with SM

This forces us to take more seriously scenarios with nothing or almost nothing below Planck scale Content To assess possible implications of future ng scales and patterns measu Ng Symmetry or no symmetry measurements Ind Hass Hierarchy, CP-phase and steriles 4. Theoretical relevance and urgency





Scales and frameworks Electrowea (\bigtriangleup)

High scale seesaw Quark- lepton symmetry /analogy GUT

Low scale seesaw, Radiative mechanisms, High dimensional operators

Dimene

Scale of neutrino masses themselves

- Relation to dark Energy, MAVAN?

High scale seesaw, unification

$$m_v = -m_D^T \frac{1}{M_R} m_D$$

similarity: $m_D \sim m_q \sim m_l$

 $M_R = M_{GUT} \sim 10^{16} \text{ GeV}$ For the heaviest in the presence of mixing

 $\rm M_R$ ~ 10^8 - 10^14 GeV

double seesaw

 $M_{\rm R} \sim 10^{16} - 10^{17} \, GeV$

many heavy singlets (RH neutrinos) $\rm N \sim 10^2$...string theory



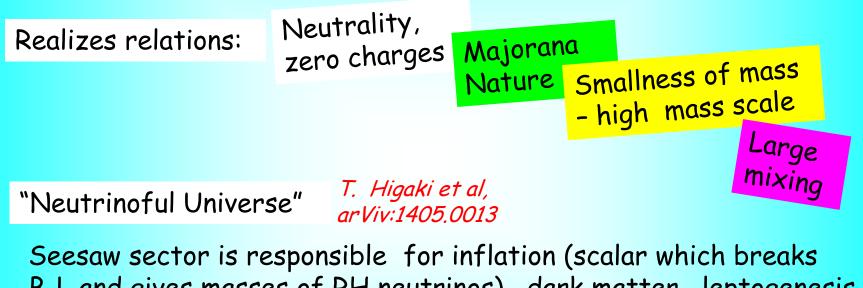
Neutrino mass as an evidence of Grand Unification?

Gauge coupling unification BICEP-II ?

Leptogenesis: the CP-violating out of equilibrium decay



Natural, minimalistic, in principles



B-L and gives masses of RH neutrinos), dark matter, leptogenesis

Testable?

- Proton decay

- Majorana masses

A GUT scenario

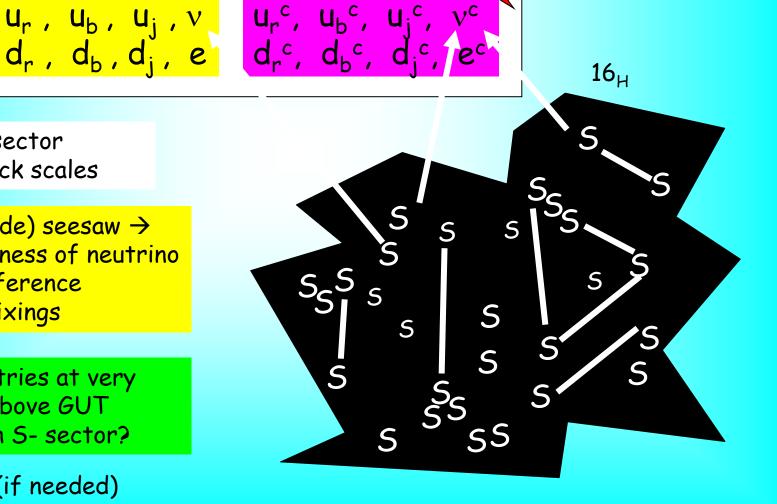
SO(10) GUT + hidden sector + flavor symmetries

with Hidden sector at GUT - Planck scales

Double (cascade) seesaw \rightarrow explains smallness of neutrino mass and difference of q- and I- mixings

Flavor symmetries at very high scales, above GUT Symmetries in S-sector?

Randomness (if needed)



RH-neutrino

but

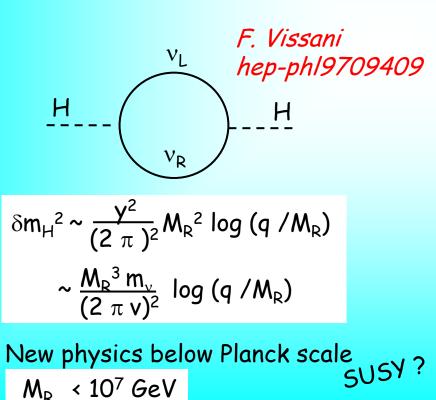
The problem

Hierarchy problem

Natural scale
$$M_R \sim m_D^2 / m_v \sim 10^{14} \text{ GeV}$$

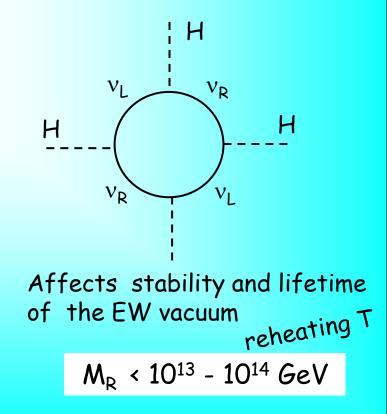
 ν_{R}

introduces new mass scale << M_{Pl} (Another indication: unification of gauge couplings)



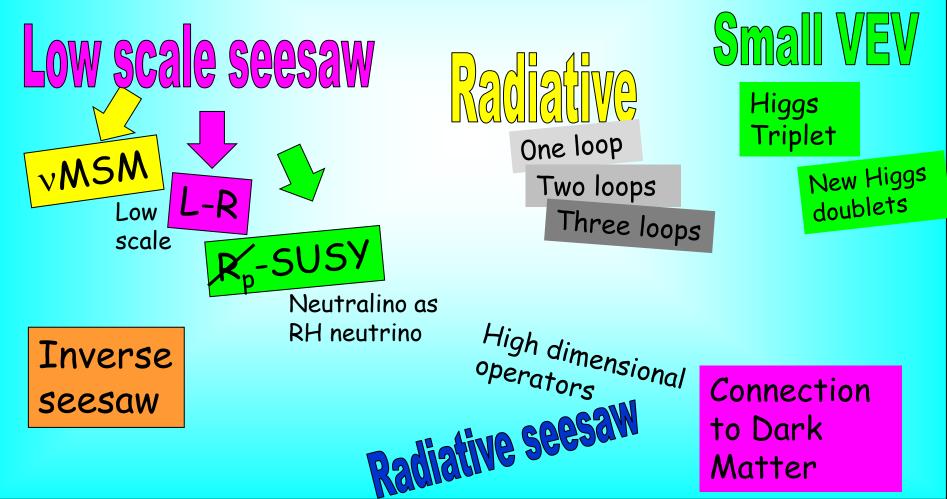
J Elias-Miro et al, 1112.3022 [hep-ph]

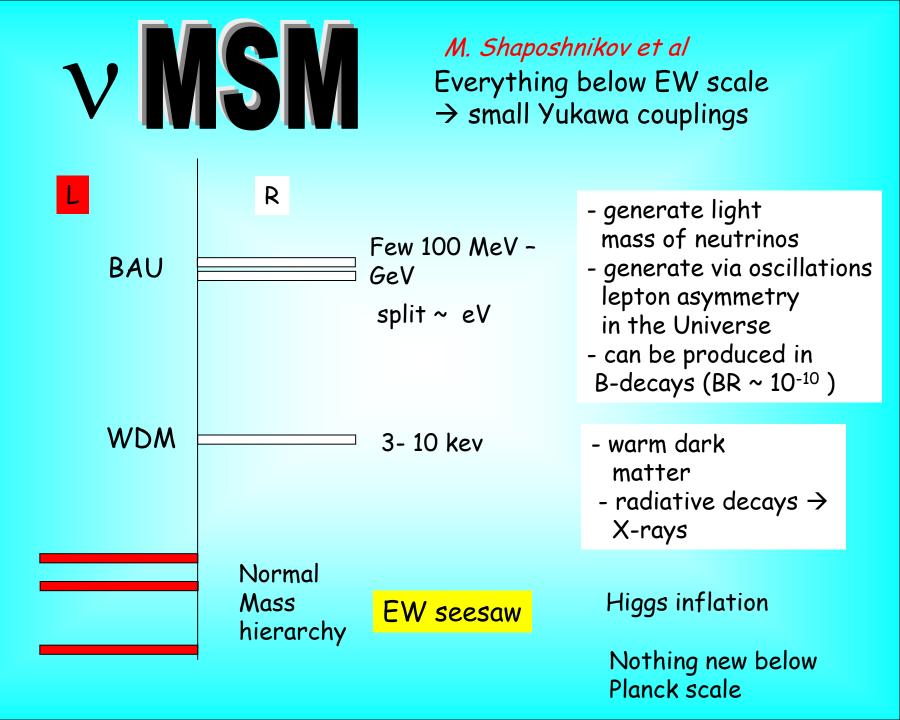
Renormalization of quartic Higgs coupling λ (making it more negative)



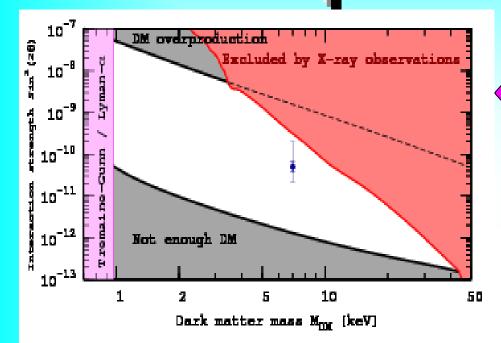
EW - LHC scale

- No hierarchy problem (even without SUSY)
- testable at LHC, new particles at 0.1 few TeV scale
- LNV decays

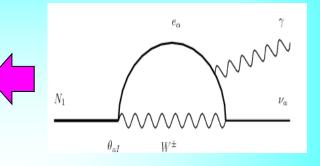




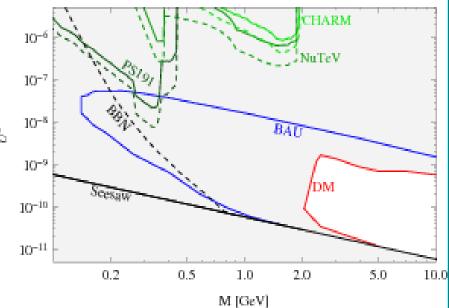
Bounds on parameters of nuNSN



The blue point: the best-fit value from M31 (Andromeda galaxy). Thick error bars are $\pm 1\sigma$ limits on the flux. Thin error bars correspond to the uncertainty in the DM distribution in the center of M31.



A Boyarsky et al, 1402.4119







Three additional singlets S which couple with RH neutrinos

$$\begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & \mu \end{pmatrix} \begin{pmatrix} v \\ v^c \\ S \end{pmatrix}$$

 $\mu << M_D$

 μ - scale of L violation

$$\mathbf{m}_{v} = \mathbf{m}_{D}^{\mathsf{T}} \mathbf{M}_{D}^{-1\mathsf{T}} \boldsymbol{\mu} \mathbf{M}_{D}^{-1} \mathbf{m}_{D}$$

- pseudo-Dirac neutrino with mass $M^{}_{\rm D}$ formed by $\nu^{\rm c}$ and S
- one light Majorana neutrino per generation

If
$$m_D \sim 100 \text{ GeV}$$
, $M_D \sim \text{few TeV} \implies \mu \sim \text{keV}$ Physics of light neutrinos here

- Violation of universality, unitarity in the light sector $\sim 10^{\text{-2}}$
- pseudoDirac neurinos at LHC

eV - sub eV scale physics

Very light sector which may include

- new scalar bosons, majorons, axions,
- new fermions (sterile neutrinos, baryonic nu),
- new gauge bosons (e.g. Dark photons)

M. Pospelov

Maybe related to Dark energy, MAVAN

Generate finite neutrino masses, usual Dirac masses can be suppressed by seesaw with $M_R = M_{Pl}$

eV scale Seesaw with RH neutrinos for sterile anomalies LSND/ MiniBooNE

A. De Gouvea

Tests:

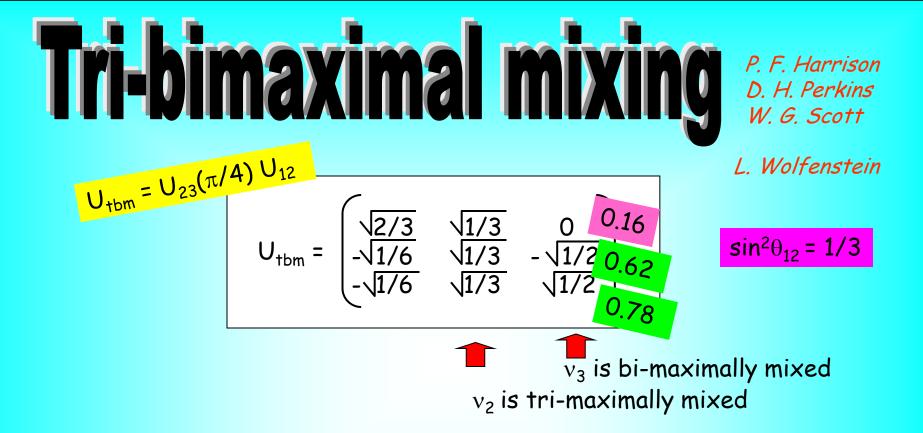
5th force searches experiments

Modification of dynamics of neutrino oscillations

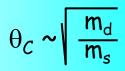
Checks of standard oscillation formulas, searches for deviations

<section-header>





Difficult if possible connect to masses Mixing decouples from masses

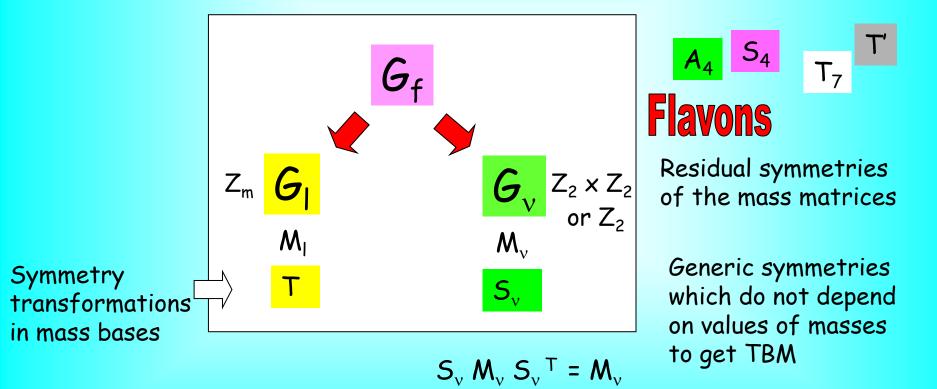


Residual symmetries approach

Acceidental?

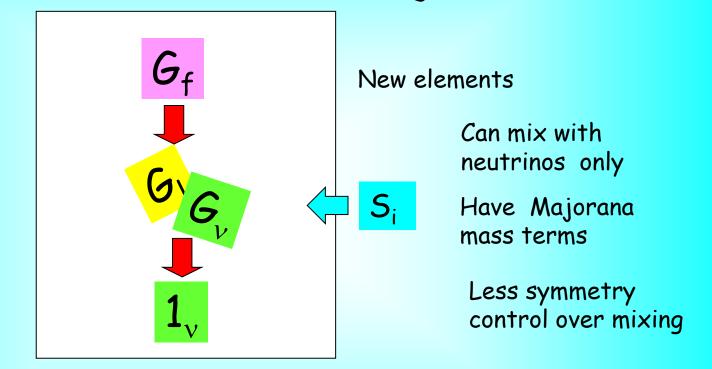


Mixing appears as a result of different ways of the flavor symmetry breaking in the neutrino and charged lepton (Yukawa) sectors.



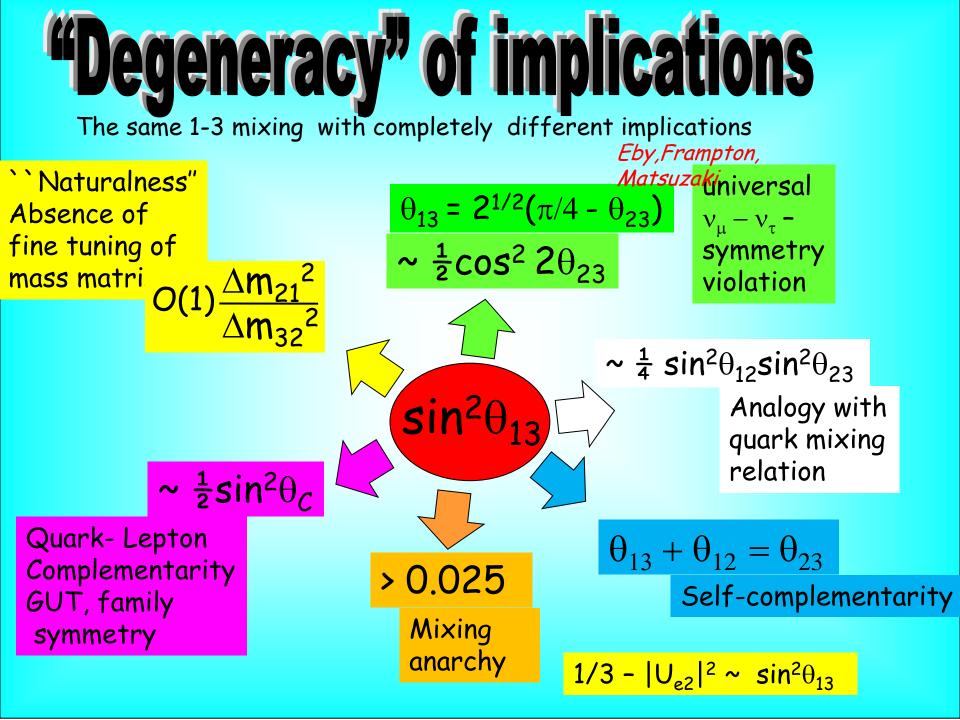
Maximal control over mixing as implied by TBM Now: broken TBM symmetry

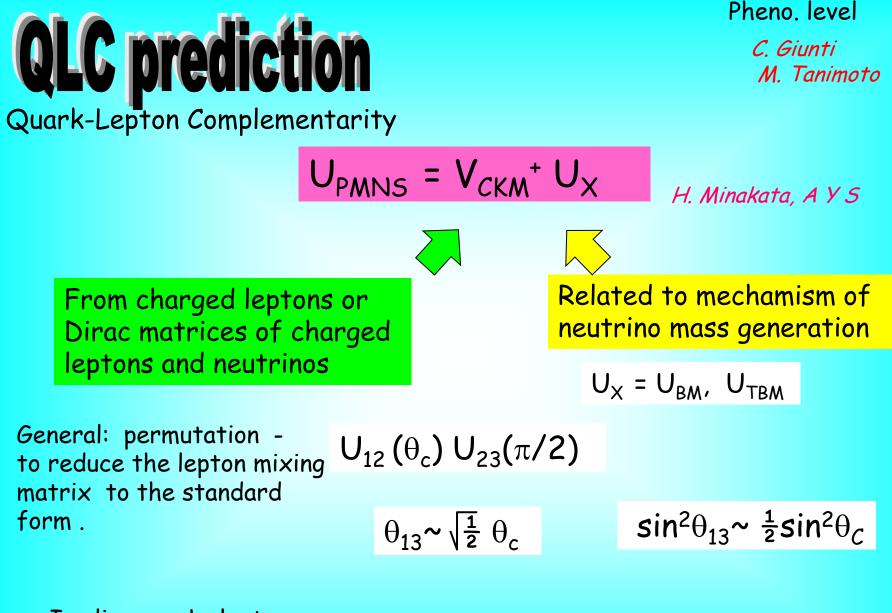
Another realizations of symmetries



Mixing originates from

- different nature of the mass terms of the charged leptons (Dirac) and neutrinos (Majorana)
- Mixing from new degrees of freedom (e.g. singlets of SM)



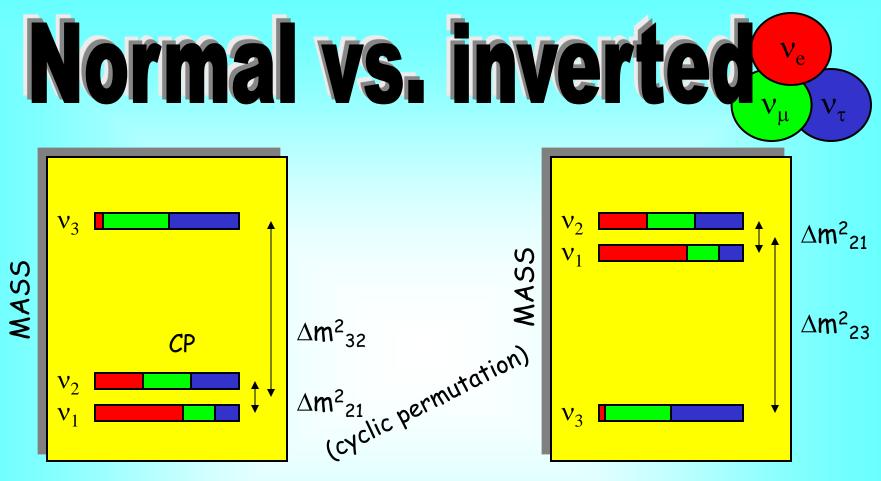


Implies quark -lepton symmetry, unification, GUT?

Should we take this seriously?

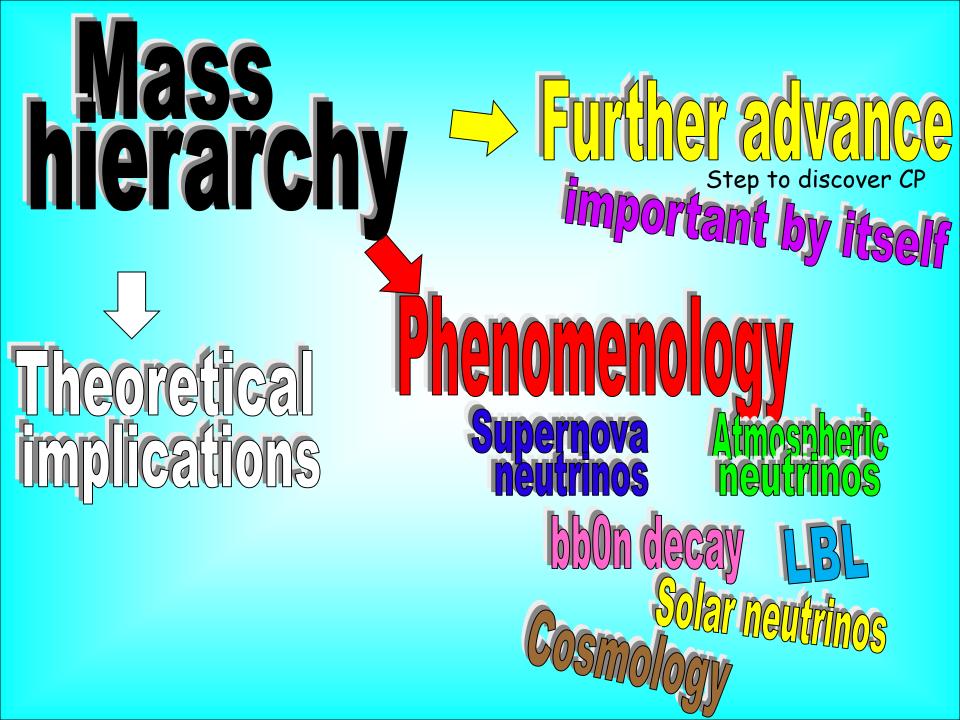
Kiass hieroty

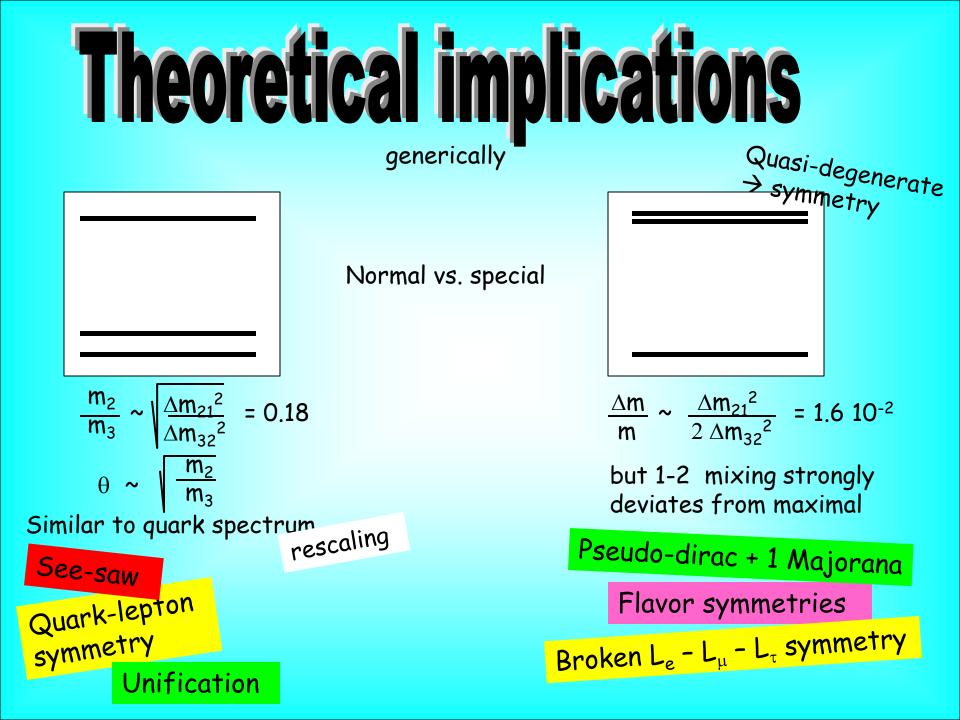




Light flavor (v_e) in the light states Weaker mass hierarchy

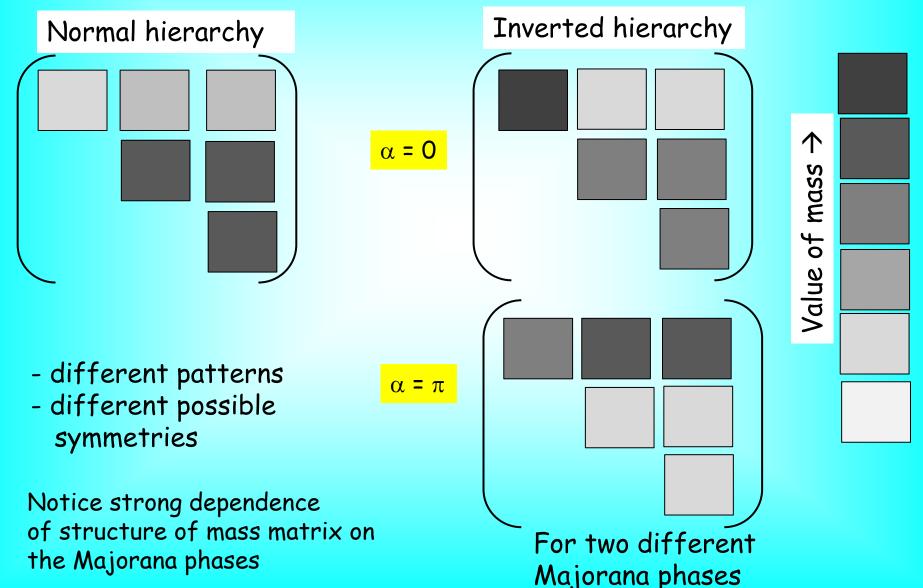
Light flavor (v_e) in the heavy states





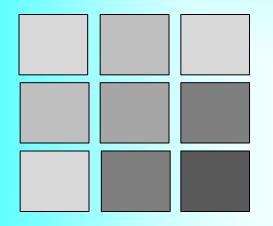
Pattern of mass matrices

For Majorana neutrinos





Values of elements gradually decrease from $m_{\tau\tau}$ to m_{ee}



corrections wash out sharp difference of elements of the dominant $\mu\tau$ -block and the subdominant e-line

This can originate from power dependence of elements on large expansion parameter $\lambda \sim 0.7 - 0.8$. Another complementarity: $\lambda = 1 - \theta_c$

Froggatt-Nielsen?



In models with residual (discrete) flavor symmetries mixing does not depend on mass at least in the symmetry limit. The same mixing pattern can be obtained for NH and IH

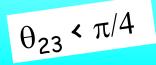
The type of hierarchy is fixed by field content (scalar sector), auxiliary symmetries, etc.

Radiative mechanisms have different preferences those which are related to charged leptons prefer NH

broken $(L_e - L_\mu - L_\tau)$ symmetry

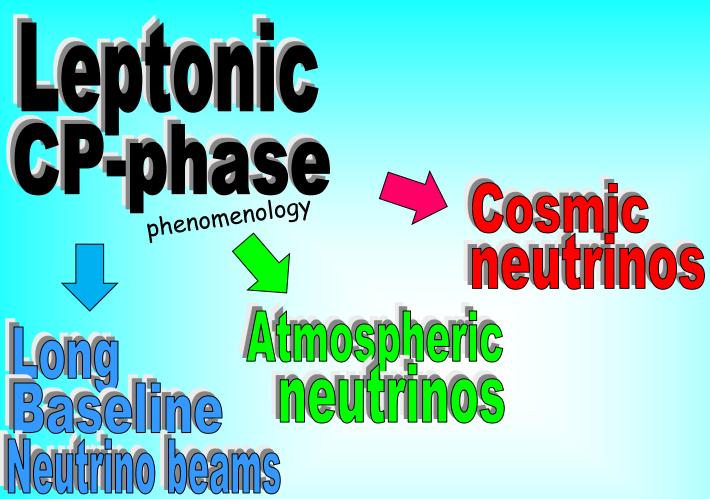
Seesaw type I with quark-lepton analogy (symmetry) leads to NH Establishing NH would favor the line with similarity with quark sector, high scale seesaw, etc.

If so, one would expect the first quadrant, no eV scale steriles





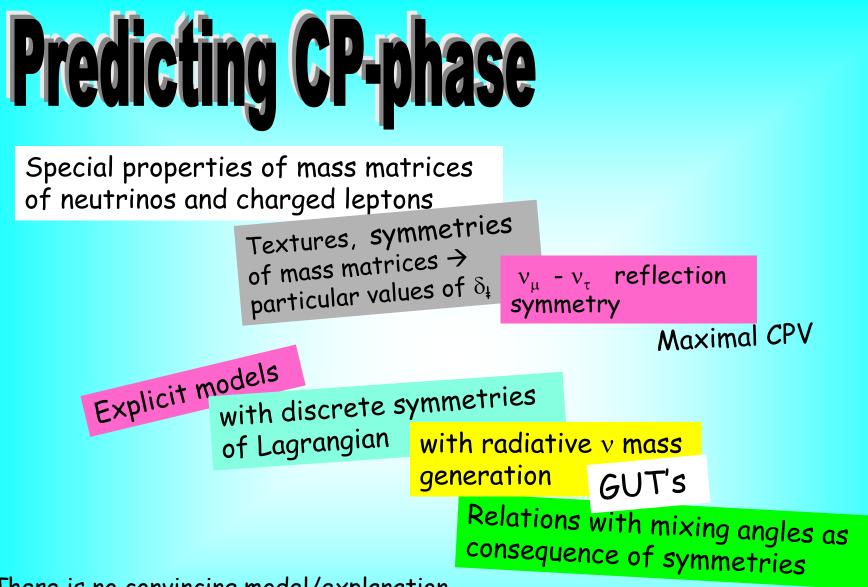




Theoretical Implications Probe of the underlying Physics, enters various test equalities

Onbb-decay Cosmology Leptogenesis





There is no convincing model/explanation of the value of phase in guark sector

Can we predict the phase in lepton sector where situation is more complicated due to additional elements producing smallness of neutrino mass?



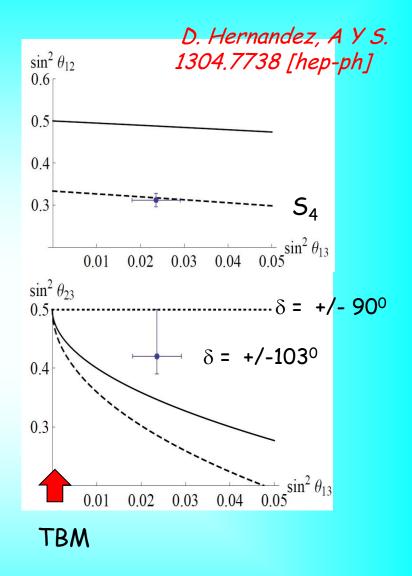
In the residual symmetry approach For column of the mixing matrix:

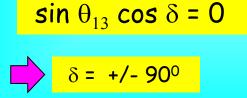
$$|U_{\beta i}|^{2} = |U_{\gamma i}|^{2} \qquad k_{\alpha} = 0$$

$$|U_{\alpha i}|^{2} = \frac{1+\alpha}{4 \sin^{2}(\pi k/m)}$$

k, m, p integers which determine symmetry group

From ν_{μ} - $\nu_{\tau}{}^{c}$ reflection symmetry of the mass matrix





W. Grimus , L. Lavoura, Y. Farzan, A.S.

Relating to mass degeneracy

Symmetry which left mass matrices invariant for specific mass spectra:

Partially degenerate spectrum $m_1 = m_2$, m_3

D. Hernandez, A.S.

Transformation matrix $S_v = O_2$ $G_v = SO(2) \times Z_2$

Relation:
$$sin^2 2\theta_{23} = +/-sin \delta = cos \kappa = \frac{m_1}{m_2} = 1$$
maximal $+/-\pi/2$ Majorana phase

1-2 mixing is undefined

Small corrections to mass matrix lead to 1-2 mass splitting and 1-2 mixing



If δ_{I} is known we can at least compare it with δ_{q}

The closest quark-lepton connection

GUT or/and common flavor symmetry Seesaw type-I Similarity of the Dirac mass matrices

 $\mathbf{m}_{\mathrm{D}^{\mathrm{v}}} \sim \mathbf{m}_{\mathrm{D}}^{\mathrm{q}}$

B. Dasgupta, A.S.

In general $U_{PMNS} = U_L U_X$ $U_L \sim V_{CKM}^*$ Has similar hierarchical structure determined (as in Wolfenstein parametrization) by powers of

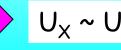
 $\lambda = \sin \theta_{C}$

As in QLC

Related to (any) mechanism that explains smallness of neutrino mass

Should be fixed to reproduce correct Lepton mixing angles

 $V_{CKM} \sim I$



Leptonic CP from CKN

If $U_L \sim V_{CKM}^* (\delta_q)$ is the only source of CP violation -as in the quark sector, U_X is real

$$\begin{split} s_{13} \sin \delta_{CP} &= (-c_{23}) s_{13}^{q} \sin \delta_{q} \\ \sin \delta_{CP} &\sim \lambda^{3} / s_{13} \sim \lambda^{2} \\ \sin \delta_{CP} &\sim 0.046 \\ \delta_{q} &= 1.2 + / - 0.08 \text{ rad} \\ \delta_{q} &= 0.046 \\ \delta_{q} &= 0.046$$

If other value of phase is observed → contributions beyond CKM (e.g. from the RH sector) or another framework



neglecting terms of the order ~ λ^3

$$\sin \delta_{CP} = s_{13}^{-1} \left[sin(\alpha_{\mu} + \delta_{X}) V_{ud} | X_{e3} | - sin \alpha_{e} | V_{cd} | X_{\mu 3} \right]$$

here α_{μ} , δ_{X} and α_{e} are $% \beta_{\text{P}}$ parameters of the RH sector

Some special values of δ_{CP} can be obtained under certain assumptions

if
$$X_{e3} = 0$$
 sin $\delta_{CP} \sim -\sin \alpha_{e}$

and if $\alpha_e = \pi / 2$ $\delta_{CP} \sim 3\pi / 2$

One can find structure of the RH sector which lead to these conditions

In the Seesaw type I

B Dasgupta A.S

 U_x is the matrix diagonalizes

 $M_X = -m_D^{diag} U_R^+ (M_R)^{-1} U_R^* m_D^{diag}$

Here $m_D = U_L (m_D^{diag}) U_R^+$

In contrast to quarks (Dirac fermions) for Majorana neutrinos the RH rotation that diagonalizes m_D becomes relevant and contributes to PMNS

CPV from U_R

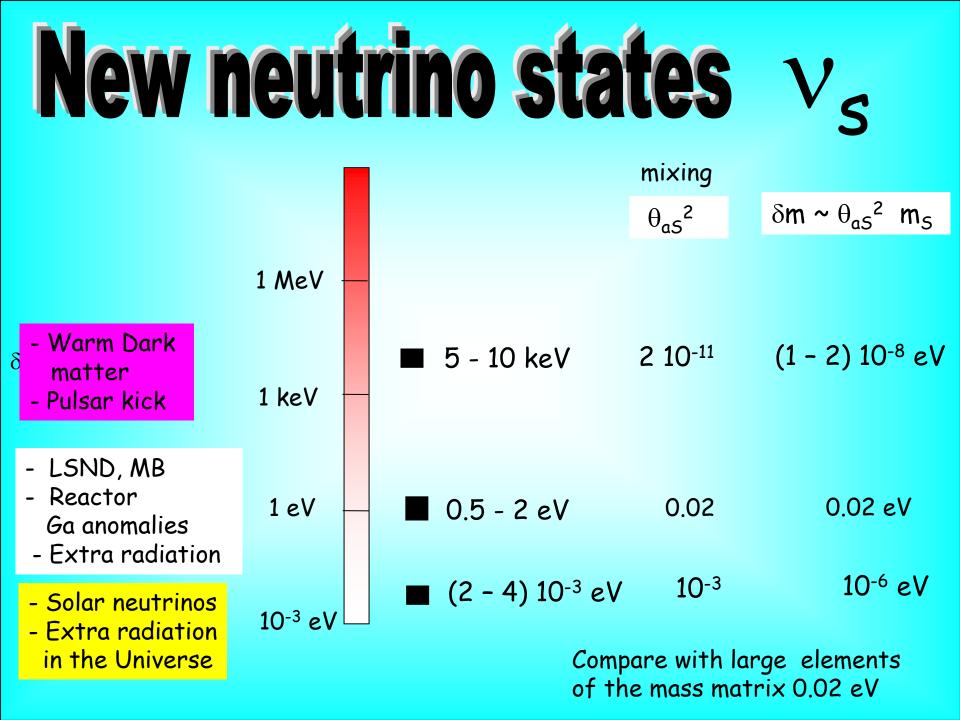
Minimal extension is the L-R symmetry:

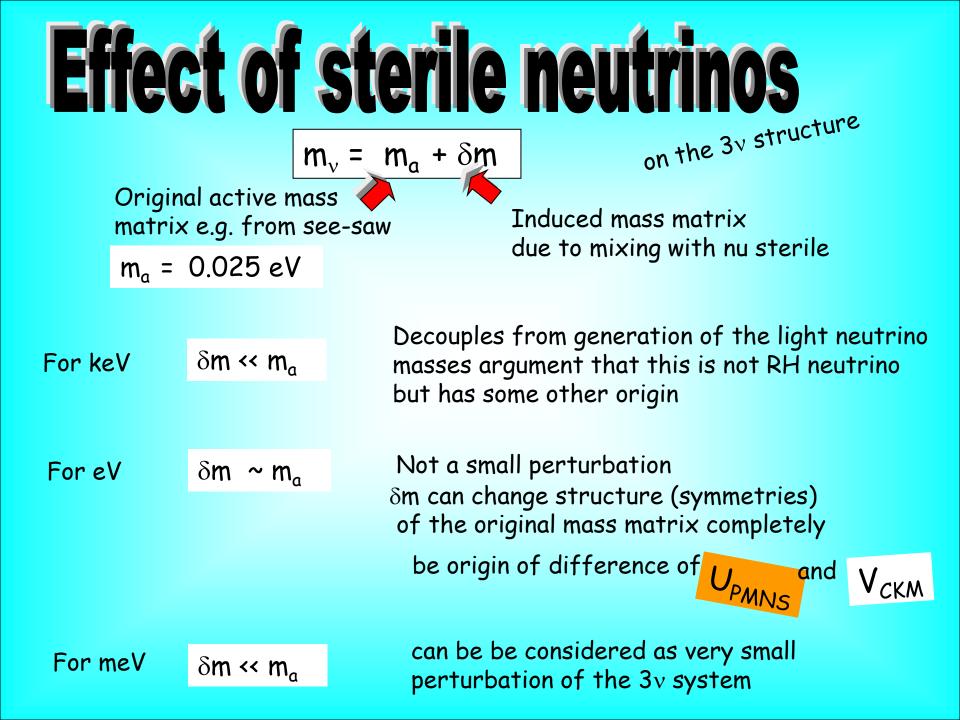
 $U_R = U_L \sim V_{CKM}^*$ and no CPV in M_R

Seesaw can enhance this small CPV effect so that resulting phase in PMNS is large

Sterile heutrinos









Theoretical relevance and urgency



LSND 1 eV steriles , controversial, not favored... still if exist change theory substantially – not a small perturbation, 3 Dirac CP phases etc

Bound on mixing should be $\theta_{aS}^2 < 10^{-3}$

)-3 urgen

Other steriles (7 kev , meV) - small/negligible perturbation of the 3v theory from other sectors ment



Crucial for SN neutrinos and $\beta\beta 0\nu$ decay, important for cosmology and atmospheric neutrinos Knowledge of H facilitates determination of δ

Will exclude some specific models, will favor certain approach to understand neutrino mass:

NH: q-l similarity, unification, high scale seesaw, But also some radiative mechanisms

IH: unusual, low (EW TeV) scale mechanisms, also - radiative, implies symmetry

Theoretical relevance and urgency

CP VIOLATION which are p framework

 δ_{CP} enters various test equalities, sum rules, etc., which are probes of underlying physics in certain frameworks and under certain assumptions

Specific values like $0, \pi, \pi/2$ may have more straightforward implications (still not unique) +/- $\pi/2$ can be related (by symmetry) with maximal 2-3 mixing, quasi-degeneracy of mass states ...

Comparison with quark phase will be interesting Even in unification approach they can be very different. Substantial deviation of δ_{CP} from 0, π , will testify for new sources of CP in lepton sector

Majorana vs. Dirac

Majorana - favored, seesaw , unification line

Theoretical relevance and urgency

2-3 Quadrant

 $\theta_{23} = 45^{\circ}$ is special. More important in first place – value of deviation from 45° – that may be Even more substantial probe of the underlying physics.

For NH: θ_{23} < 45° would be in favor of similarity and unification, otherwise strange

For IH both θ_{23} < 45° and θ_{23} > 45° are possible



Exclude (discover) completely degenerate spectrum



Progress in understanding the underlying physics may come from Non-neutrino experiments LHC, Dark matter



I \square

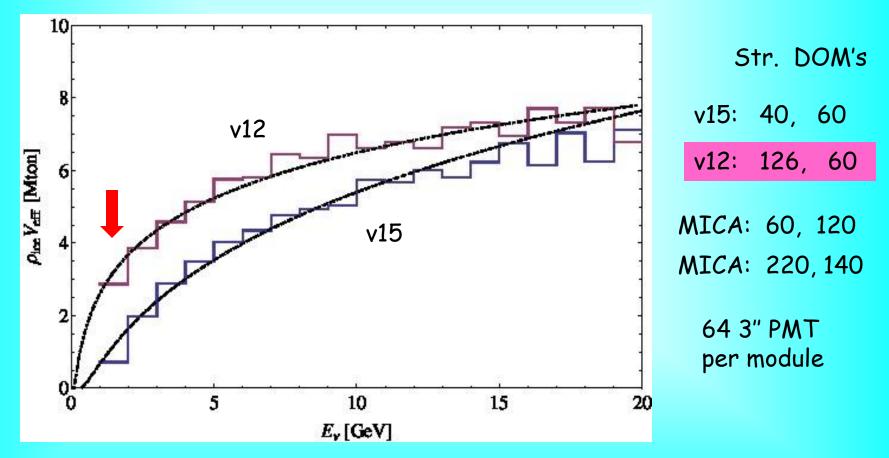
S. Razzaque, A.Y.S. arXiv: 1406.1407 hep-ph







Effective volume



Lagre volume at small energies

Distinguishability and CP-difference

Quick estimator (metric) of discovery potential

E. Kh. Akhmedov, S. Razzaque, A. Y. S. arXiv: 1205.7071

For each ij- bin relative CP-difference

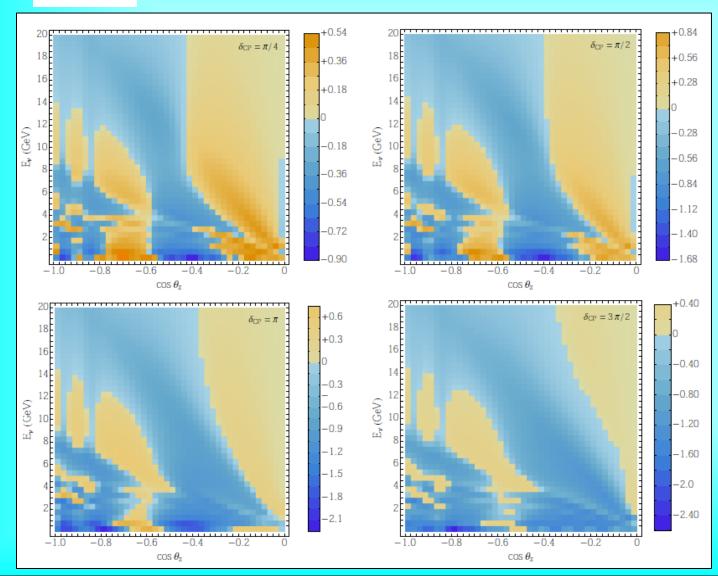
ce
$$S_{ij} = \frac{N_{ij}^{\delta} - N_{ij}^{\delta=0}}{\sqrt{N_{ij}^{\delta=0}}}$$

no fluctuations

If is true value $\rightarrow N_{ij}^{\delta}$ corresponds to ``true" value of events $\rightarrow N_{ij}^{\delta=0}$ ``measured" number of events

$$S^{tot} = [\Sigma_{ij} S_{ij}^2]^{1/2}$$

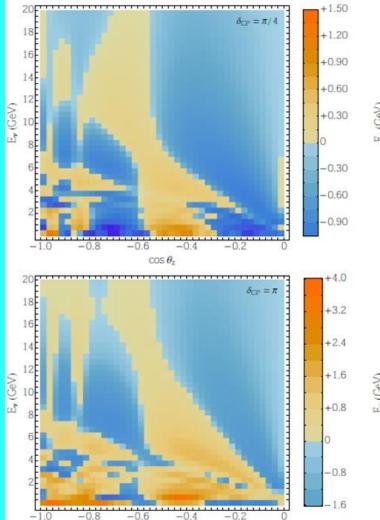
Relative CP differences tracks



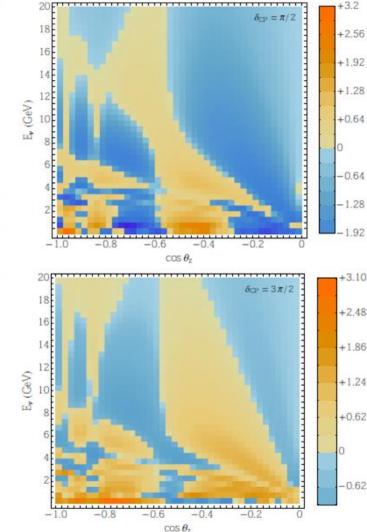
S-distributions for different values of δ

Normal mass hierarchy

Relative CP differences

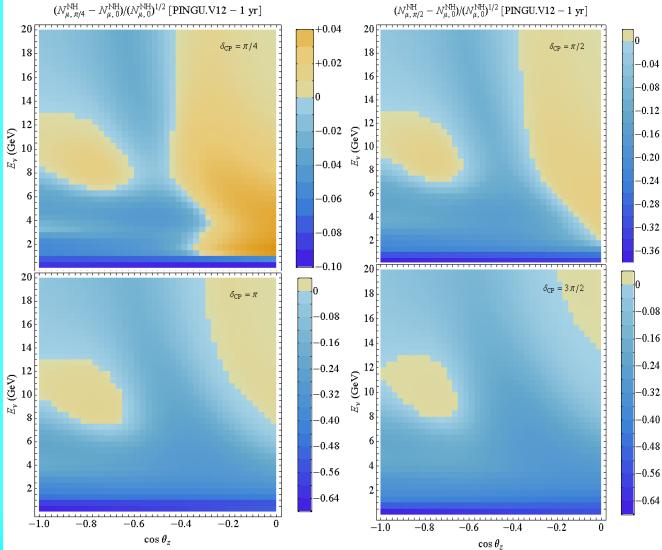


 $\cos \theta_{\tau}$

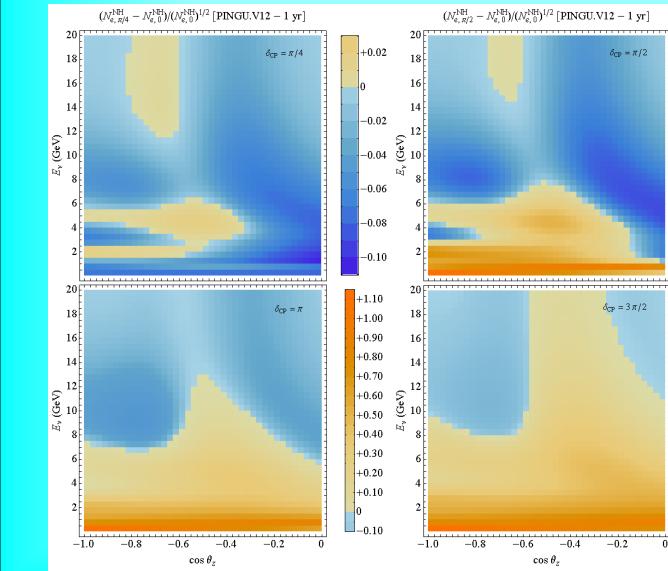


S-distributions for different values of δ









Normal mass hierarchy

+0.24

+0.20

+0.16

+0.12

+0.08

+0.04

-0.04

-0.08

+0.90

+0.80

+0.70

+0.60

+0.50

+0.40

+0.30

+0.20

+0.10

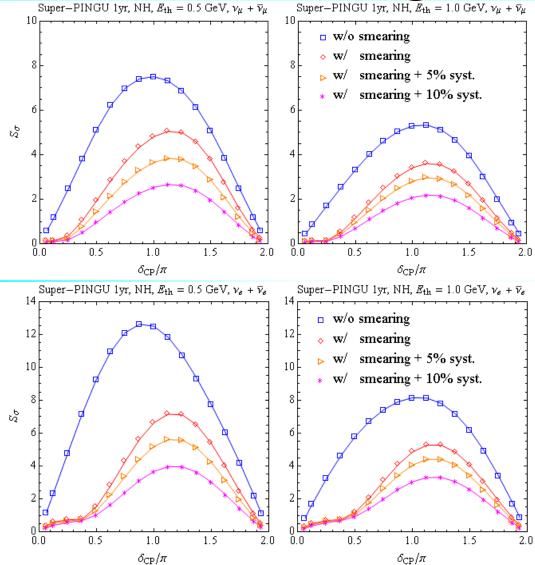
-0.10

0

0

Solar magic line and interference phase lines determine structure





δ from O

Results and further improvements

Super-PINGU with 0.5 GeV threshold

After 4 years of operation with 5% uncorrelated error

Distinguishability

 π /2 from 0 S_{tot} = 4 - 5 (depending on contribution of cascades)

 π from 0 S_{tot} = 6 - 7

Lowering threshold $1.0 \rightarrow 0.5 \text{ GeV}$ 50 % improvement

Further improvements:

```
Lowering threshold 0.5 \rightarrow 0.2 GeV 20 - 30 % improvement
```

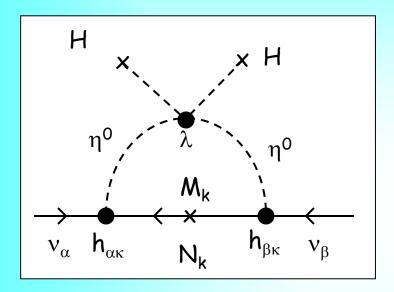
Partial separation of the neutrino and antineutrino events

Larger effective volume at ~ 1 GeV denser DOMs array

Better flavor identification

Increase of exposure time

One loop mechanism



If H gives mass to charged leptons leptons E. Ma, hep-ph 0601225

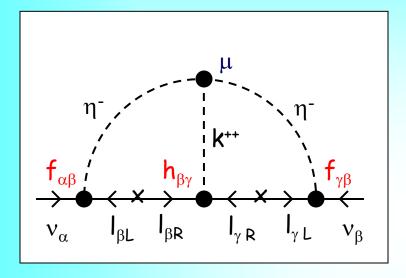
No RH neutrinos new higgs doblet (η⁺ , η⁰) and fermionic singlets N_k

odd under discrete symmetry Z₂ SM particles are Z₂ even η⁰ has zero VEV

If η^0 or is exact η^0 or lightest $\,N_k\,\,$ are stable and can be Dark Matter particles

Neutrino mass - DM connection

Zee-Babu mechanism



Features:

- the lightest neutrino mass is zero
- neutrino data require inverted hierarchy of couplings h

 $m_v \sim 8 \mu f m_l h m_l f I$

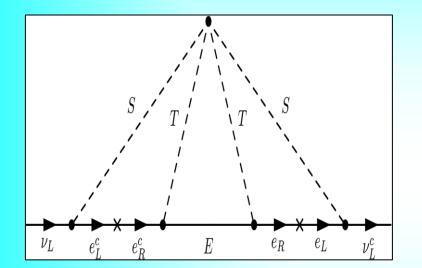
$$m_l = diag (m_e, m_\mu, m_\tau)$$

f and h are matrices of the couplings in the flavor basis

Testable:

- new charged bosons
- decays $\mu \rightarrow \gamma e$, $\tau \rightarrow 3 \mu$ within reach of the forthcoming experiments





A. Ahriche et al, 1404.2696 hep-ph

Z₂ symmetry

S~(1,1,2) and T~(1,3,2) are scalars E~(1,3,0) is a fermionic triplet. There are three distinct diagrams with the sets {T+, E^0 ,T-}, {T⁺,(E+)c,T0} and {T0,E+,T--} propagating in the inner loop.

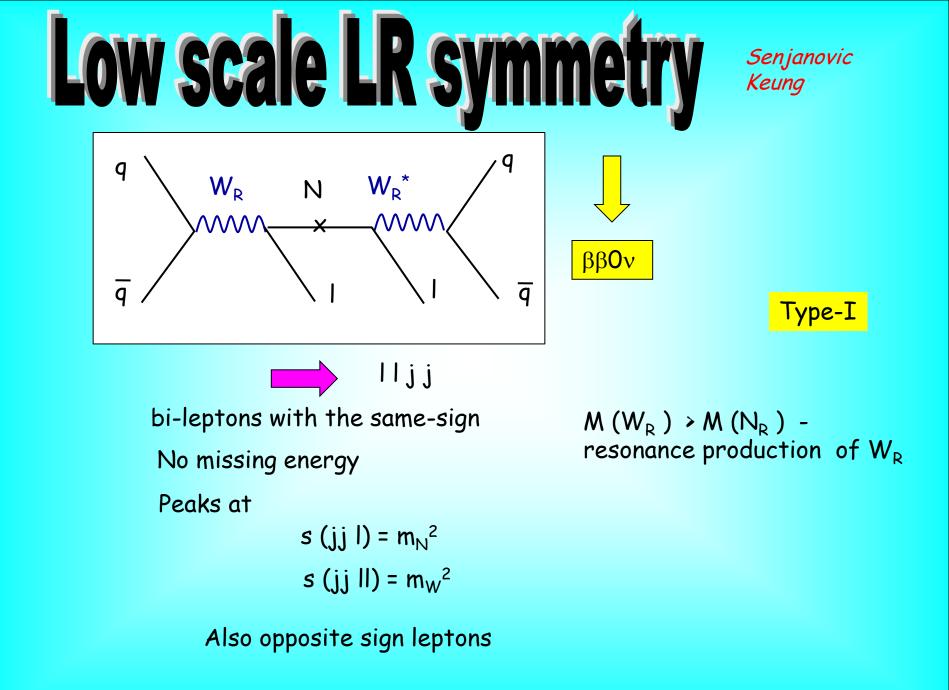
Low scale L-R symmetry

$SU(2)_L \times SU(2)_R \times U(1)$

M (W_R) ~ few TeV M (N_R) ~ 0.5 - few TeV low scale restoration of the L-R symmetry

Several contributions to light neutrino mass

See-saw type I with small Yukawa couplings Higgs Triplet mechanism



Two loop mechanism

ν ν

K S Babu, E Ma

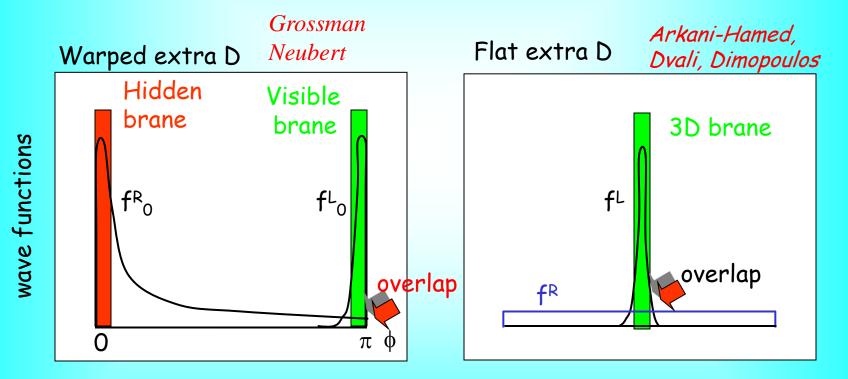
If usual neutrinos mix with heavy Majorana lepton N

 4^{th} generation of fermions \rightarrow main contribution

Overlap in extra dimensions

Right handed components are localized differently in extra dimensions

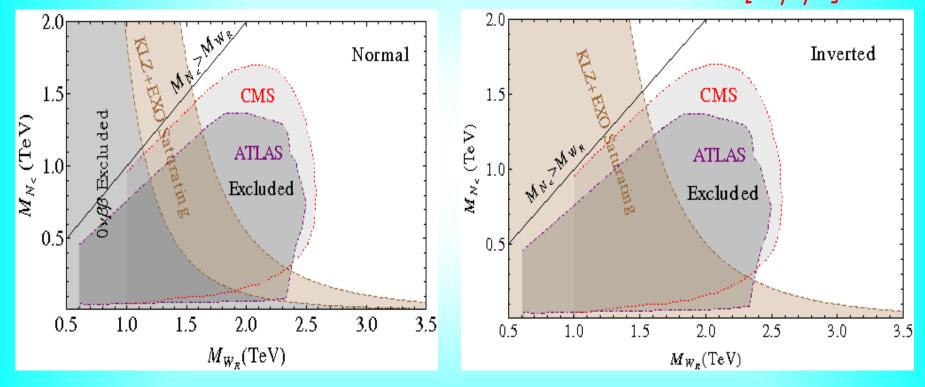
small Dirac masses due to overlap suppression:



 $m \varepsilon f^{L} f^{R} + h. c.$

amount of overlap in extra D

Bounds on L-R model P.5 Bhupal Dev, et al, 1305.0056 [hep-ph]





Origins: KK states of extra dimensions

B. Feldstein, W. Klemm arXiv: 1111.6690

Statistical distribution of Yukawa couplings y

$$\rho(y) \sim \frac{1}{y^{1+\delta}}$$
 $Y = 10^{-6} - 1$ reasonable fit
for charged leptons

M = 10¹⁴ - 10¹⁶ GeV Linear scale Seesaw type I

Random simulations, random phases

Anarchical spectrum



As zero order structure

$$U_{bm} = U_{23}^{m} U_{12}^{m}$$
Two maximal rotations
F. Vissani V. Barger et al

$$U_{bm} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

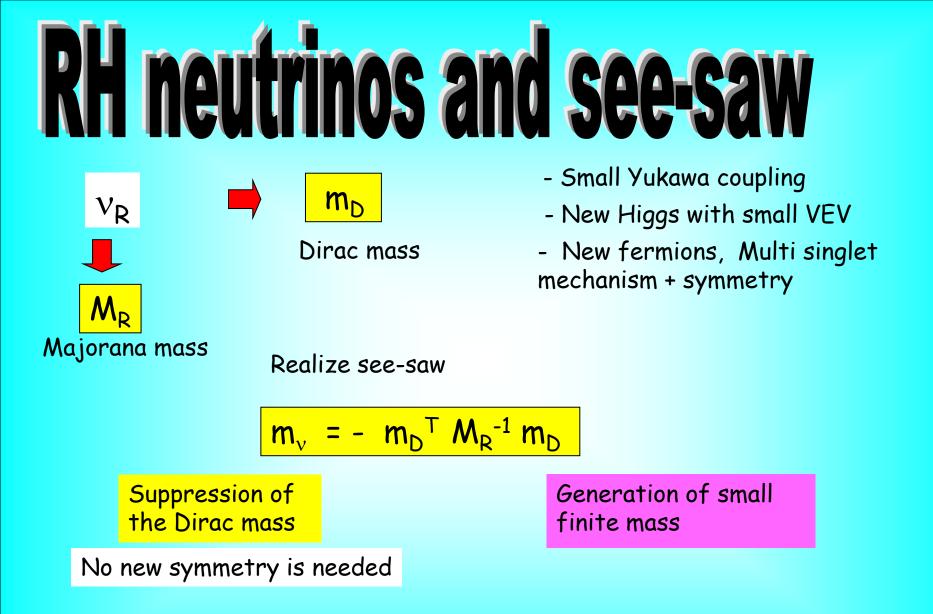
- maximal 2-3 mixing
- zero 1-3 mixing
- maximal 1-2 mixing
- no CP-violation

Scenario:

In the lowest approximation:

$$V_{quarks} = I$$
, $U_{leptons} = V_{bm}$
 $m_1 = m_2 = 0$

- mass splitting
- CKM and
- deviation from bi-maximal



Seesaw – only suppression, e.g. if the RH masses are at Planck scale. The dominant contribution – from another mechanism



There is no convincing model/explanation of the value of phase in quark sector

Can we predict the phase in lepton sector where situation is more complicated due to additional elements producing smallness of neutrino mass?

At least compare with the phase in quark sector

Can the phases be equal? sin $\delta_{\rm l} \sim \sin \delta_{\rm q}$

Connected in some way?

Can phases be unrelated to masses or even mixing?

```
Take special value like \pi/2?
```

Recall Global fit: $\delta~$ ~ 1.39 π

Parametrization

and notations

Majorana phases

 $\mathbf{U}_{\mathsf{PMNS}}^{\mathsf{std}} = \mathbf{U}_{23}\mathbf{I}_{\delta}\mathbf{U}_{13}\mathbf{I}_{-\delta}\mathbf{U}_{12}$

In general

 $U_{PMNS} = D(\phi) U_{PMNS}^{std} D(\beta)$

D - diagonal matrices with phase factors

Parametrizing U_L and U_X in similar way we have

 $U_{PMNS}^{std} = D(\gamma) U_{L}^{std} (\delta_{q}) D(\alpha) U_{X}^{std} (\delta_{x}) D(\eta)$

 $D(\alpha) = diag(e^{i\alpha_e}, e^{i\alpha_{\mu}}, e^{i\alpha_{\tau}})$ phases from the RH sector

 $D(\beta) = diag(e^{i\beta_1}, e^{i\beta_2}, 1)$

 γ and η should be fixed by standard parametrization conditions Arg {U_{e1}} = Arg {U_{e2}} = Arg {U_{µ3}} = Arg {U_{τ3}} = 0 $|U_{e1}|$ Im $U_{µ2} = |U_{e2}|$ Im $U_{µ1}$

Symmetry group relation

Transformations should be taken in the basis where CC are diagonal

$$(S_{iU} T)^{p} = I$$
 $(S_{iU} T)^{p} = (W_{iU})^{p} = I$

In flavor basis

Explicitly

$$(U_{PMNS} S_i U_{PMNS} T)^p = I$$

D. Hernandez, <mark>A.S.</mark> 1204.0445

The main relation: connects the mixing matrix and generating elements of the group in the mass basis

Equivalent to

$$\begin{array}{l} \text{Tr}\left(U_{\text{PMNS}}S_{i}U_{\text{PMNS}}^{+}T\right) = a \\ a = \sum_{j}\lambda_{j} \\ \lambda_{j}^{\text{P}} = 1 \quad j = 1,2,3 \end{array} \\ \begin{array}{l} \text{Tr}\left(W_{iU}\right) = a \\ \lambda_{j} - \text{three eigenvalues of } W_{iU} \end{array}$$



$$V_{ub} = \frac{1}{2} V_{us} V_{cb}$$

 $\sin\theta_{13} \sim \frac{1}{2} \sin\theta_{12} \sin\theta_{23}$

The same coupling strength between generations Similar Ansatz for structure of mass matrices

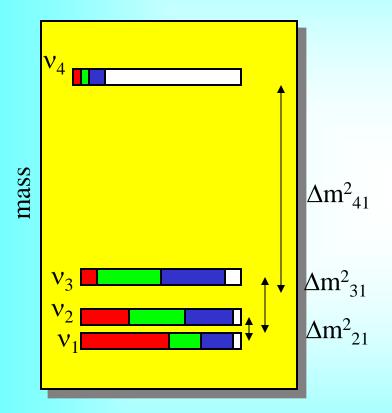
Fritzsch Anzatz similar to quark sector, RH neutrinos with equal masses



Normal mass hierarhy, Right value of 13 mixing

Relations between masses and mixing Flavor ordering in mass matrix





LSND/MiniBooNE: vacuum oscillations

$$P \sim 4|U_{e4}|^2|U_{\mu4}|^2$$

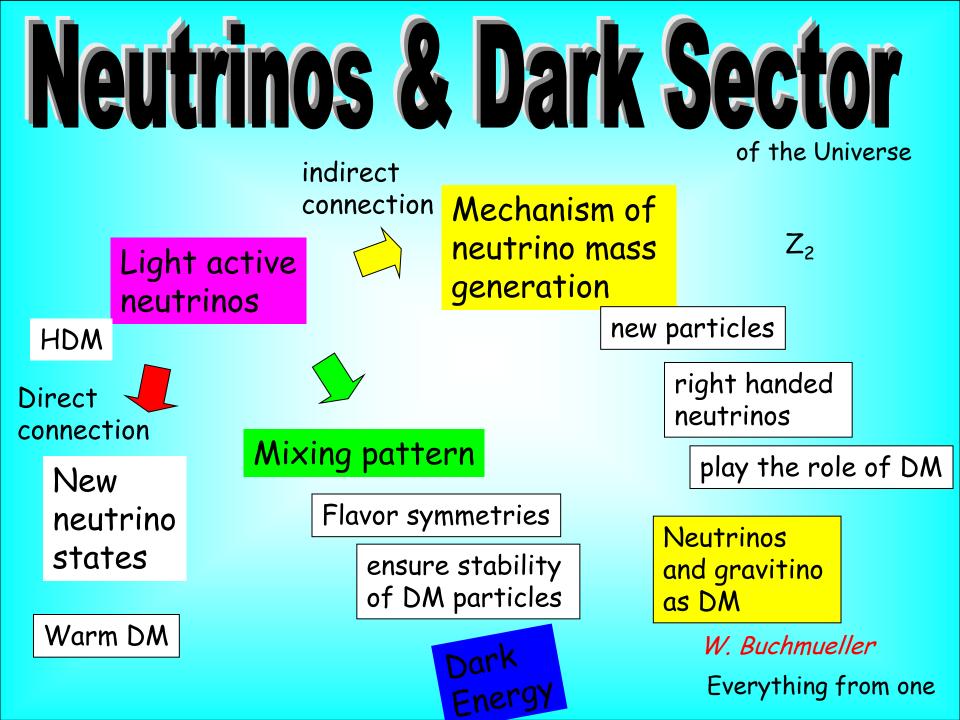
restricted by short baseline exp. BUGEY, CHOOZ, CDHS, NOMAD

For reactor and source experiments $P \sim 4|U_{e4}|^2(1 - |U_{e4}|^2)$

additional radiation in the universe
 bound from LSS?

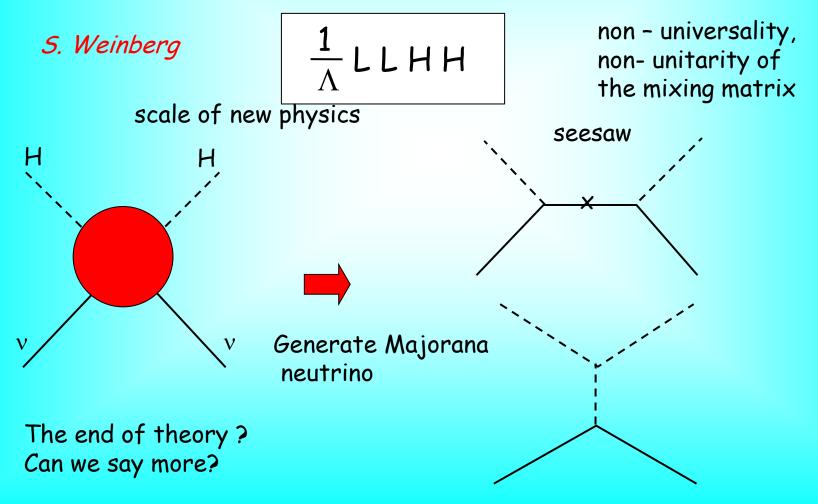
 Δm_{41}^2 = 1.78 eV² (0.89 eV²) U_{e4} = 0.15 U_{µ4} = 0.23

With new reactor data:



SM and Weinberg operator

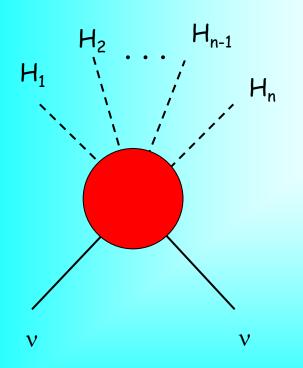
If no new particles at the EW scale, after decoupling of heavy degrees of freedom \rightarrow set of non-renormalizable operators





D=5 operator can be suppressed by symmetry

 $H \rightarrow i H$



$$\frac{1}{\Lambda^{n-1}}$$
 LLHⁿ

allows to reduce the scale of new physics responsible for neutrino mass generation

$$m_{v} = \frac{\langle H \rangle^{n}}{\Lambda^{n-1}}$$

$$m_{v} = \frac{1}{\Lambda^{n-1}} \prod_{i=1...n} \langle H \rangle^{n-1}$$

m

Leptonic CP from CKN

where $o = (S_{13}^q / S_{13}) C_{23} \sin o_q$

Assume $U_L \sim V_{CKM}^* (\delta_q)$ is the only source of CP violation -as in the quark sector, the LH rotation that diagonalizes the Dirac mass matrix U_X is real

framework

ed

Effective field theory approach