Depolarization at CLIC (IP)

Tony Hartin

- ✗ Need depolarization from upstream polarimeter through to IP collisions – so BDS and IP depol
- ✗ Vanilla CAIN/GP results for IP depolarization are uncertain due to theoretical corrections
- ✗ Calculation of beamstrahlung with kinematic approximations
- ✗ Higher order beam-beam effects, estimation of theoretical uncertainty in depolarization
- ✗ IP Depolarization + uncertainty for CLIC 3TeV

The 'usual' IP depolarization

There is depolarization (spin flip) due to the QED process of Beamsstrahlung, given by the Sokolov-Ternov equation

$$\frac{dW}{d\omega_f} = \frac{\alpha m}{\sqrt{3}\pi\gamma^2}\int_z^\infty K_{5/3}(z)\,dz + \frac{y^2}{1-y}K_{2/3}(z)$$

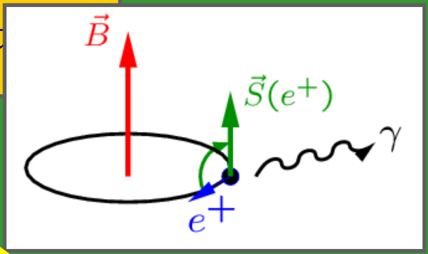
$$\text{where}\quad z=\frac{2}{3\Upsilon}\,\frac{y}{y-1}\ ,\quad y=\frac{\omega_f}{\epsilon_i}$$

The fermion spin can also precess in the bunch fields. Equation of motion of the spin given by

$$\frac{d\vec{S}}{dt} = -\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T+(a+1)\vec{B}_L-\gamma\left(a+\frac{1}{\gamma+1}\right)\frac{1}{c^2}\vec{v}\times\vec{E}\right]\times\vec{S}$$

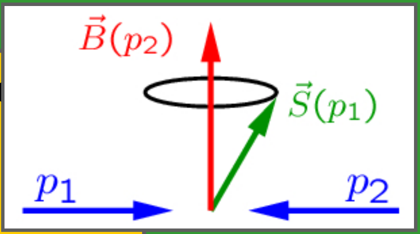
At the IP, the anomalous magnetic moment subject to radiative corrections in the presence of the bunch field

Stochastic spin diffusion from photon emission: Sokolov-Ternov effect



Parameter set	Depolarization ΔP_{lw}		
	T-BMT	S-T	total
Nominal	0.08%	0.02%	0.10%
low Q	0.04%	0.02%	0.06%
large Y	0.17%	0.02%	0.19%
low P	0.15%	0.09%	0.24%
TESLA	0.11%	0.03%	0.14%

Classical spin prec in inhomogeneous external fields: T-BMT equation.



Depolarization at the ILC IP

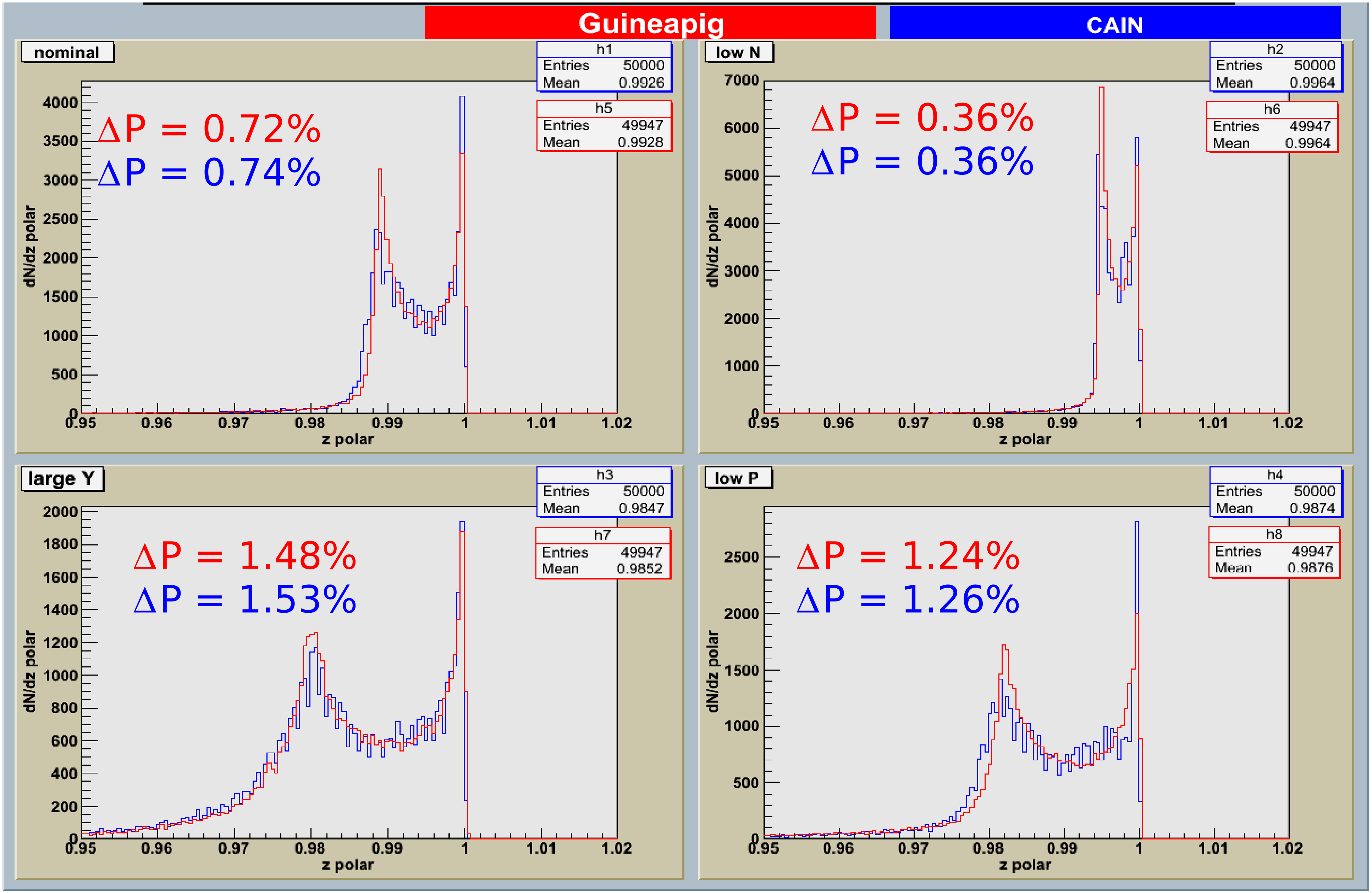
- We need an accurate value (to 0.1%) of the luminosity-weighted depolarization
- 2 'usual' sources T-BMT spin precession and Sokolov-Ternov spin-flip
- Uncertainty comes from neglect of theoretical refinements
- From Yokoya & Chen paper, ΔP_{lw} ~$0.273\Delta P$ for T-BMT process
- LAL(GP++) uses $\Delta P = 1 - \mathrm{sqrt}(\langle Sz1 * Sz2 \rangle)$, Sz being the statistical population polarized +1 per macro-particles

Lumi Depolarization in GP++[CAIN]	Nominal	LowN	LargeY	LowP
T-BMT only	0.17 [0.17]	0.08	0.41	0.28
T-BMT+spin-flip	0.22 [0.22]	0.12	0.46	0.41
$\Delta P_{lw}/\Delta P$ for T-BMT	0.270	0.276	0.295	0.269

- Uncertainty comes from neglect of theoretical refinements
  - Eg: S-T assumes classical dynamics of electron and no radiation angle
  - HO Corrections to anomalous magnetic moment -> T-BMT
  - Higher order intense field QED processes

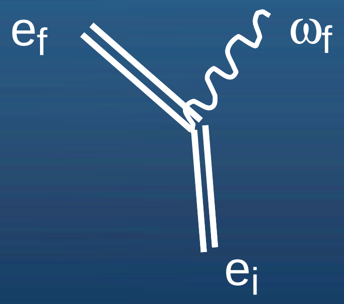
Comparison of CAIN & GP++ total depolarization for e⁻ after beam-beam interaction: $\Delta P = 1 - \langle P \rangle$

(P. Bambade, F. Blampuy (summer intern), G. Le Meur, C. Rimbault – LAL Orsay)



Generalization of Sokolov-Ternov

- ✗ Generally beam field is constant crossed electromagnetic field
- ✗ Use exact solutions of Dirac equation in the bunch field and include them at Lagrangian level
- ✗ Check agreement of full result with S-T in suitable limit



Solution of Dirac equation in Beam field A^e

$$\left[(p-eA^e)^2-m^2-\frac{ie}{2}F^e_{\mu\nu}\sigma^{\mu\nu}\right]\psi_V(x,p)=0$$

$$\psi_V(x,p)=u_s(p)F(\phi)$$

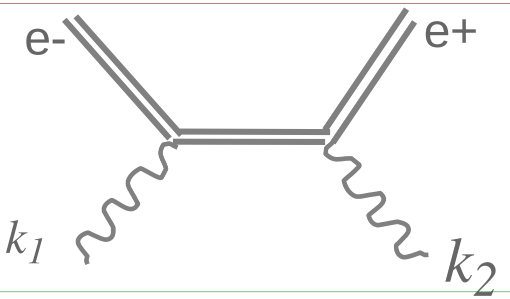
Substitution of the general solution for ψ_V yields a first order differential equation. whose solution can be expanded in powers of k, A^e

$$\psi_V(x,p)=\left[1+\frac{e}{2(kp)}\gamma^\mu k_\mu \gamma^\nu A^e_\nu\right]\exp\left[F(k,A^e)\right]\,e^{-ipx}u_s(p)$$

- make Fourier transform to get exponential of linear term in x

- n external field photons contribute
- Fermion momentum gains $\frac{\nu^2}{kp}k$
- Leads to fermion mass shift $m^2+\nu^2$
- F_2 are
  - Bessel functions for circular polarized A^e
  - Airy functions for constant crossed A^e

Usual solution in the absence of A^e



fermion solutions represented by double straight lines

Beamstrahlung in an external field (Sok-Ter) – Nikishov & Ritus (1964)

Calculation first performed in a linearly polarized field

$A_\mu = a_\mu \cos(k.x)$

Volkov solutions introduce complicated functions B (l external field photons)

$$B_n(l,\alpha,\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos^n k.x\, e^{f(k.x)} \quad \text{where} \quad f(k.x) = i\alpha\sin(k.x) - i\beta\sin(2k.x) - il(k.x)$$

External field strength expressed by dimensionless parameter ν which has a direct relationship to field potential or strength and an inverse relationship to the field frequency ω

$$\nu = \frac{ea}{m} \propto \frac{B}{\omega}$$

Constant field calculation performed for $\nu \to \infty$ $(\omega \to 0)$

Saddle point approximation used to write B_n as a function of Airy functions and the phase ψ of the slowly alternating external field

$$B_n \propto \frac{1}{\nu\sin\psi}\frac{Ai(y)}{\sqrt{y}} \quad \text{where} \quad y = \left(\frac{\nu}{\sin\psi}\right)^{2/3}$$

other approximations also made

Transformation to constant crossed field using solutions of a Schlömilch eqn

$$\text{if} \quad W(B) = \frac{2}{\pi}\int_0^{\pi/2} F(B\sin\psi)\,d\psi \quad \text{then} \quad F(B) = W(0) + B\int_0^{\pi/2} W'(B\sin\psi)\,d\psi$$

Clearly it would be better to do the calculation directly in the constant field, for arbitrary n and without approximations – work in progress

Beamstrahlung in the bunch field (no kinematic approximations)

Without going into details of the calculation the final results of the differential transition rate can be compared

Sokolov-Ternov

$$\frac{dW}{d\omega_f} = \frac{\alpha m}{\sqrt{3}\pi\gamma^2}\int_z^\infty K_{5/3}(z)\,dz + \frac{y^2}{1-y}K_{2/3}(z) \quad where \quad z=\frac{2}{3\Upsilon}\frac{y}{1-y} \quad , \quad y=\frac{\omega_f}{\epsilon_i}$$

Full calculation

$$\frac{dW}{du}(1+u)^2 = \frac{\alpha m}{\sqrt{3}\pi\gamma^2}\int_z^\infty K_{5/3}(z)\,dz + \frac{y^2}{1-y}K_{2/3}(z) \quad where \quad z=\frac{2}{3\Upsilon}\frac{y}{1-y} \quad , \quad y=\frac{\omega_f(1-\cos\theta_f)}{\epsilon_i-|\vec{p}_i|\cos\theta_i-\omega_f(1-\cos\theta_f)}$$

In the limit of ultra-relativistic e+/e-, $\epsilon_i \approx |\vec{p}_i|$, $\cos\theta_i \approx \cos\theta_f$ and the full calculation reduces to the Sokolov-Ternov equation

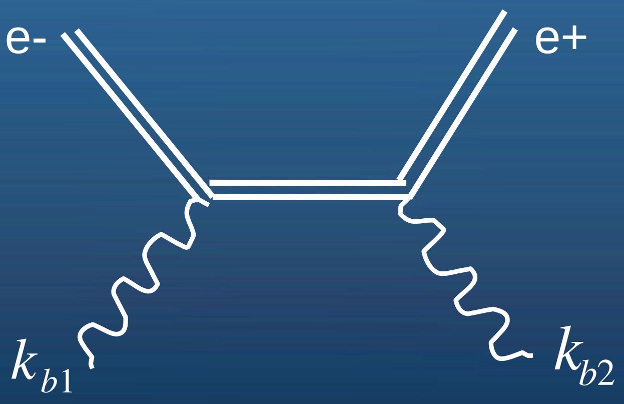
Spin flip rate has a similar dependence to that shown above

SENSITIVITY: whenever z is small i.e. when the radiated photon has low energy and significant angle

Numerical studies underway

Higher order effects

- ✗ Vertex correction leading to different anomalous magnetic moment
- ✗ Compton effect in the bunch fields
- ✗ Work in progress



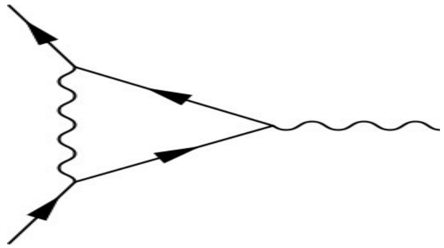
Anomalous magnetic moment in a strong field (IPPP - Durham)

Needed in T-BMT equation to calculate the rate of depolarization due to beam-beam effect

$$\vec{\Omega}=-\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T+(a+1)\vec{B}_L-\gamma\left(a+\frac{1}{\gamma+1}\right)\frac{\beta}{c}\vec{e}_v\times\vec{E}\right]$$

Main contrib^n from vertex diagram

$$a=\frac{\alpha}{2\pi}+O(\alpha^2)$$



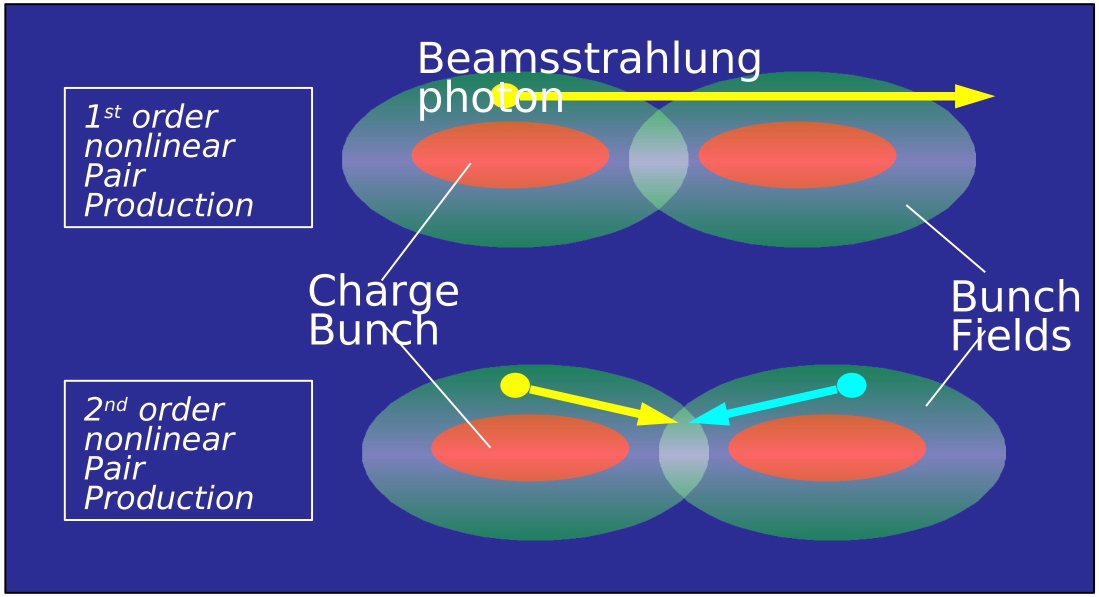
when fermion is embedded in a strong external field characterised by $\Upsilon=\nu^2\frac{(k.p)}{m^2}$ the anomalous magnetic moment develops a dependence on Υ and is given by (Baier-Katk)

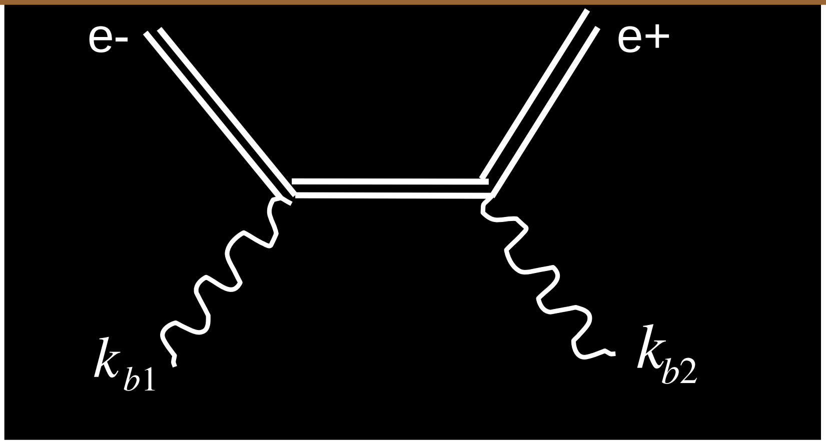
$$a(\Upsilon)=-\frac{\alpha}{\pi\Upsilon}\int_0^\infty\frac{x}{(1+x)^3}dx\int_0^\infty\sin\left[\frac{x}{\Upsilon}\left(t+\frac{1}{3}t^3\right)\right]dt$$

However…we can envisage

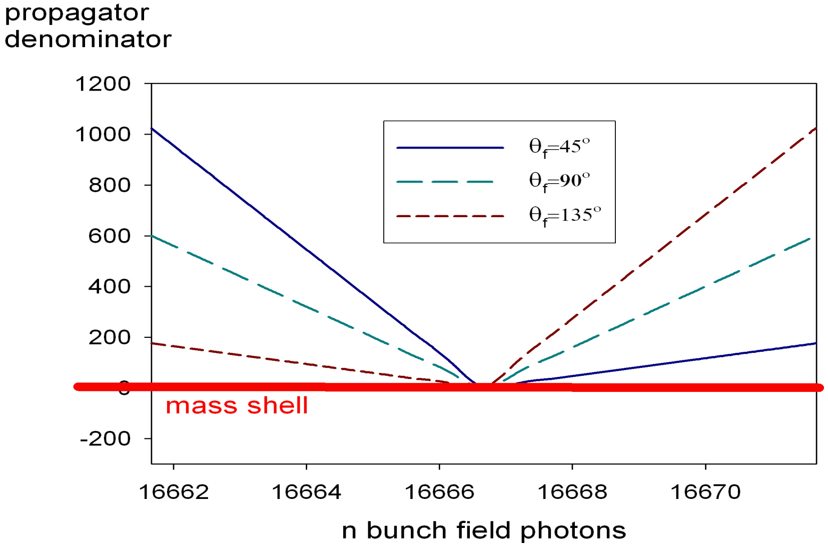
- recalculating the vertex diagram in BIP with Volkov solutions replacing all fermion lines
- Making mass correction (including self-energies)

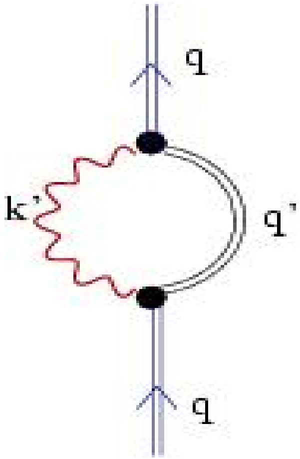
2nd order external field process: Coherent Breit-Wheeler (CBW) process





- 2nd order process contains twice as many Volkov E_p
- Double integrals over products of 4 Airy functions – mathematical challenge!
- spin structure same as ordinary Breit-Wheeler





fermions recieve a mass shift due to bunch field and the propagator can reach mass shell whenever $r\,\omega \sim \omega_b$

RESULTS - Depolarization at the IP

(I Bailey, A Hartin, G Moortat-PickEUROTeV-Report-2008-026)

	ILC baseline	CLIC-G
$\sqrt{s}$/GeV	500	3000
N $/10^{10}$	2	0.37
n_B	2625	312
β_x^*/mm	20	4
β_y^*/mm	0.4	0.09
σ_x^*/nm	640	40
σ_y^*/nm	5.7	$\sim$1
σ_z/μm	300	45
D_x	0.17	
Υ	0.048	
L/10^{34}cm^{-2}s^{-1}	2	2

Parameter set	Depolarization ΔP_{lw}		
	ILC 100/100	ILC 80/30	CLIC-G
T-BMT	0.17%	0.14%	0.10%
S-T	0.05%	0.03%	3.4%
incoherent	0.00%	0.00%	0.06%
coherent	0.00%	0.00%	1.3%
total	0.22%	0.17 %	4.8%

For CLIC (80%,60%),
Uncertainty estimated at 1.5%

- theoretical refinements will reduce uncertainty
  - S-T assumes classical dynamics of electron and no radiation angle
  - HO Corrections to anomalous magnetic moment -> T-BMT
  - Higher order intense field QED processes

Summary & Future work

(1) Full polarization treatment (Sok-Tern, T-BMT) and pair processes has been implemented in CAIN and Sok-Tern and T-BMT in GP++ - good agreement so far

(2) Depolarization for CLIC parameters **significant** and theoretical uncertainties need to be reduced

(3) Present Sokolov-Ternov equation assumes small Upsilon, but larger values (CLIC) require more exact calculation using Volkov solutions

(4) Previous Volkov solution calculations (1964) use several approximations – calculation with no approximations in progress

(5) T-BMT equation sensitive to anomalous magnetic moment calculation – use Volkov solutions

(6) Higher order IFQED processes being examined

UPCOMING WORKSHOP

Please come!