Intra-beam Scattering Studies for the CLIC damping Rings
Status and Plan

Michel Martini, CERN

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Injectors and Damping rings Working Group

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Plan for CLIC Damping Ring IBS studies

- The aim
  - Estimate the equilibrium beam emittance in the CLIC damping rings for intra-beam scattering (IBS) and radiation damping dominated emittances
- The issue
  - From the material collected in the IBS mini-workshop at Cockcroft Institute (Aug 07) classical and novel approaches for IBS and related computer codes have been reviewed
  - Research of schemes to compute the equilibrium phase space distribution in lepton storage rings in the case of strong IBS is a challenge for CLIC performance
    - » Conventional Gaussian-beam models, Fokker-Planck approach for arbitrary distributions, molecular dynamics method for particle-particle interaction ...
    - » No ready to use solution clearly exists to quantify the effect of IBS for non-Gaussian beams in the presence of radiation damping
    - » Development efforts on IBS in progress in theory and numerical tools

Plan for CLIC Damping Ring IBS studies

- CLIC damping ring and IBS considerations (in brief)
  - Beam parameters: energy 2.424 GeV, bunch population 4.1×10^9, max. extracted hor/ver & longitudinal normalized emittances 550/5 nm & 5000 eVm
  - Presently IBS growth times calculations are based on the modified Piwinski formalism
  - Numerical/analytical approach for effect of strong IBS yielding non-Gaussian tails with radiation damping not available so far (codes handling non-Gaussian beams exist but do not include the damping effect of wigglers

Plan for CLIC Damping Ring IBS studies

➢Toward a solution: explore other ways to solve the IBS problem

1. Theory & numerical tools: P. Zenkevich et al.

- ➢ Use existing codes, e.g. MOCAC "MOnte CArlo Code" for simulations
  - » "Kinetic effect in multiple intra-beam scattering", P. Zenkevich, O. Boine-Frankenheim, A. Bolshakov

2. Theory: C. Benedetti et al.

- ➢ Investigate for a stochastic-diffusion approach of IBS beyond the conventional models
- – "Time series analysis of Coulomb collisions in a beam dynamic simulation", C. Benedetti, G. Turchetti, A. Vivoli
  - » IBS theory can be based on the Landau collision integral yielding collision effects in a mean field framework as a stochastic process. Data obtained from integration of the equation of motion for a 2D-model of transverse beam dynamics are analyzed, and a suitable stochastic process is added to the mean field equations to describe the dynamics
- – "Collisional effects in high intensity beams", C. Benedetti, COULOMB'05
- – "Models of anomalous diffusion based on Continuous Time Random Walk", A. Vivoli, PhD thesis, 2006

Kinetic analysis of IBS (1) P. Zenkevich et al.

- FPE in coordinate-momentum space
  - The evolution of the beam distribution (coordinate-momentum space) induced by IBS (multiple small angle Coulomb scattering) is based on the solution of a FPE.
  - Introducing the beam distribution $\Phi(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{t})$, the friction and diffusion terms $F(\mathbf{r}, \mathbf{p}, t)$ and $D(\mathbf{r}, \mathbf{p}, t)$ (F and D are averaged over the field particles, denoted by ') and the Coulomb logarithm L_C, the FPE in 6D phase-space can be written as

$$\vec{r} = \begin{pmatrix} z - z_s \\ x \\ y \end{pmatrix} \quad \vec{p} = \begin{pmatrix} \gamma^{-1} \dfrac{\Delta p}{p} \\ x' = p_x / p \\ y' = p_y / p \end{pmatrix} \quad \frac{\partial \Phi}{\partial t} = -\sum_m \frac{\partial}{\partial p_m}(F_m \Phi) + \frac{1}{2} \sum_{m,m'} \frac{\partial^2}{\partial p_m \partial p_{m'}} (D_{m,m'} \Phi)$$

  - The friction force due to IBS and the diffusion coefficients can be cast into the form

$$\vec{F} = -\frac{2\pi c r_0^2}{\beta^3 \gamma^5} \int L_C(\vec{p}, \vec{p}') \frac{\vec{p} - \vec{p}'}{|\vec{p} - \vec{p}'|^3} \Phi(\vec{p}, \vec{p}', t) d^3 \vec{p}'$$

$$\overline{D}_{m,m'} = \frac{\pi c r_0^2}{\beta^2 \gamma^5} \int L_C(\vec{p}, \vec{p}') \frac{\delta_{m,m'} |\vec{p} - \vec{p}'|^2 - (p_m - p'_m)(p_{m'} - p'_{m'})}{|\vec{p} - \vec{p}'|^3} \Phi(\vec{p}, \vec{p}', t) d^3 \vec{p}'$$

Kinetic analysis of IBS (1) P. Zenkevich et al.

FPE in invariant space

- The 6 variables in the FPE in coordinate-momentum space can be reduced to 3 by reformulation in the space of invariants: energy (for the longitudinal motion) and Courant-Snyder invariants (for the transverse motion)
- Using the action-angle variables J_m, Ψ_m, components of the invariant and phase vectors, yielding the Courant-Snyder invariant :

$$J_m = \gamma_m \tilde{r}_m^2 + 2\alpha_m \tilde{r}_m \tilde{p}_m + \beta_m \tilde{p}_m^2$$

- Particle coordinate-momentum are expressed via the action-angle variables

$$\tilde{\vec{r}} = \begin{pmatrix} z - z_s \\ x - D_x \Delta p/p \\ y \end{pmatrix} \quad \tilde{\vec{p}} = \begin{pmatrix} \frac{1}{\gamma}\frac{\Delta p}{p} \\ x' - D'_x \Delta p/p \\ y' \end{pmatrix} \qquad \begin{aligned} \tilde{r}_m &= \sqrt{\beta_m J_m}\cos\psi_m \\ \tilde{p}_m &= -\frac{\alpha_m}{\beta_m}\tilde{r}_m - \sqrt{\frac{J_m}{\beta_m}}\sin\psi_m \end{aligned}$$

- $\alpha_{2,3}$, $\beta_{2,3}$, $\gamma_{2,3}$ are the Twiss parameters, $\alpha_1=0$, $\beta_1=1$; $\gamma_1=0$ for coasting beams and $\gamma_1=Q_s^2/\gamma^2\,[(\gamma^{-2} - \gamma_t^{-2})R]^2$ for bunched beams
- For uniform phase distributions over [0, 2π] the FPE can be written as

$$\frac{\partial \Phi}{\partial t} = -\sum_m \frac{\partial}{\partial J_m}(\tilde{F}_m \Phi) + \frac{1}{2}\sum_{m,m'} \frac{\partial^2}{\partial J_m \partial J_{m'}}(\tilde{D}_{m,m'}\Phi)$$

Kinetic analysis of IBS (1) P. Zenkevich et al.

Solution of the FPE

- The beam distribution Φ and the coefficients in the FPE depend on the 3 invariants J_m and t. The FPE coefficients can be expressed as follows, with friction and diffusion kernels K^F_m and $K^D_{m,m}$

$$\tilde{F}_m(\vec{J},t) = -\frac{2\pi c r_0^2}{\beta^3\gamma^5}\int \tilde{K}^F_m(\vec{J},\vec{J}')\Phi(\vec{J},t)d^3\vec{J}'$$

$$\tilde{D}_{m,m'}(\vec{J},t) = \frac{\pi c r_0^2}{\beta^3\gamma^5}\int \tilde{K}^D_{m,m'}(\vec{J},\vec{J}')\Phi(\vec{J},t)d^3\vec{J}'$$

- Classical grid based methods for the numerical solution of the FPE are too difficult to put into practice
- A convenient method to solve the FPE is to use the “Binary collisions” map model (**BCM**)
- An approximate model (**AM**) of the FPE was derived to reduce the macro-particle number presuming that most of the IBS interactions happen in the beam core
  - **AM** supposes (i) Gaussian beams, (ii) constant components of the diffusion coefficients and friction kernel, (iii) constant Coulomb log
  - The **AM** of the FPE is solved by means of the Langevin equation
  - The **AM** usually needs only ~10^2 to 10^3 macro-particles, instead of more than 10^4 macro-particles for the **BCM** model

Kinetic analysis of IBS (1) P. Zenkevich et al.

➢ Solution of the FPE with BCM multi-particle algorithm

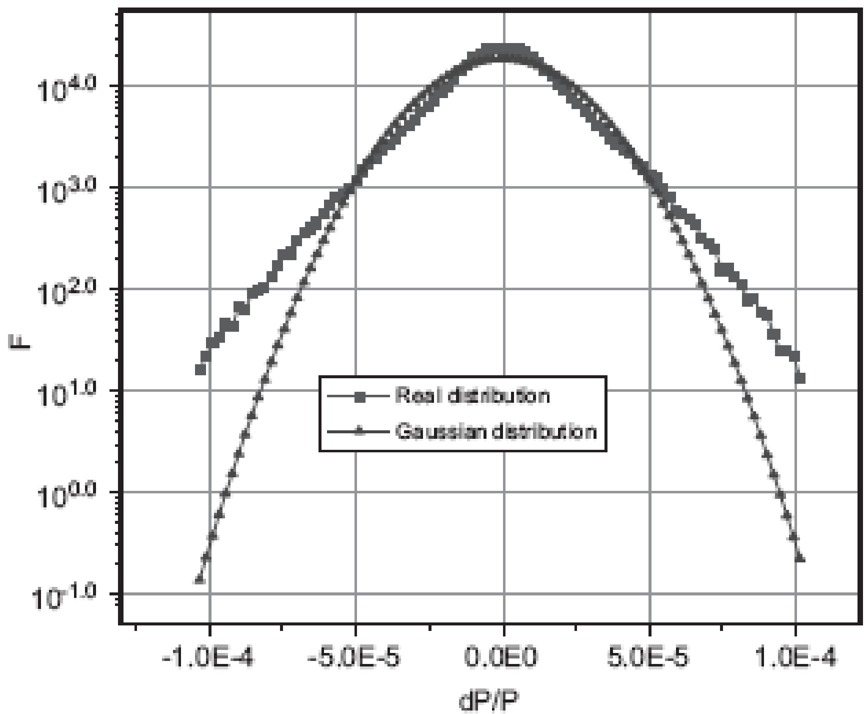
- **BCM** is implemented in the **MOCAC** code (MOnte-CArlo Code) for IBS simulations
  - » For two colliding macro-particles the "collision angle" $\Psi^{i,j}$ is computed and the momentum change of each interacting particle is derived (ρ_0 is the particle density)

$$\sin\left(\frac{\Psi^{i,j}}{2}\right) = \sqrt{\frac{2A\rho_0 L_C^{i,j}\Delta t}{N\left|\vec{p}^{\,i} - \vec{p}^{\,j}\right|^{3/2}}}$$

- The beam volume is divided into cells on a grid. The algorithm over time Δt is :
  - » Form an initial macro-particle set with random phases and compute the particle momenta and coordinates (the beam is characterized by a set of macro-particles with given invariants)
  - » Allocate particles in cells and link a particle to each particle in the cell
  - » Compute the collision map in each cell for each particle
  - » Derive the new invariant and check the boundary conditions
  - » Collect the final macro-particle set
- Besides IBS **MOCAC** includes further processes: electron cooling, target interactions … but **radiation damping is missing**

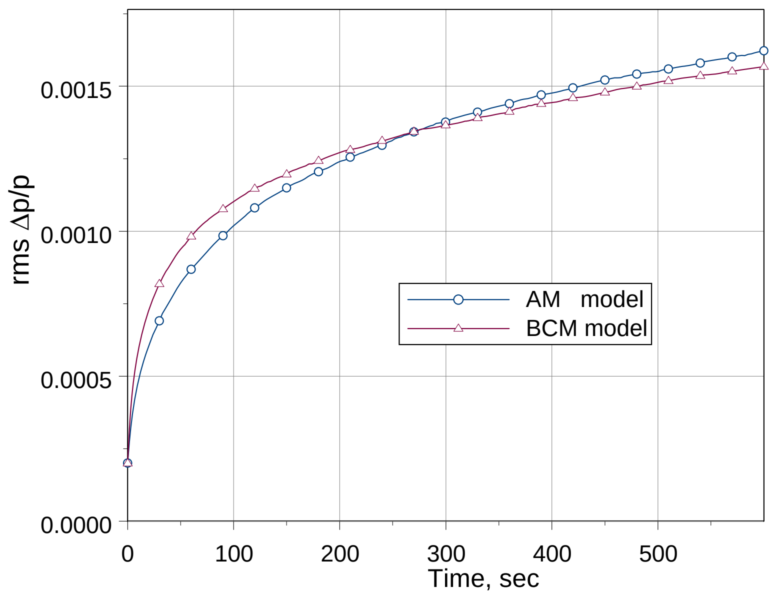
Kinetic analysis of IBS (1) P. Zenkevich et al.

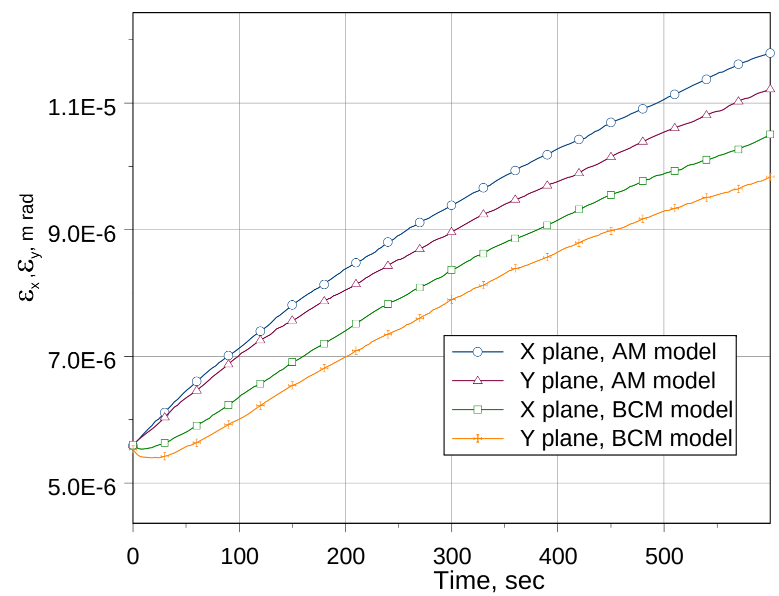
- Results of numerical IBS modeling for HESR ring (GSI)
  - Study of the formation of non-Gaussian beam tails
  - Beam momentum distribution computed using MOCAC in the presence of IBS, e^- cooling and beam target interaction dependence on equilibrium r.m.s. momentum spread. The tails appear to be mostly due to IBS



Kinetic analysis of IBS (1) P. Zenkevich et al.

- Results of numerical IBS modelling for TWAC storage ring (ITEP)
  - Time-evolution of the r.m.s. momentum spread and beam emittances for Al_{27}^{+13} coasting ion beams at 620 MeV/u (10^{12} ions)
  - Simulations parameters : 20000 macro-particles, 0.3 s time-step, 38 azimuthal points, 100 transverse cell (BCM model)
  - Approximate model (AM) results are very close to those obtained using the "binary collision" map (BCM)
  - AM results also match the results of the Bjorken-Mtingwa model





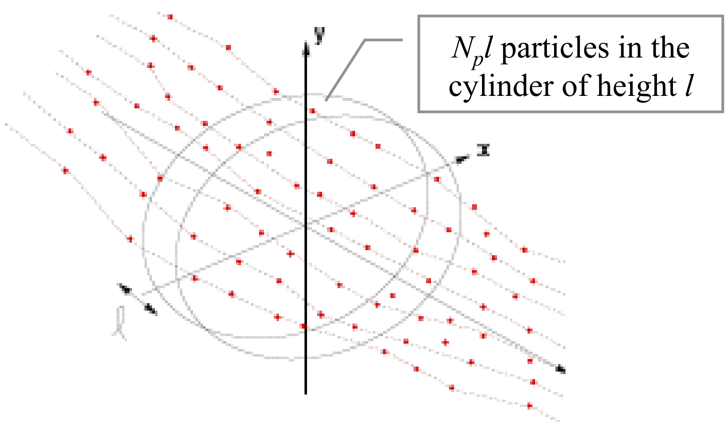
Kinetic analysis of IBS (2) C. Benedetti et al.

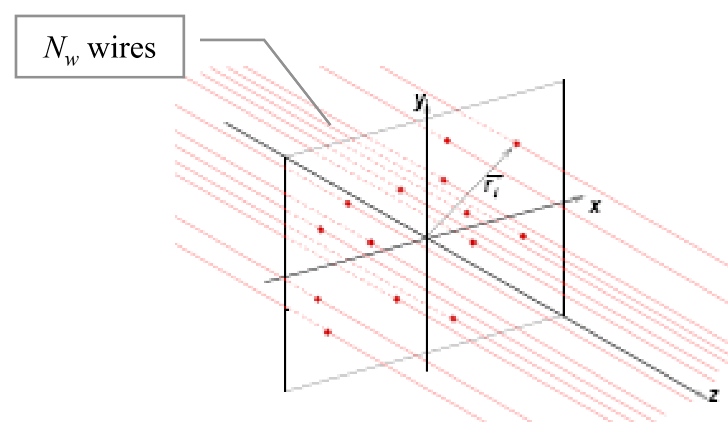
Coulomb oscillators (2D model)

- Coulomb interaction effects (**space charge**) significant for intense protons or ions beams at intermediate energies: **non-relativistic** energies are assumed
- **Very long bunches** are supposed to rotate in storage rings: **coasting beams** are assumed (2D model)
- Consider a coasting beam with N_p charged particles per unit length, R_B is the mean beam radius, the mean particle density is $\rho_B \sim N_p(\pi R_B)^{-1}$.
- Define $l \sim \rho_B^{-1/3}$ and associate a charged "wire" to each particle in a cylinder of radius R_B and height l, the number of wires N_w is

$$N_w = \pi^{1/3}\left(N_p R\right)^{2/3} \equiv N_p l$$

- $N_w \sim 10^6$ for $N_p \sim 10^{11}$ particles per unit length and $R_B \sim 10$ mm. Only $\sim 10^4$ "wires" can be simulated in practice, so **scaling laws** are needed to make the right extrapolations





Kinetic analysis of IBS (2) C. Benedetti et al.

Equation of motion

- The (non-relativistic) Hamiltonian describing the transverse dynamics of the oscillators system is, using the logarithmic potential

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2} + \omega_0^2 \vec{r}_i^{\,2}}{2} + \frac{\xi}{N_w} \sum_{1 \le i < j \le N_w}^{N_w} \log\left|\vec{r}_i - \vec{r}_j\right|$$

- $\boldsymbol{r}_i$, $\boldsymbol{p}_i$ are the position and momentum of the i^{th} "wire" (refer to as particles), ω_0 is the phase advance per meter, ξ the perveance, N_w the number of particles
- Changing N_w changes the collisionality level (**scaling laws**)

Landau's equation

- The collisions (IBS) can be introduced in a **mean field** framework and modeled as a random process as long as they are instantaneous, frequent and soft.
- In the mean field framework the evolution of a **collisionless** single particle ("wire") phase space distribution $\Phi(\boldsymbol{r}, \boldsymbol{p})$ (assumed to be continue) is defined by the Vlasov-Poisson equations

$$\frac{\partial \Phi}{\partial s} + \left[\Phi, H\right] = 0$$

$$\Delta U(\vec{r}) = -4\pi \int \Phi(\vec{r}, \vec{p}) d\vec{p} \;\; \text{where } U = -\iint \log\left|\vec{r} - \vec{r}\,'\right| \Phi(\vec{r}, \vec{p})\, d^2\vec{r}\,' d^2\vec{p}$$

Kinetic analysis of IBS (2) C. Benedetti et al.

- The (test) particle momentum change is

$$\Delta\mathbf{p} = -(\partial H/\partial\mathbf{r})\Delta s + \Delta_c\mathbf{p} \quad \text{(with } \Delta s \sim v_0 \Delta t\text{)},$$

- The first term is due to the **mean field**, the second to **collisions** (IBS) and is assumed to be a Wiener (i.e. Gaussian) stochastic process
- Hence, the evolution of the single (test) particle ("wire") phase space distribution $\Phi(\boldsymbol{r}, \boldsymbol{p})$ is the solution of the Vlasov-Poisson-Focker-Planck-Landau equation (VPFPL)

$$\frac{\partial\Phi}{\partial s} + [\Phi, H] = -\sum_{m=1}^{2} \frac{\partial}{\partial p_m}(F_m\Phi) + \frac{1}{2}\sum_{m,n=1}^{2} \frac{\partial^2}{\partial p_m \partial p_n}(D_{m,n}\Phi)$$

- $F(\mathbf{r}, \mathbf{p})$ and $D(\mathbf{r}, \mathbf{p})$ are the friction (or drift) and diffusion coefficients (averaged over the field particles)

$$F_m = \left\langle \frac{\Delta_c p_m}{\Delta s} \right\rangle \qquad D_{m,n} = \left\langle \frac{\Delta_c p_m \Delta_c p_n}{\Delta s} \right\rangle$$

- The friction term can be rewritten as

$$\vec{F} = -\frac{N_w}{2}\int d\vec{p}'\Phi(\vec{r}, \vec{p}')(\sigma^{(0)} - \sigma^{(1)})|\vec{p} - \vec{p}'|(\vec{p} - \vec{p}') \text{ with } \sigma^{(k)} = \int \frac{d\sigma}{d\theta}\cos^k\theta d\theta$$

- $d\sigma/d\theta$ is the cross section for a 2D binary collision between particles ("wires") and θ is the scattering angle

Kinetic analysis of IBS (2) C. Benedetti et al.

- Simulations
  A. Direct numerical integration of the Hamilton's equations of a 2D system of particles describing the transverse dynamics of the beam has been done
  B. Numerical simulations via the mean field equations with the addition of a Wiener process in order to model Coulomb collisions has been done
  C. The data of both simulations have been compared, analysing the found differences as a time series of a stochastic process describing the Coulomb collisions between the particles
  D. Replacement of the Wiener process in the mean field equations by a non-Gaussian stochastic process to model the Coulomb collisions has been investigated (A. Vivoli)

Further studies are needed to find a full theory of the stochastic process describing Coulomb collisions in more general cases

Appendix: workable non-Gaussian distributions for IBS ?

Beam phase-space distributions

- Gaussian model (Bjorken-Mtingwa, Piwinski)
  - Gaussian phase-space distribution p(x,x',y,y',δ,s) expressed in terms of transverse and longitudinal phase-space coordinates writes (with $\delta = \Delta p/p$, N particle number)

$$\mathrm{p}(x,x',y,y',\delta,s) = N\frac{\exp\left(-S^{(H)}(x,x') - S^{(V)}(y,y') - S^{(L)}(\delta,s)\right)}{\int \exp\left(-S^{(H)}(x,x') - S^{(V)}(y,y') - S^{(L)}(\delta,s)\right) dx\, dx'\, dy\, dy'\, d\delta\, ds}$$

$$\mathrm{S}^{(\mathrm{H})}(x,x') = \frac{1}{2\varepsilon_x}\left(\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2\right)$$

$$S^{(V)}(y,y') = \frac{1}{2\varepsilon_y}\left(\gamma_y x^2 + 2\alpha_y y y' + \beta_y x'^2\right)$$

$$S^{(L)}(\delta,s) = \frac{\delta^2}{\sigma_\delta^2} + \frac{(s-s_0)^2}{2\sigma_s^2} \quad \text{(bunched beam)}$$

$$\varepsilon_x = \frac{\sigma_x^2}{\beta_x} \quad \varepsilon_y = \frac{\sigma_y^2}{\beta_y} \quad \sigma_\delta = \frac{\sigma_\mathrm{p}}{\mathrm{p}_0}$$

- Non-Gaussian model
  - In the presence of non-Gaussian tail how would it be possible to substitute non-Gaussian to Gaussian distributions into the "classical" Gaussian model? (e.g. **L-stable distributions**, **quasi-polynomials distributions** …)

Appendix: workable non-Gaussian distributions for IBS ?

L-stable distributions

- Characteristic function $\Pi(t)$ (no analytical $p(x)$ in general)
  - L-stable laws $S(\alpha, \beta, \gamma, \mu)$ have parameters:
    - index tail $0<\alpha\leq 2$,
    - skewness $-1\leq\beta\leq 1$,
    - scale $\gamma>0$ (determines the width)
    - location μ (determines the peak)
  - $S(2, 0, \gamma, \mu)$ is a Gaussian law (with $\gamma=\sigma^2/2$, σ^2 is the variance)
  - $S(1, 0, \gamma, \mu)$ is a Cauchy law
  - For $\alpha<2$ the variance is infinite; for $\alpha>1$ the mean exists and is equal to μ

$$\log\Pi(t) = \begin{cases} i\mu t - \gamma|t|^{\alpha}\left(1 + i\beta(t/|t|)\tan(\pi\alpha/2)\right) & \alpha \neq 1 \\ i\mu t - \gamma|t|^{\alpha}\left(1 + 2i\beta(t/|t|)\log(t)/\pi\right) & \alpha = 1 \end{cases} \qquad \mathrm{p}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\Pi(t)\,dt$$

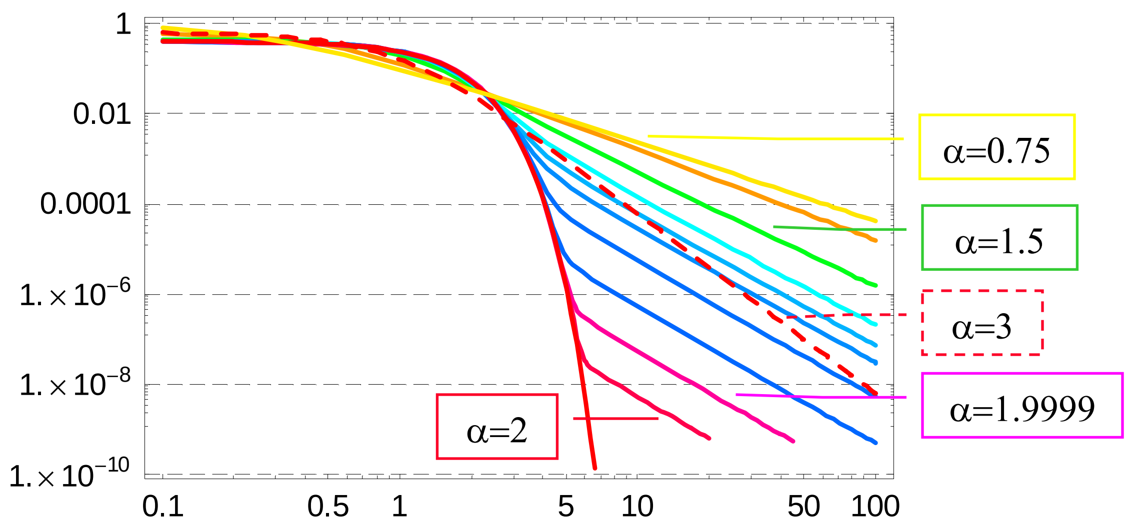
- Tail behavior
  - For $\alpha<2$ the tails converge toward a Pareto law, i.e.
  - $p(x)\sim x^{-1-\alpha}$ as $x\to\infty$, where $p(x)$ is the probability density function
  - $\mathrm{Prob}(X>x)=1-F(x)\sim x^{-\alpha}$ as $x\to\infty$, $F(x)$ being cumulative probability function of $p(x)$

Appendix: workable non-Gaussian distributions for IBS ?

– Plots
  - » Log-log plot of symmetric (β=0, μ=0) L-stable distribution functions p(x) for α=0.75, 1, 1.5, 1.8, 1.9, 1.95, 1.99, 1.999, 1.9999, 1.99999 and 2, with γ=1/2
  - » Pareto power tails are clearly visible for α<2. The Gaussian (α=2) tail decays as a parabola in the log-log plot
  - » Laws converging asymptotically toward Pareto laws with α>2 have fat tailed character but are not L-stable, e.g. $p_\sigma(x)$ with variance σ^2

$$p_\sigma(x) = \frac{2\sigma^3}{\pi(x^2+\sigma^2)^2}$$

  - » $p_\sigma(x)$ falls off as $p_\sigma(x) \sim x^{-4}$ as $x\to\infty$ yielding a tail index α=3 (red dotted line).



Appendix: workable non-Gaussian distributions for IBS ?

- Quasi-polynomials distributions (E. Métral, A. Verdier)
  - Distribution with heavier tail than the Gaussian
    - Let $p(J_x,J_y)$ a bivariate distribution of "invariant-space" variables J_x, J_y extending up to 6σ (e.g. for truncated beam distributions due to collimation)
    - Suitably choosing the parameters a, b, n, p (with n>15, p<15, and $b=18\sigma^2$) of $p(J_x,J_y)$ yields projected distributions $p_x(x)$ with fatter tails than a Gaussian (when approaching the cutoff point 6σ)
    - Tails of quasi-polynomials laws do not converge toward a Pareto law due to the truncated nature of the distributions

$$\mathrm{p}(J_x,J_y) = a\left(1-\frac{J_x+J_y}{b}\right)^n + d\left(1-\frac{J_x+J_y}{b}\right)^p$$

$$\mathrm{p}_x(x) = \frac{1}{9\pi(n-p)\sqrt{2b}}\left\{\begin{array}{l}\dfrac{(n+2)(n+3)(15-p)\left(2^{n+1}(n+1)!\right)^2}{(2n+3)!}\left(1-\dfrac{x^2}{2b}\right)^{n+\frac{3}{2}}\\ +\dfrac{(p+2)(p+3)(n-15)\left(2^{p+1}(p+1)!\right)^2}{(2p+3)!}\left(1-\dfrac{x^2}{2b}\right)^{p+\frac{3}{2}}\end{array}\right.$$

Appendix: workable non-Gaussian distributions for IBS ?

- Plots
  - Left: Quasi-polynomials law tails p(x) near 6σ for n=16 and p=1, 2, 3, 4, 5, 6, with σ=1. Close to the cutoff point 6σ the tails become slimmer than the Gaussian tail
  - Right: Semi-logarithm plot of the above distribution functions p(x)