

# Linear and non-Linear Correction using Turn-by-Turn BPM Data



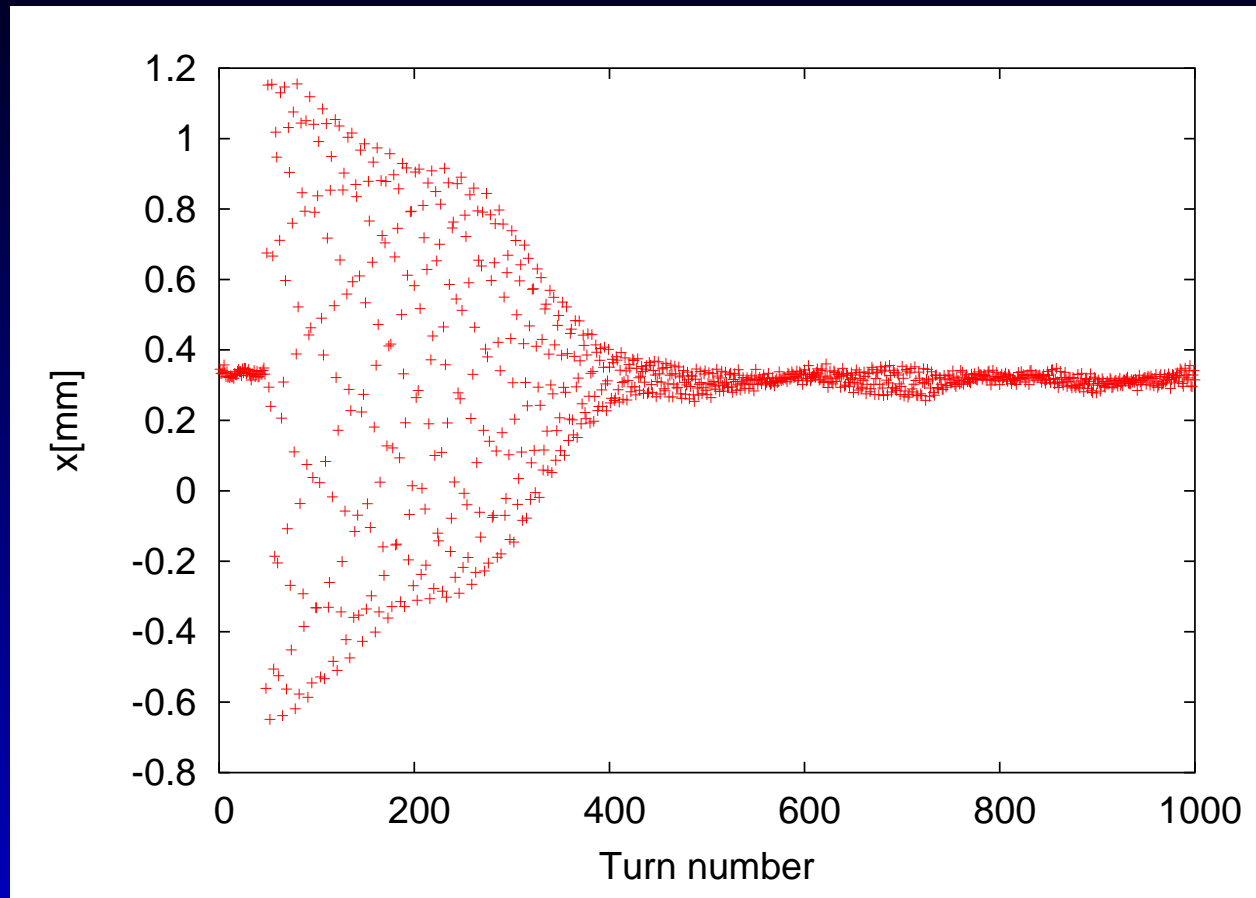
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Thanks to: M. Aiba, R. Bartolini, H. Braun,  
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# Contents

- Introduction and Math
- Examples of linear optics applications:
  - SPS optics correction
  - LHC first 90 turns
- Examples of non-linear dynamics:
  - SPS missing sextupole
  - ATFDR
  - DIAMOND experience by R. Bartolini
- Summary and Outlook

# Turn-by-turn BPM data



→ A single-turn kick excites betatron motion.

→ Filamentation damps the centroid oscillation.

# Measurement from BPM data

Momentum reconstruction from 2 BPMs:

$$p_{12}(N) = (x_1(N) + x_2(N) \sin \delta) / \cos \delta$$

Description of the motion:

$$x_1(N) - ip_{12}(N) = \sqrt{\beta_{x1}} \left\{ \sqrt{2I_x} e^{i(2\pi\nu_x N + \psi_{x1})} - 2i \sum_{jklm} j f_{jklm}^{(1)} (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} \times e^{i[(1-j+k)(2\pi\nu_x N + \psi_{x1}) + (m-l)(2\pi\nu_y N + \psi_{y1})]} \right\}$$

→  $\psi_{x1}$  and  $f_{jklm}^{(1)}$  can be inferred from the FFT

# What is $f_{jklm}$ ?

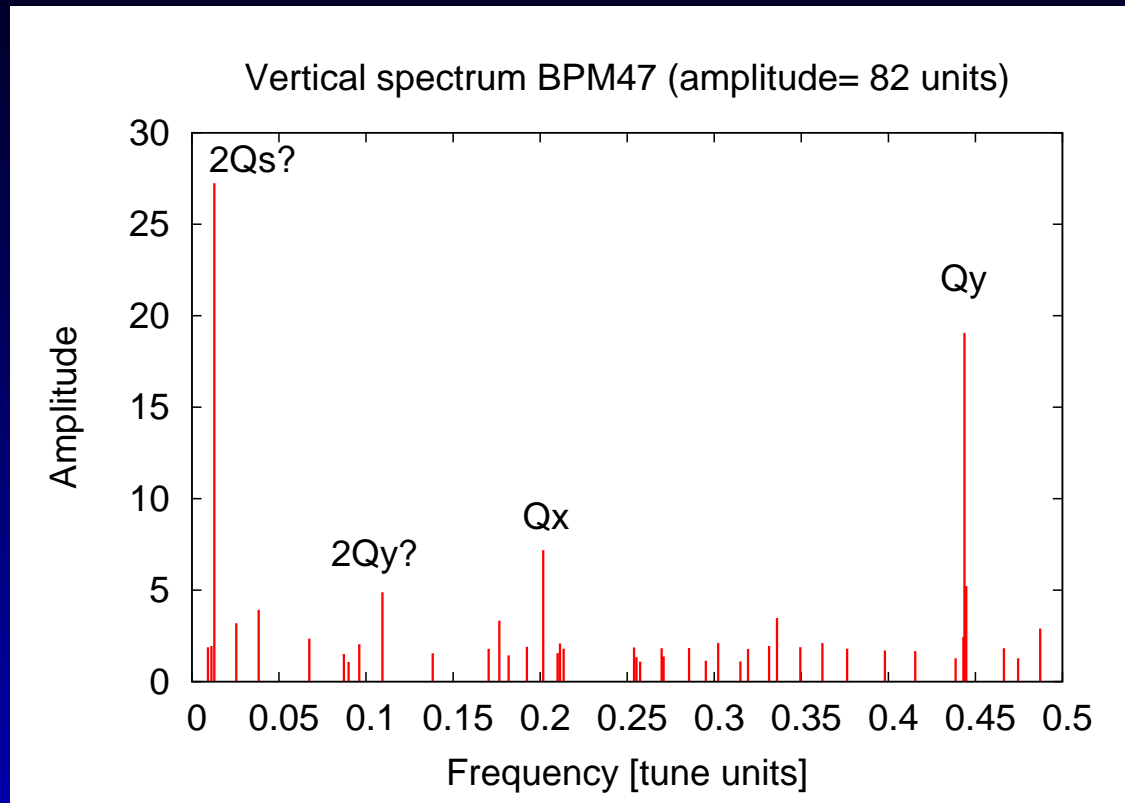
It is proportional to the Hamiltonian term  $h_{jklm}$ :

$$f_{jklm} = \frac{h_{jklm}}{1 - e^{-i2\pi[(j-k)Q_x + (l-m)Q_y]}} .$$

It drives resonances and spectral lines:

Term	Resonance	Type	Line	Plane
$f_{1001}$	(1,-1)	skew	$-Q_y$	H
$f_{3000}$	(3,0)	norm.	$-2Q_x$	H
$f_{0210}$	(2,1)	skew	$2Q_x$	V
$f_{0030}$	(0,3)	skew	$-2Q_y$	V

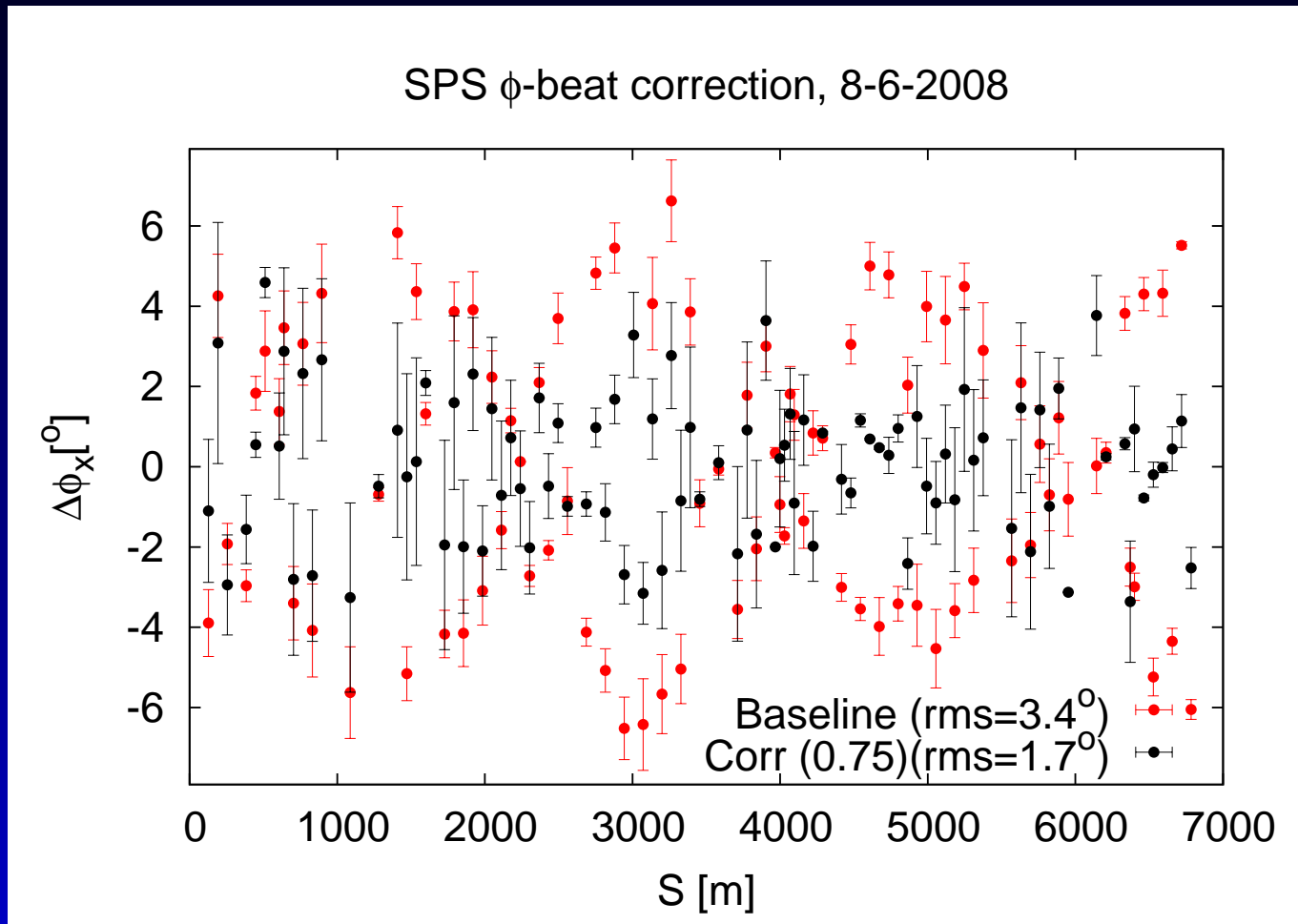
# Spectrum example



$Q_x$  line comes from linear coupling.

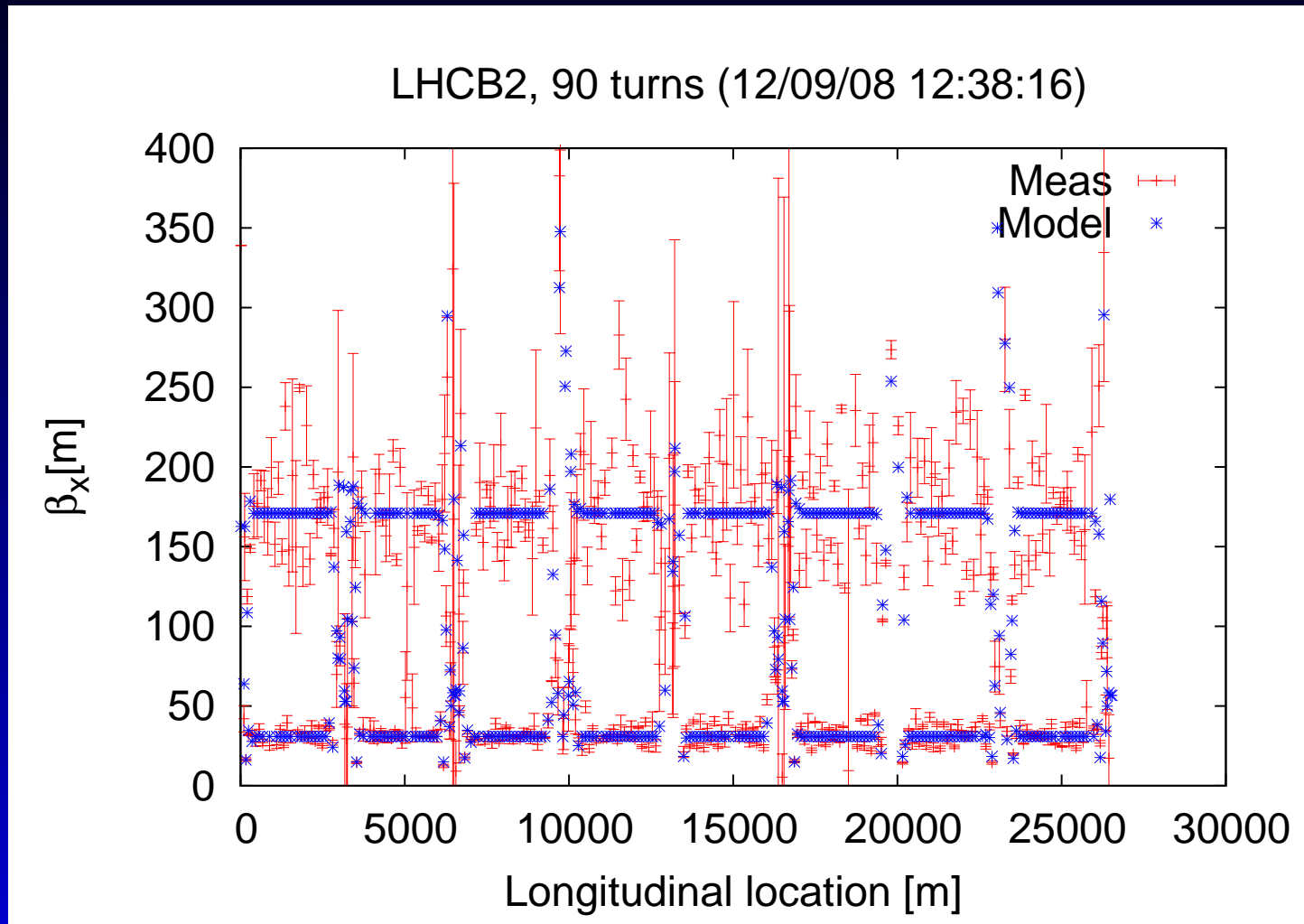
$2Q_y$  line related to resonance (0,3). This resonance is driven by skew sextupoles.

# SPS phase-beating correction



$\phi$ -beating decreased by a factor of two after a single  $\phi$  measurement and correction in the SPS

# LHC (beam2) first 90 turns



$\beta$  obtained from  $\Delta\phi$  between 3 BPMs

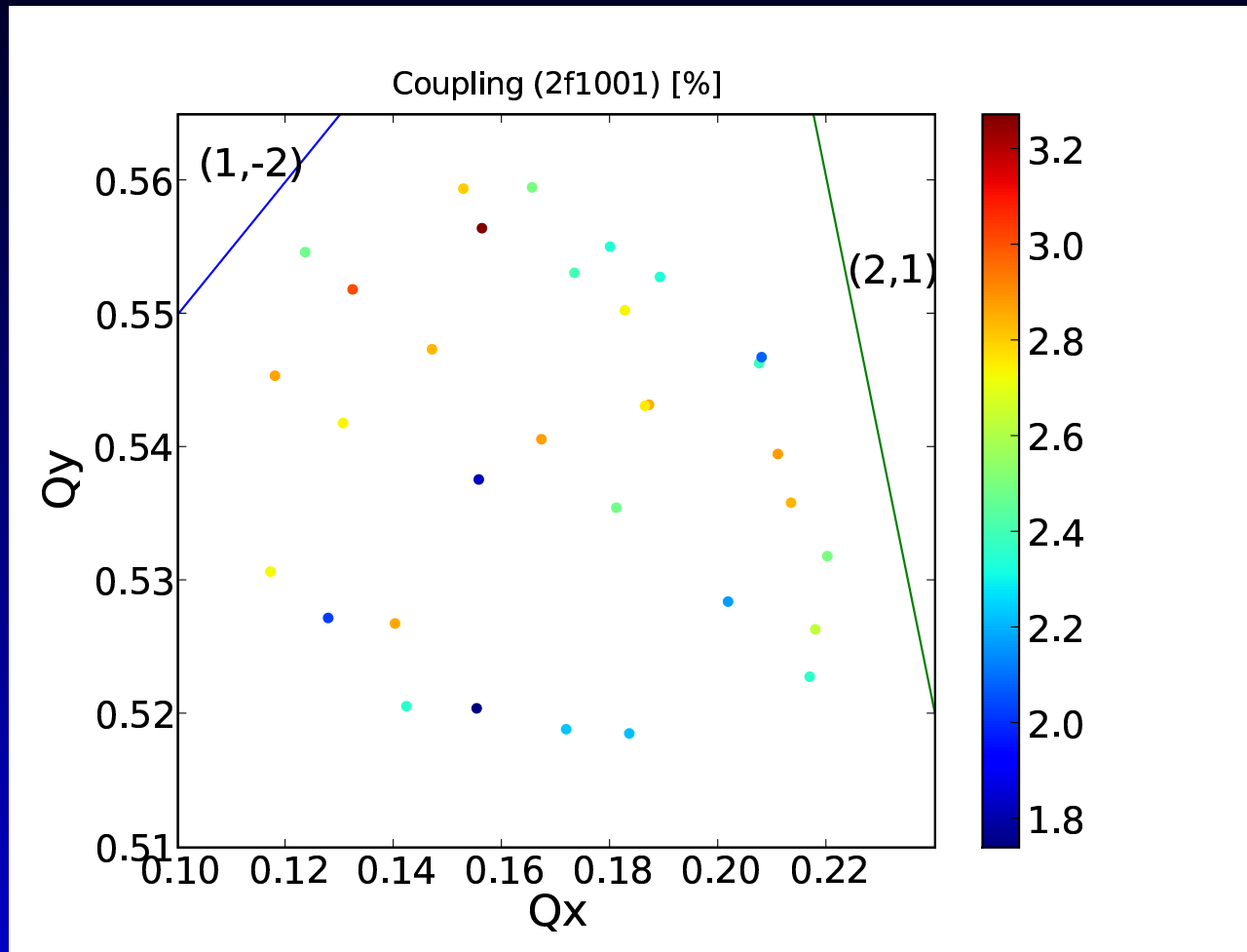


# Coupling measurement: $f_{1001}$

$$2|f_{1001}| = \sqrt{\frac{\text{line}(0, 1)_h \text{line}(1, 0)_v}{\text{line}(1, 0)_h \text{line}(0, 1)_v}}$$

- Calibration independent
- Kick independent
- Model independent

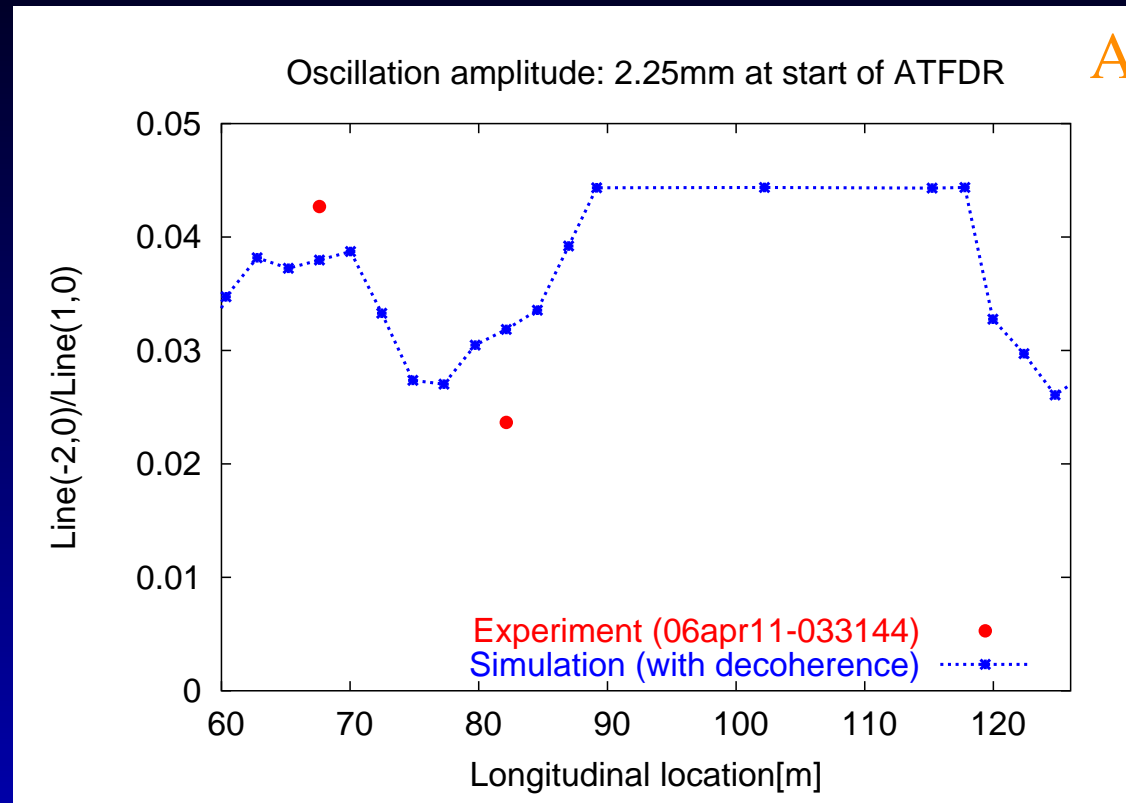
# Measuring $f_{1001}$ in ATF (2007)



Average coupling =  $2.5\% \pm 0.3\%$ , (quite flat).

# Measuring (3,0) Resonance

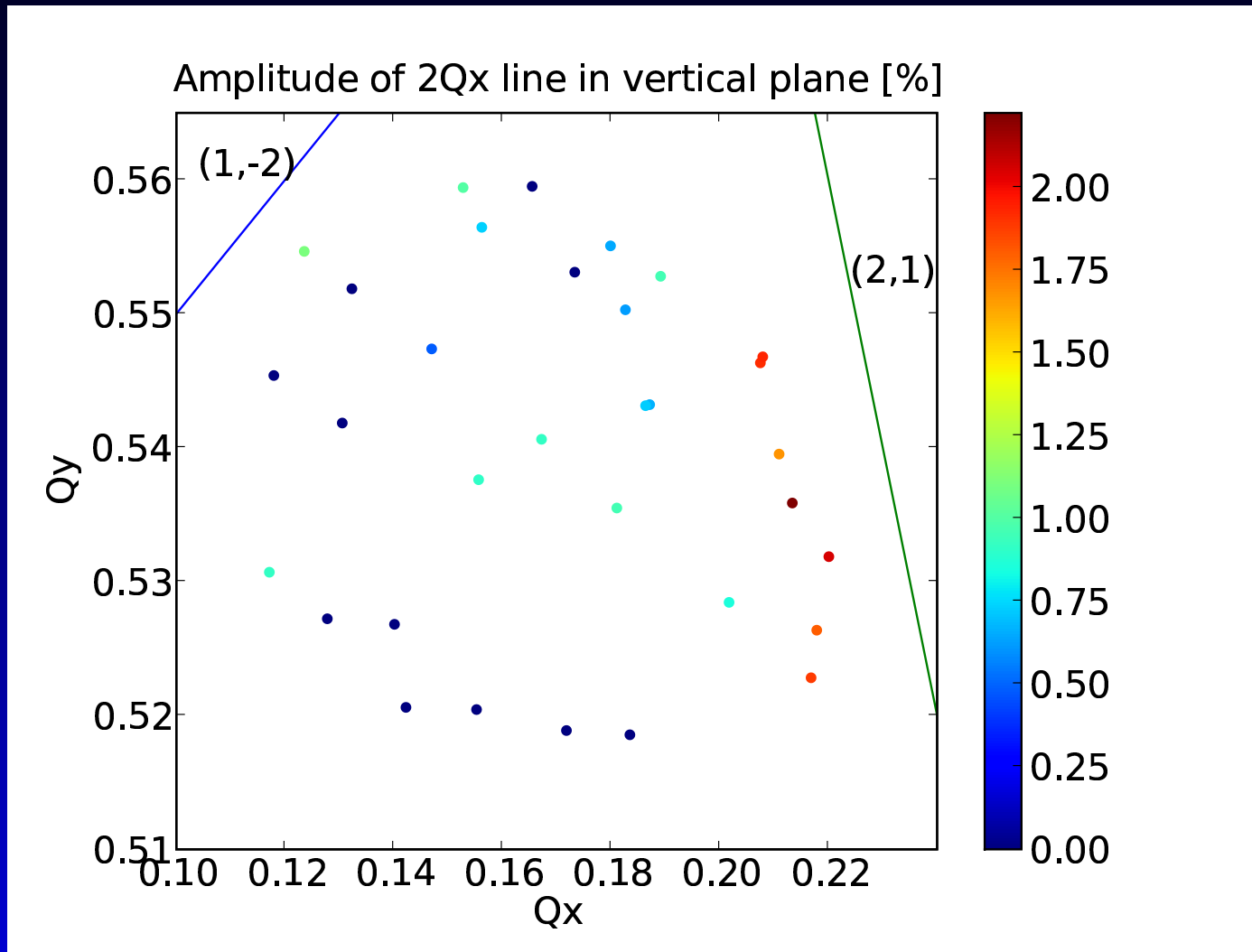
ATF report 06-08



Horizontal resonance (3,0) successfully probed via spectral line  $-2Q_x$ .

However vertical plane totally unprobed.

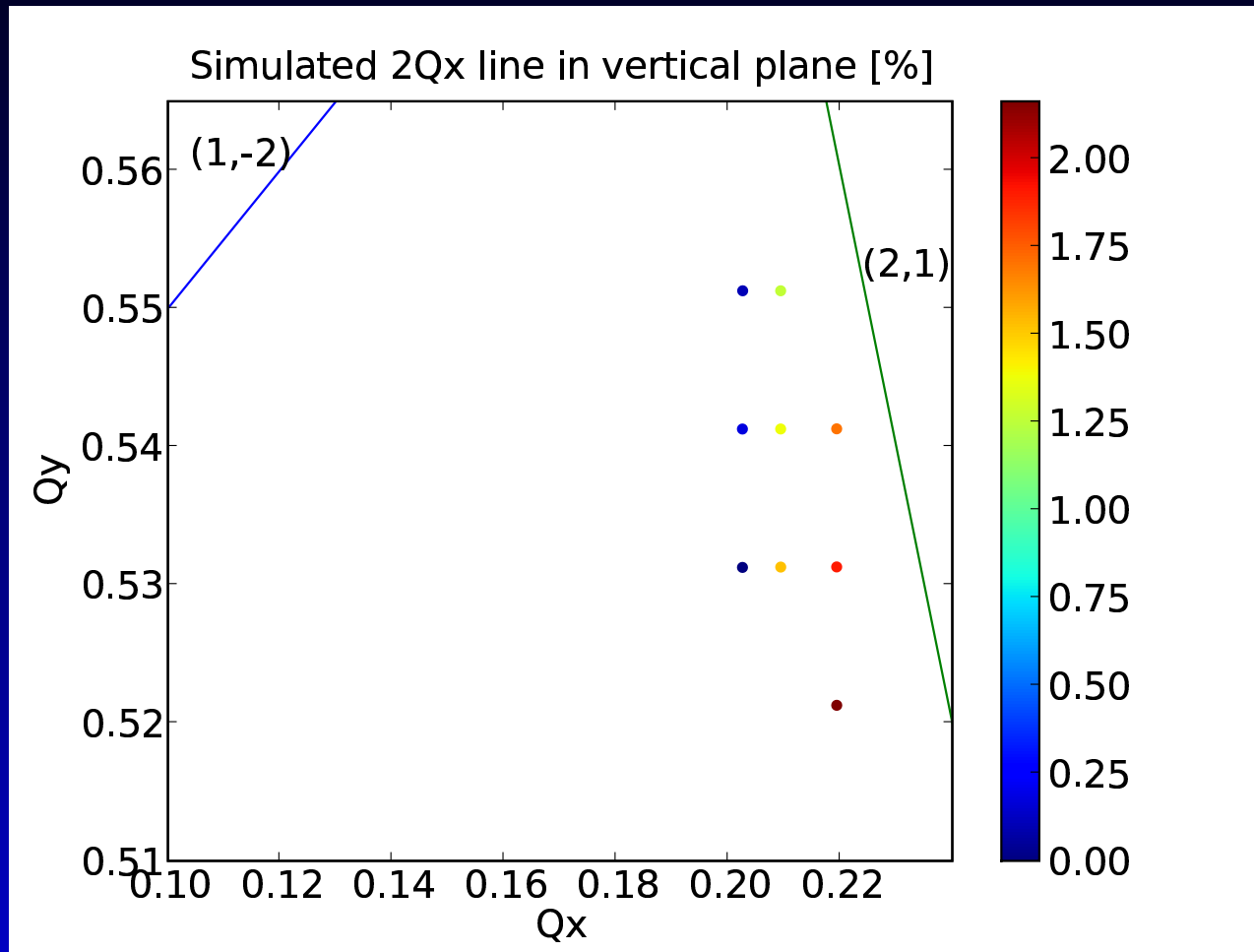
# Measuring skew resonance (2,1)



→ Clear correlation between line and resonance!

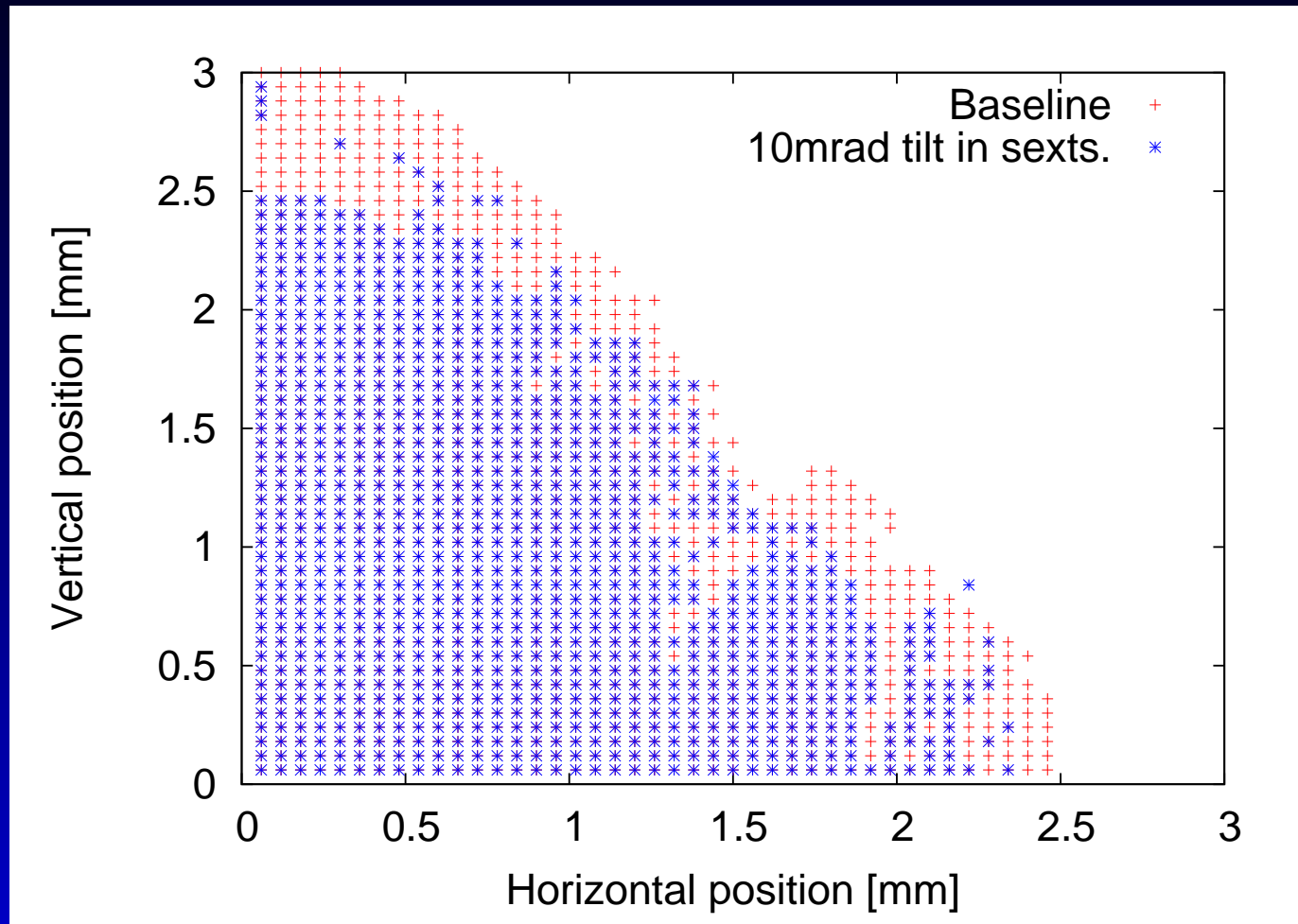
→ And it seems to be large!

# Simulating resonance (2,1)



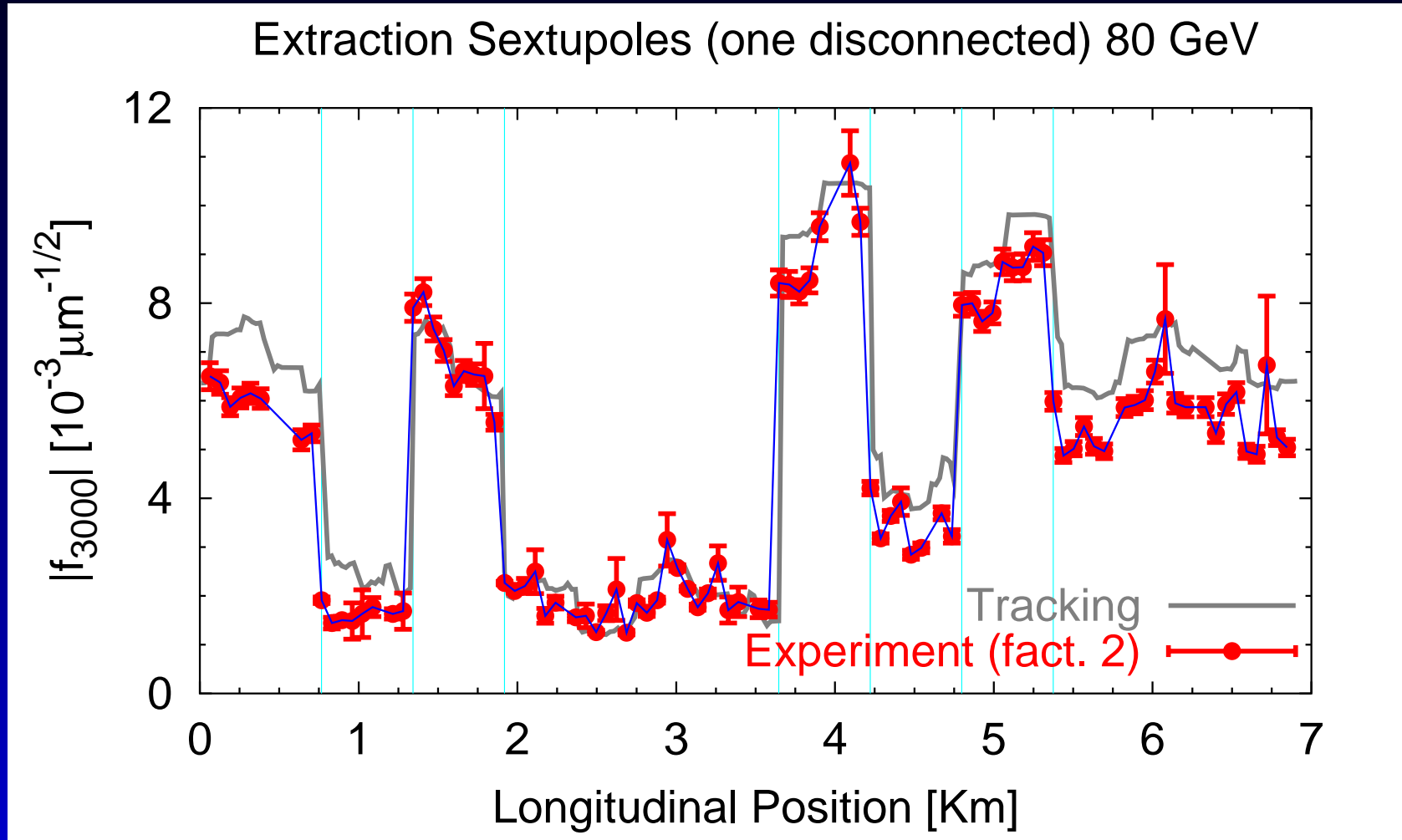
→ 10mrad random tilts at the sextupoles are required to reproduce the measurement.

# Computing DA with sextupole tilts



→ 10mrad tilts certainly have an impact on DA!

# SPS missing sextupole



[http://www.tesisenxarxa.net/TESIS\\_UV/AVAILABLE/TDX-0219104-131907/rogelio.pdf](http://www.tesisenxarxa.net/TESIS_UV/AVAILABLE/TDX-0219104-131907/rogelio.pdf)

# DIAMOND experience

- Measurement of sextupolar spectral lines amplitudes at all BPMs
- Correction based on matrix inversion until amplitudes match simulation
- 10% lifetime increase in DIAMOND !
- EPAC 08 - THPC053



# Conclusions

- Turn-by-turn BPM data provides all linear and non-linear information on the beam dynamics
- Mature and precise algorithms have been developed over years in many accelerators
- First promising tests in ATF2 using only 4 BPMs
- Hope that many turn-by-turn BPMs be made available in ATF2
- DIAMOND proved non-linear correction using these techniques!