Main linac feedback studies

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Introduction

- Trajectory feedback necessary to counteract effect of ground motion and other dynamic imperfections.
- Trajectory feedback systems use BPM readings to determine suitable dipole kicks.
- Emittance preservation with traditional feedback systems rely on correlation between BPM readings and particle distribution at end of linac, but do not optimally exploit the correlation.
- Formalism developed for design of optimized feedback.
- Formalism also useful for efficient simulations and analytical studies of dynamic imperfections.
- Reported in *Phys. Rev. ST Accel. Beams* **11**, *051003 (2008)*

Response matrices

- BPM readings denoted:

$$\mathbf{b} = (b_1, b_2, \ldots, b_q)^T$$

- Macro-particle coordinates at end of linac denoted:

$$\tilde{\mathbf{y}} = (y_1, y_2, \ldots, y_p, y'_1, y'_2, \ldots, y'_p)^T$$

- BPM and coordinate response matrices denoted:
  - $\tilde{\mathbf{R}}_{cq}$ - for response of **c**oordinates to **q**uadrupole displacements.
  - $\tilde{\mathbf{R}}_{Bq}$ - for response of **B**PM readings to **q**uadrupole displacements.
  - $\tilde{\mathbf{R}}_{cs}$ - for response of **c**oordinates to accelerating **s**tructure displacements.
  - Etc. . .
  - First column of $\tilde{\mathbf{R}}_{cq}$ is the change in $\tilde{\mathbf{y}}$ due to a unit displacement of the first quadrupole.

Emittance function and normalized coordinates

- Normalized emittance of macro-particle beam is

$$\epsilon_N = \gamma_r[\langle y^2\rangle\langle y'^2\rangle - \langle yy'\rangle^2]^{1/2}$$

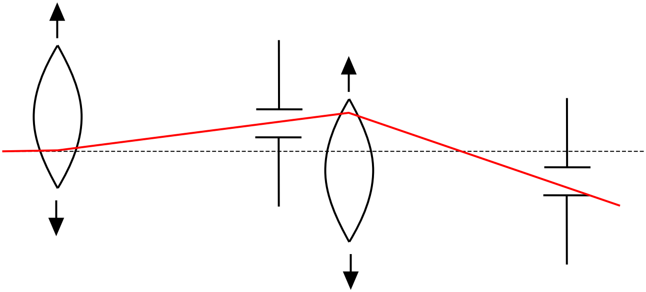
  where $\langle\cdot\rangle$ denotes a weighted average including second moments of macro-particles.
- To second order $\Delta\epsilon_N = \epsilon_N - \epsilon_{N0} \approx \frac{1}{2}\tilde{\mathbf{y}}^T\mathbf{H}\tilde{\mathbf{y}}$.
- Eigenvalue decomposition of symmetric matrix $\mathbf{H} \Rightarrow$

$$\Delta\epsilon_N \approx \tilde{\mathbf{y}}^T\mathbf{M}^T\mathbf{M}\tilde{\mathbf{y}} = \mathbf{y}^T\mathbf{y} = |\mathbf{y}|^2$$

- Coordinate normalization ($\mathbf{y} = \mathbf{M}\tilde{\mathbf{y}}$) simplifies emittance function and analytical treatment of dynamic imperfections and feedback performance.

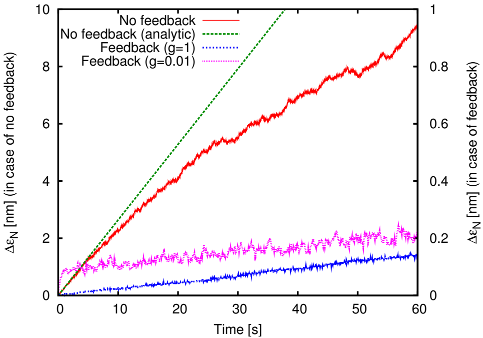
Traditional trajectory feedback

- Steers beam to the centre of selected BPMs using dipole correctors.
- Often a least-square solution is necessary.



Traditional trajectory feedback

- Example: 40 communicating feedbacks each consisting of 2 correctors and 3 BPMs. Feedbacks equally spaced along the linac. ATL ground motion, $A = 0.5 \cdot 10^{-6}\ \mu\text{m}^2/\text{s}/\text{m}$.



- If only a fraction (gain g) of the calculated corrector change is applied each pulse, ground motion will cause more emittance growth.
  - Low gain may however be required to reduce effect of, for example, finite BPM resolution.

Analytical treatment of ground motion

- Emittance growth caused by ground motion after P pulses without feedback:

$$\langle \Delta\epsilon_N \rangle \approx P\sigma_{gm}^2 ||\mathbf{R}_{cg}||_F^2$$

  where $\sigma_{gm}^2 = A \times \Delta T \times \Delta L \approx 7.4 \cdot 10^{-3}$ nm^2.
- The matrix $\mathbf{R}_{cg}$ describes the coordinate response to relative **g**irder displacements (see article for details).
- For the 2005 CLIC parameters, emittance growth rate $\epsilon_g = \frac{d\langle \Delta\epsilon_N \rangle}{dP} \approx 0.26$ nm/s.

Analytical treatment of ground motion and feedback

- Corrector change of traditional trajectory feedback determined from $\mathbf{b} + \mathbf{R}_{Bd}\Delta\mathbf{d} = 0 \quad \Rightarrow \quad \Delta\mathbf{d} = -[\mathbf{R}_{Bd}^T\mathbf{R}_{Bd}]^{-1}\mathbf{R}_{Bd}^T\mathbf{b} = \mathbf{R}_{dB}\mathbf{b}$.
- If gain is taken into account: $\Delta\mathbf{d} = g\mathbf{R}_{dB}\mathbf{b}$.
  - $\Delta\mathbf{y} = g\mathbf{R}_{cd}\mathbf{R}_{dB}\mathbf{b} = g\mathbf{R}_{cB}\mathbf{b}$.
- Matrix algebra and geometric sums $\Rightarrow$

$$\langle\Delta\epsilon_N\rangle \approx \sigma_{gm}^2[Pe_{gm,0} - \frac{2}{g}e_{gm,1} + \frac{1}{g(2-g)}e_{gm,2}]$$

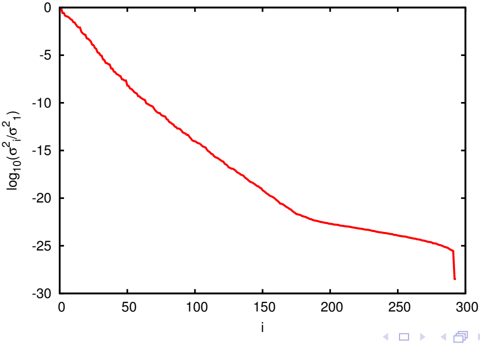
  for large P.
- Scaling factors $e_{gm,i}$ may be determined from response matrices, $e_{gm,0} = ||\mathbf{R}_{cg} + \mathbf{R}_{cB}\mathbf{R}_{Bg}||_F^2$.
- For a traditional feedback system with 80 correctors and 1324 BPMs, emittance growth rate is $\epsilon_g = \frac{d\langle\Delta\epsilon_N\rangle}{dP} \approx 2.5 \cdot 10^{-3}$ nm/min.

Optimized feedback system

- By minimizing $\epsilon_g \approx \sigma^2_{gm} ||\mathbf{R}_{cg} + \mathbf{R}_{cB}\mathbf{R}_{Bg}||^2_F$, an optimal feedback response $\mathbf{R}_{cB,opt}$ may be determined.
  - $\Rightarrow$ $\epsilon_g \approx 1.75 \cdot 10^{-4}$ nm/min. $\sim$ 14 times lower than for traditional feedback.
- Matrix $\mathbf{R}_{cB,opt}$ offers the best achievable prediction of the macro-particle coordinates for certain BPM readings.
- Determination of $\mathbf{R}_{cB,opt}$ does not mean that a new feedback system is constructed!

Optimized feedback system

- By studying the squared norms of the columns of $\mathbf{R}_{cB,opt}$ it is seen that certain BPM readings predict large emittance growth (remember $\Delta\epsilon_N \approx |\mathbf{y}|^2$).
- SVD of $\mathbf{R}_{cB,opt}$ reveals that a few linear combinations (patterns) of BPMs contain all information needed to predict the corresponding coordinate changes.
- Now BPM space is transformed and only most important patterns used for determination of correction.

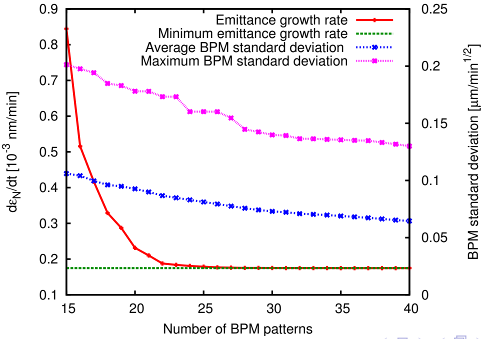


Optimized feedback system

- Construction of optimized feedbacks can be achieved by solving:

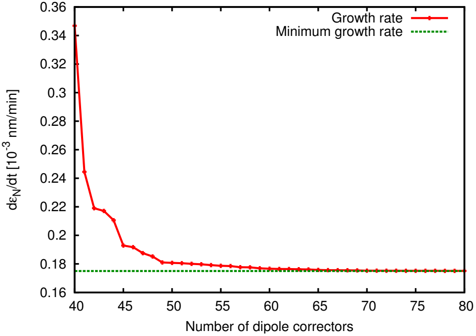
$$\begin{pmatrix} \mathbf{R}_{cd} \\ \mathbf{R}_{B'd} \end{pmatrix} \mathbf{X} = \begin{pmatrix} \mathbf{R}_{cB',opt} \\ -\mathbf{I} \end{pmatrix}$$

- Matrix **X** denotes the optimal corrector changes for certain BPM pattern readings.
- No more than the 30 most important are required for efficient emittance preservation.



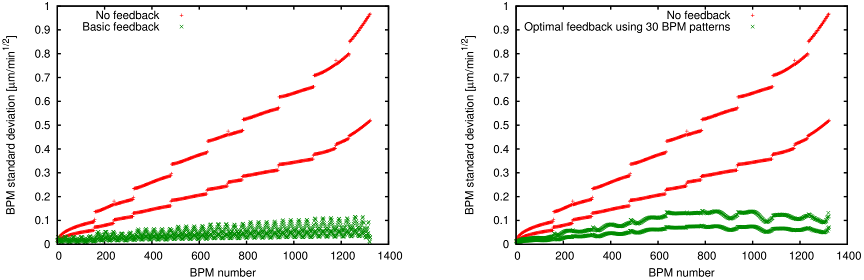
Optimized feedback system

- A limited number of correctors may be used to contruct the 30 vectors describing the optimal feedback response (as few as 60 sufficient).



Optimized feedback system

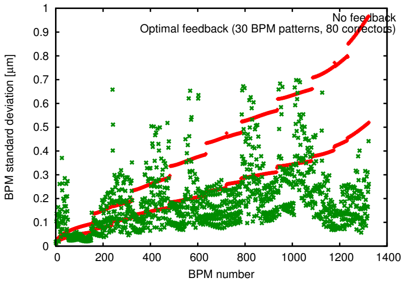
- However, no control of the trajectory components along less important BPM patterns ($\Rightarrow$ increased trajectory deviations).
- Information from 30 most important BPM patterns is in theory sufficient to control trajectory equally well as with traditional feedback system using 1324 BPMs and 80 correctors.



Optimized feedback system

- Improved trajectory control requires that the 30 most important BPM patterns are used to predict the less important. A modified equation gives a better solution:

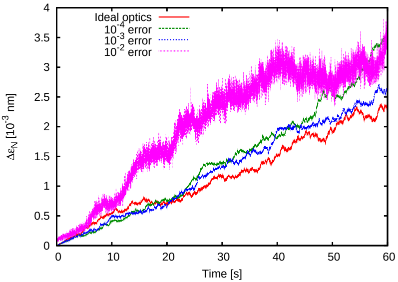
$$\begin{pmatrix} \mathbf{R}_{cd} \\ \mathbf{R}_{B'd} \\ \mathbf{R}_{B''d} \end{pmatrix} \mathbf{X} = \begin{pmatrix} \mathbf{R}_{cB',opt} \\ -\mathbf{I} \\ \mathbf{R}_{B''B',opt} \end{pmatrix}$$

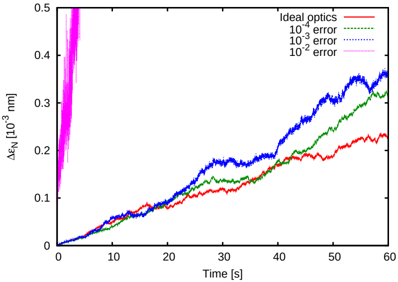


- Emittance growth rate nearly unchanged: $\epsilon_g \approx 1.8 \cdot 10^{-4}$ nm/min.
- After 14 hours: $\Delta\epsilon_N \approx 0.15$ nm ,20 μm trajectory deviations.

Imperfect system knowledge

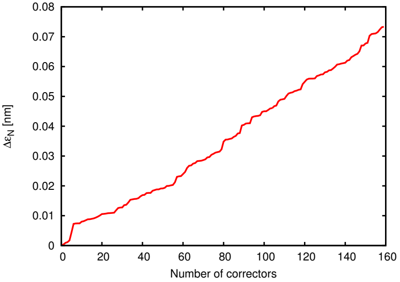
- Design of feedback system relies on good knowledge of machine optics, in particular for optimized system.
- Basic and optimized trajectory feedbacks compared on machines with optics errors for quadrupole strength (0.01%, 0.1%, and 1%).





Corrector imperfections

- Step size imperfections limit the number of correctors.
- For a step size error of 10 nm (correctors simulated as quad displacements with a minimum step of 10 nm.), the use of 1324 quads $\Rightarrow \Delta\epsilon_N \approx 0.72$ nm.

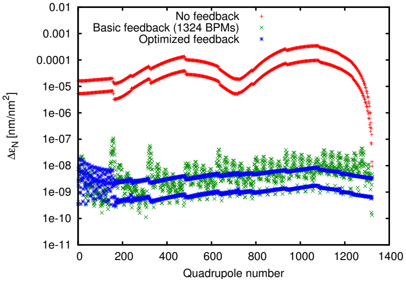


- Calibration error and corrector strength noise (indep. from pulse to pulse) cause little $\Delta\epsilon_N$ (emittance stated in [10^{-3} nm]).

Feedback setup	Constant error $\sigma = 1\%$	$\sigma = 3\%$	$\sigma = 5\%$	Noise $\sigma = 1\%$
80 correctors	0.29	2.4	6.7	1.2
60 correctors	0.53	4.2	11	1.9
80 "weak" correctors	0.22	3.0	9.4	0.82

Further imperfections

- A slowly drifting quadrupole is very severe without feedback
  - For most harmful quad, 21 nm displacement $\Rightarrow$ $\Delta\epsilon_N \approx 0.15$ nm.
- Both feedback designs efficiently reduce sensitivity.



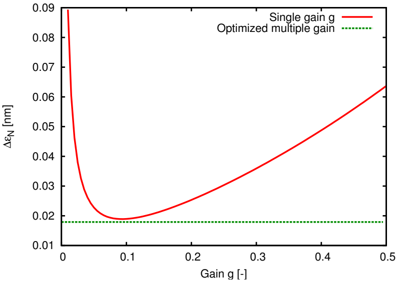
- Sensitivity to drifts of the injected beam is also significantly reduced by feedbacks (in particular optimized feedbacks).

Feedback setup	$\Delta\epsilon_N/\Delta d^2$ [nm/μm^2]
No feedback	5.13
Basic feedback (1324 BPMs)	2.20×10^{-2}
Optimized feedback	5.09×10^{-4}

Gain optimization

- Finite resolution of feedback BPMs cause time-independent emittance growth. Similarly the feedback system indirectly cause emittance growth due to various jitter effects. The effects are gain-dependent and favour a low gain. At the same time low gain means slow feedback and more emittance growth due to ground motion.

- Gain was optimized to minimize $\Delta\epsilon_N$. Optimal gain $g \approx 0.09$.
- Multiple gain optimization tested with only little improvement.
  - Most general approach with individual gain for each BPM pattern.



Remarks on simulations

- Matrix formalism not only facilitates design of improved feedbacks, it also allows analytical studies of a number of imperfections (see article).
- Furthermore, response matrices may be used to perform fast simulations of dynamic imperfections, through simple matrix operations.
- In addition, since the state of the machine for this purpose may be described by the BPM readings and the coordinates of the macro-particles at the end of the linac, the response matrices may be reduced significantly in size.
    - The 294×21914 response matrix $\mathbf{R}_{cs}$ may be reduced to a 294×294 matrix, by using SVD to remove intrinsic dependencies.

Summary

- Possibility of improving main linac feedback design investigated.
- Formalism developed to facilitate feedback studies.
- Optimized feedback system with one order of magnitude lower emittance growth rate $\frac{d\langle\Delta\epsilon_N\rangle}{dP}$ than for traditional feedback system.
- Optimized feedbacks more sensitive to faulty system knowledge, but does not seem to be a problem.
- Corrector imperfections cause little emittance growth.
- Formalism also useful for efficient simulations and analytical studies of dynamical imperfections.
- All details in *Phys. Rev. ST Accel. Beams* **11**, *051003 (2008)*